ADVANCED TOPICS IN STRING THEORY

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TIME: 12.30 - 2.30 PM, THURSDAYS

PLACE: 269 MERCER ST. ROOM 602

OUTLINE: INTRODUCTION TO D AND NS BRANE TECHNOLOGY, SUMMARY OF STRING THEORY BASICS

- "STRINGY" DUALITIES
- APPLICATIONS OF STRING DUALITIES TO NON PERTURBATIVE FIELD THEORY AND BLACK HOLES
- INTRODUCTION TO ADS/CFT

- "BRANE WORLD", RANDALL-SUNDRUM COMPACTIFICATIONS AND THEIR EXTENSIONS

PLANNED: LECTURES BY PROF. B. GREENE ON MIRROR SYMMETRY AND THE QUANTUM GEOMETRY OF CY SPACES

- LECTURE BY PROF. G. DVALI ON THE PHENOMENOLOGICAL ASPECTS OF "BRANE WORLD"
- LECTURES BY VOLUNTEERS (!?) ON SELECTED TOPICS

EXAM: TAKE HOME +

<u>SEE ABOVE</u>

LECTURE 1: STRINGS & D BRANES WITHOUT STRINGS (EFFECTIVE FIELD THEORY)

REF. G.T. HOROWITZ AND A. STROMINGER NUCL. PHYS. B360 (1991) 197

$\mathcal{N}=2$ SUPERSYMMETRY IN 10-D

$Q_\alpha \leftarrow$ MAJORANA (REAL) SPINOR

32 REAL COMPONENTS

$$\Gamma^{11} = \Gamma^0 \Gamma^1 \cdots \Gamma^9$$

$$Q_\alpha = Q_\alpha^+ + Q_\alpha^-$$

$$\Gamma^{11} Q_\alpha^+ = Q_\alpha^+ \qquad \Gamma^{11} Q_\alpha^- = -Q_\alpha^-$$

IIA, NON-CHIRAL SUSY

OTHER POSSIBILITY: II B

$Q_\alpha^i \qquad i=1,2 \qquad \Gamma^{11} Q_\alpha^i = Q_\alpha^i$

2 MAJORANA-WEYL (CHIRAL) SPINORS

16 + 16 = 32 REAL COMPONENTS

$\mathcal{N}=2$ SUSY (II A OR II B) UNIQUELY FIXES FIELD CONTENT AND LOW ENERGY ACTION (UP TO 2 DERIVATIVES) OF SUPERGRAVITY.

II A FIELDS:

G_{MN}	GRAVITON
B_{MN}	ANTISYMMETRIC TENSOR
ϕ	SCALAR
A_{MNO}	ANTISYMMETRIC
A_M	VECTOR

FERMIONIC PARTNERS:

ψ_M MAJORANA VECTOR-SPINOR

χ MAJORANA SPINOR

COUNTING DEGREES OF FREEDOM:

$$\underbrace{\left(\frac{8\times 9}{2}-1\right)}_{G_{MN}} + \underbrace{\frac{8\cdot 7}{2}}_{B_{MN}} + \underbrace{1}_{\phi} + \underbrace{\frac{8\cdot 7\cdot 6}{1\cdot 2\cdot 3}}_{A_{MNO}} + \underbrace{8}_{A_M}$$

$$= 128$$

$$\underbrace{\frac{8\cdot 32}{2}}_{\psi_M + \chi} = 128$$

II B

G_{MN} , B_{MN} , ϕ

+

ψ SCALAR

C_{MN} ANTISYMMETRIC

+

C_{MNPQ} SELF-DUAL 4-FORM

$$F_{MNPQR} = \partial_{[M} C_{NPQR]}$$

SELF-DUALITY : $F_{MNPQR} = \frac{1}{5!} \epsilon_{MNPQRSTUVW} F^{STUVW}$

FERMIONIC PARTNERS :

ψ_M^i , $\Gamma^{11} \psi_M^i = \psi_M^i$ MAJORANA-WEYL VECTOR-SPINORS

χ^i $\Gamma^{11} \chi^i = -\Gamma^{11} \chi^i$ MAJORANA-WEYL

HOMEWORK : SHOW THAT

BOSONIC D.O.F. = FERMIONIC D.O.F.

II A AND II B BOTH CONTAIN

G_{MN}, ϕ, F $\quad$ F = $D-2$ FORM

THE ACTION OF II A AND II B, RESTRICTED TO G, ϕ, F CAN BE WRITTEN AS

$$S = \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[R + 4(\nabla\phi)^2 \right] + - \frac{2 e^{2\alpha\phi}}{(D-2)!} F^2 \right\}$$

$$F = dC$$

LOOK FOR $(10-D)$- EXTENDED SOLITONS

ANSATZ :

$$ds^2 = e^A \, d\hat{s}^2 + e^B \, dx_i \, dx^i$$

D- DIM. METRIC

(TIME + D-1 SPACE COORD.)

ROTATIONAL INVARIANCE IN (D-1):

$$d\hat{s}^2 = -\lambda^2 dt^2 + \lambda^{-2} d\hat{r}^2 + $$
$$+ R^2 d\Omega^2_{D-2}$$

BIANCHI + E.O.M. OF F SOLVED BY

$F = Q\,\epsilon_{D-2}$ ϵ_{D-2} = VOLUME ELEMENT OF UNIT D-SPHERE

Q IS THE CHARGE OF OUR SOLUTION.

WITH THIS ANSATZ SOLUTION IS

$$ds^2 = -\left[1 - \left(\frac{r_+}{r}\right)^{D-3}\right]\left[1 - \left(\frac{r_-}{r}\right)^{D-3}\right]^{\gamma_r - 1} dt^2$$
$$+ \left[1 - \left(\frac{r_+}{r}\right)^{D-3}\right]^{-1}\left[1 - r_-/r\right]^{\gamma_r} dr^2$$
$$+ r^2\left[1 - \left(r_-/r\right)^{D-3}\right]^{\gamma_r + 1} d\Omega^2_{D-2} + \left[1 - \left(\frac{r_-}{r}\right)^{D-3}\right]^{\gamma_x} dx^i dx_i$$

$$e^{-2\phi} = \left[1 - \left(\frac{r_-}{r}\right)^{D-3}\right]^{\gamma_\phi}$$

$$\gamma_r = \delta(\alpha - 1) - (D-5)/(D-3)$$

$$\gamma_x = \delta(\alpha + 1)$$

$$\gamma_\phi = -\delta(4\alpha + 7 - D)$$

$$\delta = \left[2\alpha^2 + (7-D)\alpha + 2\right]^{-1}$$

EXTREMAL SOLUTION : $r_+ = r_-$ $\quad M = Q$

$$Q = \left[\delta(D-3)^2 (r_+ r_-)^{D-3}/2\right]^{1/2}$$

FOR EXTREMAL SOLUTION $\lambda = 1$,

SYMMETRY BECOMES $SO(D-1) \times \mathcal{P}(11-D)$

IN GENERAL $M \geq Q$.

P-BRANES IN IIA, IIB SUGRA

1) NS 5 $\quad F = dB$

$\gamma_r = -1, \quad \gamma_x = 0, \quad \gamma_\phi = 1$

$$ds^2 = -\frac{1 - r_+^2/r^2}{1 - r_-^2/r^2}\, dt^2 + \frac{dr^2}{(1 - r_+^2/r^2)(1 - r_-^2/r^2)}$$

$$+ r^2 d\Omega_3^2 + dx^i dx_i \qquad i = 5, \ldots 9$$

$$e^{-2\phi} = 1 - \frac{r_-^2}{r^2} \qquad H = Q\, \epsilon_3$$

$r_+ = r_-$ THIS SOLUTION IS NONSINGULAR

IT IS A CLASSICAL SOLITON

$$ds^2 = -dt^2 + \left(1 + \frac{r_-^2}{y^2}\right)\left[dy^2 + y^2 d\Omega_3^2\right]$$

$$+ dx_i dx^i$$

$$e^{2\phi} = 1 + r_-^2/y^2$$

II A $\quad G = dA = 2$ FORM

$F = dA^{(3)} = 4$ FORM

- 2 FORM NONZERO $\quad D = 4, \ \gamma_r = \frac{1}{2}, \gamma_x = \frac{1}{2}$

$\gamma_\phi = -\frac{3}{2}$

SOLUTION IS A SIX-BRANE

- 4 FORM NONZERO $\quad D = 6, \ \alpha = 0, \ \gamma_5 = -5/6$

$\gamma_x = \frac{1}{2} \quad \gamma_\phi = -\frac{1}{2}$

THE SOLUTION IS A 4-BRANE

DUALIZE THE VARIOUS FORMS TO GET MORE SOLUTIONS

P-FORM

$$K = e^{2\alpha\phi} * F$$

HODGE DUAL

10-P FORM

DUALITY EXCHANGES BIANCHI $\leftrightarrow$ E.O.M.

1) DUALIZE H K = 7-FORM

$\Rightarrow D=9 \quad \alpha=1, \quad \gamma_r=-\frac{2}{3}, \gamma_x=1, \gamma_\phi=1$

SPACE TRANSVERSE TO SOLITON HAS DIMENSION $7+1=8$, SOLITON IS A BLACK STRING. ($p=1$ BRANE)

2) DUALIZE G : G^D IS 8-FORM "SOLITON" IS A 9-D BLACK HOLE

3) DUALIZE F F^D IS 6-FORM SOLUTION IS A 2-BRANE

SUMMARIZING: FROM G, F, G^D, F^D 6, 4, 2, 0 BRANES

FROM H & H^D 5 & 1 BRANE

ALL EXCEPT 5-BRANE ARE SINGULAR IN THE EXTREMAL LIMIT $r_+ = r_-$

II B 3-FORM $G = dC^{(2)}$

SOLUTIONS FROM G & G^D ARE

5 BRANE AND 1-BRANE

$F^+ = *F^+$ F = 5-FORM

3-BRANE IS SELF-DUAL. ϕ = CONST.

SUMMARIZING:

FROM G, G^D 1 + 5 BRANES

FROM F^+ 3 BRANE

FROM H 1+5 BRANE

NOTICE: H UNIVERSAL (IS IN IIA & IIB)

OTHER FORMS ARE NOT.

MOREOVER, H APPEARS IN ACTION

AS $e^{-2\phi} H^2$

OTHER FORMS AS F^2 NO $e^{-2\phi}$

EXTENDED, EXTREMAL SOLUTIONS WITH NONZERO FORMS COUPLING AS F^2 IN ACTION ARE CALLED D-BRANES

F'S ARE RAMOND-RAMOND FIELDS

IN IIA   0, 2, 4, 6   D-BRANES

IN IIB   1, 3, 5, 7   D-BRANES

WHERE FROM?

FIND OUT

SOLUTIONS WITH NONZERO H ARE EITHER STRINGS OR 5-BRANES.

5-BRANE SOLUTIONS ARE REGULAR SOLITONS (NS5)

STRINGS MAY BE IDENTIFIED WITH THE <u>FUNDAMENTAL STRING</u> OF STRING THEORY !?

SINGULAR SOLUTIONS NEED EXTERNAL SOURCES.

FOR NS-1 WE CONJECTURE THE FUNDAMENTAL STRING.

REASONS: NS-1 COUPLES TO H

IT IS "ELECTRICALLY CHARGED" UNDER B $\int_{S_7} *H \neq 0$

SOLUTION HAS ZERO MODES $X^i(\sigma,\tau)$

$i=1,..8$ TRANSVERSE COORDINATES

σ,τ = LONGITUDINAL COORDINATES

$X^i(\sigma,\tau)$ DESCRIBE WAVES TRAVELING ON NS-1 STRING. SAME D.O.F. AS FUNDAMENTAL STRING.

WHAT ABOUT D-BRANES ? AND NS-5 ?

QUESTION:

CAN WE PROMOTE THE "SOLITONS" WE FOUND IN THIS LECTURE TO STRING THEORY CONFIGURATIONS ?

NOTICE:

WE MUST DO THAT SINCE SUPERGRAVITY IS NON-RENORMALIZABLE AND THUS INCOMPLETE (MODIFIED AT HIGH CURVATURE)

REVIEW OF STRING THEORY

ONE-PARTICLE STATE IN FIELD THEORY:

$$|X^\mu, a\rangle$$

IN STRINGS $\quad |X^\mu(\sigma), a\rangle \quad 0 \leq \sigma \leq \pi$

– FREE BOSONIC STRING

ACTION: $\quad S = -\frac{1}{(4\pi\alpha')}\int_\Sigma d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$

E.O.M. $\quad \Box X^\mu = 0$

$$T_{\alpha\beta} = 0$$

INVARIANT UNDER $\quad \xi^\alpha \to f^\alpha(\xi)$

AND $\quad h_{\alpha\beta} \to e^{2\omega} h_{\alpha\beta}$

USE THESE SYMMETRIES TO SET $h_{\alpha\beta} = \eta_{\alpha\beta}$

RESIDUAL INVARIANCE:

$\xi^\alpha \to \xi^\alpha + \epsilon^\alpha \quad$ SUCH THAT

$$\partial^\alpha \epsilon^\beta + \partial^\beta \epsilon^\alpha = \Lambda(\xi)\eta^{\alpha\beta}$$

IN CONFORMAL GAUGE

$$\Box X^\mu = (\partial_\sigma^2 - \partial_\tau^2) X^\mu = 0$$

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} - \frac{1}{2} \eta_{\alpha\beta} \partial_\gamma X^\mu \partial^\gamma X^\nu \eta_{\mu\nu}$$

$$= 0$$

CLOSED STRING BOUNDARY CONDITIONS

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$$

SOLUTION

$$X^\mu(\tau,\sigma) = q^\mu + 2\alpha' p^\mu \tau + \sum_{n \neq 0} \Bigg(\underbrace{\frac{\alpha_n^\mu}{n} e^{-2in(\tau-\sigma)}}_{\text{RIGHT MOVERS}} + \underbrace{\frac{\tilde{\alpha}_n^\mu}{n} e^{-in(\tau+\sigma)}}_{\text{LEFT MOVERS}} \Bigg)$$

DEFINE $\xi_+ = \tau + \sigma$, $\xi_- = \tau - \sigma$

$T_{\alpha\beta}$ IS SYMMETRIC & TRACELESS

(2 INDEPENDENT COMPONENTS)

$$T_{++} = \partial_+ X \partial_+ X \quad , \quad T_{--} = \partial_- X \partial_- X$$

EXPANDING IN MODES :

$$T_{++} \sim \sum_{n \in \mathbb{Z}} \tilde{L}_n e^{-2in(\tau+\sigma)}$$

$$T_{--} \sim \sum_{n \in \mathbb{Z}} L_n e^{-2in(\tau-\sigma)}$$

$$\tilde{L}_n = \frac{1}{2} \sum_m \tilde{\alpha}_{-m} \tilde{\alpha}_{n+m} \qquad \tilde{\alpha}_0^\mu \equiv \frac{\alpha'}{\sqrt{2}} p^\mu$$

$$L_n = \frac{1}{2} \sum_m \alpha_{-m} \alpha_{n+m} \qquad \alpha_0 \equiv \frac{\alpha'}{\sqrt{2}} p^\mu$$

CANONICAL QUANTIZATION

$$[\alpha^\mu_m , \alpha^\nu_n] = m \delta_{m+n,0} \eta^{\mu\nu}$$

$$[\tilde{\alpha}^\mu_m , \tilde{\alpha}^\nu_n] = m \delta_{m+n,0} \eta^{\mu\nu}$$

$$[\tilde{\alpha}^\mu_m , \alpha^\nu_n] = 0$$

p^μ , q^μ = MOMENTUM & COORDINATE

L_0, $\tilde{L}_0$ NEED NORMAL ORDERING

<u>DEFINE</u>

$$L_0 = \frac{\alpha'}{4} p^\mu p_\mu + \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n$$

$$\tilde{L}_0 = \frac{\alpha'}{4} p^\mu p_\mu + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_n$$

ALGEBRA OF L_m'S (VIRASORO ALGEBRA)

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{d}{12} m(m^2-1) \delta_{m+n,0}$$

SAME FOR $\tilde{L}_m$

FOCK VACUUM:

$$\alpha_m |0\rangle = 0 \qquad m > 0$$

$$\tilde{\alpha}_m |\tilde{0}\rangle = 0 \qquad m > 0$$

$$p^\mu |\bar{p}\rangle = \bar{p}^\mu |\bar{p}\rangle$$

COMBINE: $\quad |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$

PHYSICAL STATES MUST OBEY $T_{\alpha\beta} \approx 0$

I.E.

$$L_m |\text{PHYS}\rangle = \tilde{L}_m |\text{PHYS}\rangle = 0 \qquad m > 0$$

$$(L_0 - 1)|\text{PHYS}\rangle = (\tilde{L}_0 - 1)|\text{PHYS}\rangle = 0$$

NORMAL ORDERING

FROM DEFINITION OF L_0, $\tilde{L}_0$ WE HAVE

$$M^2 = -p^2 = \frac{2}{\alpha'}\left[\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n + \tilde{\alpha}_{-n}\alpha_n - 2\right]$$

LOWEST-MASS PHYSICAL STATE:

$$|T\rangle = |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$$

$$L_m|T\rangle = \tilde{L}_n|T\rangle = 0 \qquad n > 0$$

$$(L_0 - 1)\,|T\rangle = \left(\frac{\alpha'}{4}\bar{p}^2 - 1\right)|T\rangle$$

$$\Rightarrow M^2 = -4/\alpha' \quad < 0 \qquad \underline{\text{TACHION}}$$

NEXT: $\quad |\mathcal{M}\rangle = \epsilon^{\mu\nu}\,\alpha^\mu_{-1}\,\tilde{\alpha}^\nu_{-1}\,|0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$

$$(L_0 - 1)|\mathcal{M}\rangle = 0 \quad \Rightarrow \quad \frac{\alpha'}{4}\bar{p}^2\,|\mathcal{M}\rangle = 0 \qquad \bar{p}^2 = 0$$

<u>MASSLESS</u>

GO TO FRAME $\quad \bar{p} = (E, E, 0, \ldots 0)$

DEFINE LIGHT-CONE COORDINATES

$$p^+ = 2E \qquad p^- = p^i = 0 \qquad i = 2, \ldots d-1$$

$$L_{+1}|M\rangle = \frac{\alpha'}{2\sqrt{2}}\, \alpha^{-}_{+1}\, \bar{p}^{+}\, \epsilon_{\mu\nu}\, \alpha^{\mu}_{-1}\, \tilde{\alpha}^{\nu}_{-1}\, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$$

$$= \frac{\alpha'}{2\sqrt{2}}\, \bar{p}^{+} \epsilon_{+\mu} \tilde{\alpha}^{\mu}_{-1}\, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$$

SO $\quad \epsilon_{+\mu} = 0 \qquad$ LIKEWISE $\epsilon_{\mu +} = 0$

FROM $\tilde{L}_{+1}|M\rangle = 0$

ALL OTHER CONDITIONS TRIVIALLY SATISFIED

STATE $|\psi\rangle$ SUCH THAT $|\psi\rangle = L_{-K}|\varphi\rangle$

HAS ZERO NORM; IT IS SPURIOUS

$L_{-1}|\varphi\rangle \qquad |\varphi\rangle = \alpha\, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$

IS SPURIOUS.

NOTE $\quad \langle \text{PHYS}|\psi\rangle = \langle \text{PHYS}| L_{-1}|\varphi\rangle = 0$

TRANSVERSE TO PHYSICAL STATES.

$|\text{PHYS}\rangle \sim |\text{PHYS}\rangle + |\psi\rangle$

CHOOSE $|\psi\rangle$ SO THAT $\quad \epsilon_{-\mu} = 0 \quad \epsilon_{ij} \neq 0$

PHYSICAL STATES :

$$\alpha_{-1}^{i} \, \tilde{\alpha}_{-1}^{j} \, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$$

SYMMETRIC - TRACELESS : $\frac{8 \cdot 9}{2} - 1 = 35$

COMPONENTS: G_{MN}

ANTISYMMETRIC : $\frac{8 \cdot 7}{2} = 28$ COMPONENTS:

B_{MN}

TRACE : 1 COMPONENT : ϕ

CLOSED BOSONIC STRING CONTAINS THE GRAVITON ALSO A TACHION. TO GET RID OF IT WE DEFINE THE SUPERSTRING

COORDINATES: $X^\mu(\tau,\sigma)$, $\psi^\mu(\tau,\sigma)$

MAJORANA SPINOR IN 2-D

ACTION: $$S = -\frac{1}{4\pi\alpha'}\int d^2\xi \left(\eta^{\alpha\beta}\partial_\alpha X \partial_\beta X + - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right) \qquad \alpha = 1, 2$$

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

2-D SUSY: $$\delta X^\mu = \bar{\varepsilon}\psi^\mu$$
$$\delta\psi^\mu = -i\rho^\alpha \partial_\alpha X^\mu \varepsilon$$

DEFINE: $$\rho^3 = \rho^0\rho^1$$
$$\psi^\mu_\pm = \frac{1 \mp \rho^3}{2}\psi^\mu$$

E.O.M. $$\begin{cases} \partial_+ \psi_-^\mu = 0 \\ \partial_- \psi_+^\mu = 0 \end{cases}$$

CLOSED SUPERSTRING:

$$\psi_-^\mu(\sigma+\pi,\tau) = \eta_- \, \psi_-^\mu(\sigma,\tau)$$

$$\psi_+^\mu(\sigma+\pi,\tau) = \eta_+ \, \psi_+^\mu(\sigma,\tau)$$

$\eta_\pm = -1$ (NEVEU-SCHWARZ SECTOR)

$\eta_\pm = +1$ (RAMOND SECTOR)

$$\psi_-^\mu = \sum_t \psi_-^\mu \, e^{-2it(\tau-\sigma)} \qquad \begin{array}{ll} t \in \mathbb{Z} & R \\ t \in \mathbb{Z}+\frac{1}{2} & NS \end{array}$$

$$\psi_+^\mu = \sum_t \psi_+^\mu \, e^{-2it(\tau+\sigma)}$$

BESIDES $T_{\alpha\beta}$ THERE IS A CONSERVED SUPER CURRENT $J_\mp = \psi_\mp^\mu \partial_\mp X^\nu \eta_{\mu\nu}$

$$J_\mp \equiv \sum G_\mp^t \, e^{-2it(\tau\mp\sigma)}$$

QUANTIZING CANONICALLY:

$$\{ \psi_t^{\#\mu}, \psi_s^{\#\nu} \} = \eta^{\mu\nu} \delta_{s+t,0} \qquad \# = + \text{ OR } -$$

IN TERMS OF OSCILLATORS :

$$L_n = :\frac{1}{2}\sum_{m\in\mathbb{Z}} \alpha_{-m}\alpha_{n+m} + \frac{1}{2}\sum_t \left(\frac{n}{2}+t\right)\psi_{-t}\psi_{t+n}:$$

$$G_t = \sum_{n=-\infty}^{+\infty} \alpha_{-n}\psi_{t+n}$$

SAME FOR $\tilde{L}_m$, $\tilde{G}_t$.

- SUPER VIRASORO ALGEBRA

VIRASORO + $[L_m, G_r] = (\frac{1}{2}m + r)\, G_{r+m}$

$$\{G_r, G_s\} = 2L_{r+s} + \frac{d}{2}\left(r^2 - \frac{a}{4}\right)\delta_{r+s,0}$$

$a = 1$ IN NS

$a = 0$ IN R

- PHYSICAL STATE CONDITIONS :

$L_m |\psi\rangle = 0 \qquad m > 0$

$(L_0 - a/2)|\psi\rangle = 0$

$G_t |\psi\rangle = 0 \qquad t > 0$

SAME WITH $\tilde{L}_m$, $\tilde{G}_t$.

FROM $(L_0 - a/2)\,|\text{PHYS}\rangle = 0$ WE HAVE

$$M^2 = \frac{2}{\alpha'}\Big(\sum_{n=1}^{\infty} (\alpha_{-n}\alpha_n + \tilde{\alpha}_{-n}\tilde{\alpha}_n)$$

$$+ \sum_{t\geq 0} t\,\psi_{-t}\psi_t + t\,\tilde{\psi}_{-t}\tilde{\psi}_{-t} - a \Big)$$

GLIOZZI - SCHERK-OLIVE (GSO) PROJECTION

- DEFINE FOCK VACUUM $\psi_t\,|0\rangle = 0 \quad t>0$

$\alpha_m\,|0\rangle = 0 \quad m>0$

ASSIGN FERMION NUMBER $\underline{0}$ TO THE VACUUM $|0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$

PROJECT STATES WITH $\left[\frac{1-(-)^F}{2}\right] ; \left[\frac{1-(-)^{\tilde{F}}}{2}\right]$

KEEP ONLY STATES WITH

$$(-)^F\,|\psi\rangle = -|\psi\rangle \;,\; (-)^{\tilde{F}}\,|\psi\rangle = -|\psi\rangle$$

- CONSISTENT WITH LORENTZ $\Lambda^{\mu\nu} \sim \sum \underbrace{\psi^{\mu}_{-t}\psi^{\nu}_{t}}_{\text{EVEN}} + \ldots$

KILLS THE POTENTIAL TACHION $|0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle$

FIRST STATES THAT SURVIVE PROJECTION IN NS-NS SECTOR:

$$\epsilon_{\mu\nu}\, \psi^{\mu}_{-\frac{1}{2}}\, \tilde{\psi}^{\nu}_{-\frac{1}{2}}\, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle \equiv |\mathcal{M}\rangle$$

$$\frac{\alpha'}{2} M^2\, |\mathcal{M}\rangle = \left(\tfrac{1}{2}\psi_{-\frac{1}{2}}\psi_{\frac{1}{2}} + \tfrac{1}{2}\tilde{\psi}_{-\frac{1}{2}}\tilde{\psi}_{\frac{1}{2}} - 1\right)|\mathcal{M}\rangle$$

$$= 0$$

MASSLESS

NONTRIVIAL AUXILIARY CONDITIONS:

$$G_{-\frac{1}{2}}\, |\mathcal{M}\rangle = \tilde{G}_{-\frac{1}{2}}\, |\mathcal{M}\rangle = 0$$

IMPLY $\quad p^{\mu}\epsilon_{\mu\nu} = 0 \qquad \epsilon_{\mu\nu}\, p^{\nu} = 0$

AND $\quad \epsilon_{\mu\nu} \sim \epsilon_{\mu\nu} + p_{\mu}\, \epsilon_{\nu} + p_{\nu}\, \eta_{\mu}$

AS IN THE BOSONIC CASE PHYSICAL D.O.F. IN FRAME $p^{\mu} = (p^{+}, 0, \ldots 0)$ ARE

$$\psi^{i}_{-\frac{1}{2}}\, \psi^{j}_{-\frac{1}{2}}\, |0\rangle \otimes |\tilde{0}\rangle \otimes |\bar{p}\rangle \qquad i,j = 2, \ldots 9$$

$$G_{MN} + B_{MN} + \phi$$

THERE ARE 3 OTHER SECTORS :

$|R\rangle \otimes |NS\rangle$, $|NS\rangle \otimes |R\rangle$, $|R\rangle \otimes |R\rangle$

VACUUM OF $|R\rangle$ IS DEGENERATE :

$$\psi^\mu_t |0\rangle_R = 0 \quad t > 0$$

$|0\rangle'_R \equiv \psi^\mu_0 |0\rangle_R$ ALSO OBEYS $\psi^\mu_t |0\rangle'_R = 0$.

NOTICE :

$$- \quad \{\psi^\mu_0 , \psi^\nu_0\} = \eta^{\mu\nu}$$

γ - MATRIX ALGEBRA !

$$\sqrt{2}\, \psi^\mu_0 \equiv \gamma^\mu$$

DEFINE

$$d_i^\pm = \frac{1}{\sqrt{2}} \left(\psi_0^{2i} \pm i \psi_0^{2i+1} \right) \quad i = 1, \dots, 4$$

$$d_0^\pm = \frac{1}{\sqrt{2}} \left(\psi_0^1 \pm \psi_0^0 \right)$$

$$\{d_i^+ , d_j^-\} = \delta_{ij}$$

RAISING / LOWERING OPERATORS

$d_i^- |0\rangle = 0$ DEFINES A STATE

(CLIFFORD VACUUM) $i = 0, 1, 2, 3, 4$

$\prod_A d^+_{i_A} |0\rangle$ SPAN A FOLK SPACE

OF DIMENSION 2^5 $(d = 10)$

IT IS A SPINOR OF 10-D LORENTZ:

$$S^{\mu\nu} = -i \sum \psi_{-r}^{[\mu} \psi_r^{\nu]} - i\, \psi_0^{[\mu} \psi_0^{\nu]} + \ldots$$

ON A RAMOND VACUUM $|0\rangle_R$

$$S^{\mu\nu} |0\rangle_R = \psi_0^{[\mu} \psi_0^{\nu]} |0\rangle_R$$

$$= \frac{1}{2} \gamma^{[\mu} \gamma^{\nu]} |0\rangle_R$$

LORENTZ ON SPINOR

$S^{\mu\nu}$ EVEN IN ψ I.E. EVEN IN d^+

$|\alpha\rangle_R$ = RAMOND VACUUM α = SPINOR INDEX

DIM $|\alpha\rangle_R = 32 = \underset{\text{EVEN}}{16} + 16 \leftarrow$ ODD

ON $|\alpha\rangle_R$ $\quad (-)^F |\alpha\rangle_R = \Gamma_{11} |\alpha\rangle_R$

FERMION PARITY = CHIRALITY.

GSO : KEEP ONLY EITHER

$$(-)^F |\psi\rangle = |\psi\rangle$$

OR $\quad (-)^F |\psi\rangle = -|\psi\rangle$

ALSO $\quad (-)^{\tilde{F}} |\psi\rangle = |\psi\rangle$

OR $\quad (-)^{\tilde{F}} |\psi\rangle = -|\psi\rangle$

2 INEQUIVALENT CHOICES :

$$(-)^F |\psi\rangle = (-)^{\tilde{F}} |\psi\rangle \qquad \text{II B}$$

OR $\quad (-)^F |\psi\rangle = -(-)^{\tilde{F}} |\psi\rangle \qquad \text{II A}$

THE OVERALL SIGN IS CONVENTIONAL AND DEPENDS ON THE FERMION PARITY OF THE CLIFFORD VACUUM (CONVENTIONAL)

PHYSICAL STATES IN RAMOND SECTOR

$$G_0 \,|\alpha\rangle_R \;=\; p_\mu \psi^\mu_0 \,|\alpha\rangle_R$$

IN FRAME $\bar{p}^\mu = (E, E, 0, \ldots, 0)$

$\bar{p}^+ = 2E \qquad \bar{p}^-, \bar{p}^i = 0$

$$G_0 |\alpha\rangle_R = 2E\, d_0^- \,|\alpha\rangle_R = 0$$

PHYSICAL STATES:

$$|0\rangle, \quad d_i^+|0\rangle, \quad d_i^+ d_j^+ |0\rangle \ldots \quad d_1^+ d_2^+ d_3^+ d_4^+ |0\rangle$$

CLIFFORD VACUUM $\qquad i = 1, 2, 3, 4$

16 STATES. GSO : 8

EITHER $|0\rangle, d_i^+ d_j^+ |0\rangle, \ldots$

OR $d_i^+|0\rangle \ldots$

AS SO(8) REPRESENTATIONS THIS MEANS

EITHER 8_c OR 8_s

$(NS + R) \times (NS + R)$

$(8_v + 8_s) \otimes (8_v + 8_c)$ II A

$(8_v + 8_s) \otimes (8_v + 8_s)$ II B

$8_v \otimes 8_v = G_{MN} + B_{MN} + \phi$

$8_s \otimes 8_c = 8_v \oplus 56_t = A_M + A_{MNP}$

$8_s \otimes 8_s = 1 \oplus 28 \oplus 35_+ = \psi + C_{MN} + C^+_{MNPQ}$

WE HAVE FOUND THE BOSONIC FIELDS OF II A AND II B SUPERGRAVITY

$8_v \otimes 8_s \oplus 8_v \otimes 8_c = \psi_M + \chi$

(NON CHIRAL, II A)

$8_v \otimes 8_s + 8_v \otimes 8_s = \psi_M^i + \chi^i$ (CHIRAL, II B)

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T-DUALITY

CLOSED BOSONIC STRING:

$$X^\mu = q^\mu + \sqrt{2\alpha'}\,(\alpha_0^\mu + \tilde{\alpha}_0^\mu)\tau - \sqrt{2\alpha'}\,(\alpha_0^\mu - \tilde{\alpha}_0^\mu)\sigma$$

$$+ i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n^\mu}{n} e^{-2in(\tau-\sigma)} + \frac{\tilde{\alpha}_n^\mu}{n} e^{-i2n(\tau+\sigma)} \right)$$

MOMENTUM: $\quad p^\mu = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)$

COMPACTIFY: $\quad X \simeq X + 2\pi R$

1D $\quad q^\mu \simeq q^\mu + 2\pi R$

WAVE FUNCTION: $\quad e^{i\bar{p}_\mu q^\mu} |\bar{p}_\mu\rangle \otimes$ OSCILLATORS

$$e^{i\bar{p}_\mu(q^\mu + 2\pi R)} = e^{i\bar{p}_\mu(q^\mu)}$$

$$\bar{p}_\mu = \frac{n}{R} \qquad n \in \mathbb{Z}$$

STRING CLOSED:

$$X^\mu(\sigma+\pi) = X^\mu(\sigma) + 2\pi m R$$

$$\pi\sqrt{2\alpha'}\,(\alpha_0^\mu - \tilde{\alpha}_0^\mu) = 2\pi m R$$

$$\alpha_0 = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} + \frac{mR}{\alpha'}\right) \qquad \tilde{\alpha}_0 = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} - \frac{mR}{\alpha'}\right)$$

$$L_0 = \frac{\alpha'}{4}\vec{p}^{\,2} + \frac{\alpha'}{4}\left(\frac{n}{R} + \frac{mR}{\alpha'}\right)^2 + \sum_{n=1}^{\infty} \alpha_{-n}\alpha_n$$

UNCOMPACTIFIED

$$\tilde{L}_0 = \frac{\alpha'}{4}\vec{p}^{\,2} + \frac{\alpha'}{4}\left(\frac{n}{R} - \frac{mR}{\alpha'}\right)^2 + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n}\tilde{\alpha}_n$$

$$M^2 = \frac{2}{\alpha'}\left(L_0 + \tilde{L}_0 - 2\right)$$

$$= \frac{2}{\alpha'}\left[\sum_{n=1}^{\infty}\left(\alpha_{-n}\alpha_n + \tilde{\alpha}_{-n}\tilde{\alpha}_n\right) - 2\right] +$$

$$+ \left(\frac{n}{R}\right)^2 + \left(\frac{mR}{\alpha'}\right)^2$$

SYMMETRY OF SPECTRUM:

$$m \longleftrightarrow n \qquad R \longleftrightarrow R_D \equiv \frac{\alpha'}{R}$$

(T-DUALITY)

T-DUALITY ON OSCILLATORS

$$\alpha_n \to \alpha_n \qquad \tilde{\alpha}_n \to -\tilde{\alpha}_n$$

ON COORDINATES:

$$X = \frac{1}{2}\left(X_- + X_+\right)$$

$$X_- = q + 2\sqrt{2\alpha'}(\tau-\sigma)\,\alpha_0 + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-2in(\tau-\sigma)}$$

$$X_+ = q + 2\sqrt{2\alpha'}(\tau+\sigma)\,\tilde{\alpha}_0 + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\tilde{\alpha}_n}{n}e^{i2n(\tau+\sigma)}$$

$$X_- \to X_- \qquad X_+ \to -X_+$$

$$X_D = \frac{1}{2}\left(X_- - X_+\right)$$

NOT ONLY THE SPECTRUM BUT THE THEORY ITSELF IS INVARIANT UNDER T-DUALITY:

T-DUALITY IS A GAUGE SYMMETRY

AT $R = R_D = \alpha'/R \Rightarrow R_{S.D.} = \sqrt{\alpha'}$

WE HAVE STATES WITH

$$\alpha_0 = \sqrt{\frac{\alpha'}{2}}\left(\pm\frac{1}{\sqrt{\alpha'}} \mp \frac{\sqrt{\alpha'}}{\alpha'}\right) = 0$$

$$\tilde{\alpha}_0 = \sqrt{\frac{\alpha'}{2}}\left(\pm\frac{1}{\sqrt{\alpha'}} \pm \frac{\sqrt{\alpha'}}{\alpha'}\right) = \pm\sqrt{2}$$

AND

$$\alpha_0 = \pm\sqrt{2} \quad , \quad \tilde{\alpha}_0 = 0 \, .$$

STATES

$$\tilde{\alpha}_{-1}^{\hat{\mu}} \, |0\rangle \otimes |\tilde{0}\rangle \otimes |\pm\sqrt{2}, 0\rangle$$

TRANSVERSE COORDINATE

$$\alpha_{-1}^{\hat{\mu}} \, |0\rangle \otimes |\tilde{0}\rangle \otimes |0, \pm\sqrt{2}\rangle$$

ARE <u>MASSLESS</u> GAUGE BOSONS

TOGETHER WITH $\tilde{\alpha}_{-1}^{\hat{\mu}} \alpha_{-1} \, |0\rangle \otimes |\tilde{0}\rangle \otimes |0,0\rangle$

GAUGE GROUP $SU(2) \otimes \widetilde{SU(2)}$

α_n TRANSFORMS AS THE J^3 COMPONENT OF $SU(2)$

UNDER π ROTATION AROUN J^1

$\alpha_M \to -\alpha_n \qquad \tilde{\alpha}_n \to \tilde{\alpha}_\mu$

UNDER $\widetilde{SU}(2) \qquad \alpha_n \to \alpha_n \qquad \tilde{\alpha}_n \to -\tilde{\alpha}_n$

T-DUALITY IS AN ELEMENT OF THE GAUGE GROUP $SU(2) \times \widetilde{SU(2)}$

T-DUALITY ON EFFECTIVE FIELD THEORY:

$$\int dx^D \left[\sqrt{G}\, R e^{-2\phi} + \ldots \right]$$

$D = 26$ (BOSONIC)
$D = 10$ (SUPER)

G, ϕ ETC. INDEPENDENT OF X^{D-1}

COMPACTIFIED ON S^1 OF RADIUS R

$$S_{COMP} = 2\pi R \int d^{D-1}x \sqrt{\hat{G}}\, \hat{R}(\hat{G})\, e^{-2\phi}$$

$$2\pi R e^{-2\phi} \equiv e^{-2\hat{\phi}}$$

INVARIANT UNDER T-DUALITY

THE D-DIM DILATON, INSTEAD, IS NOT INVARIANT

$$2\pi R e^{-2\phi} = 2\pi \frac{\alpha'}{R} e^{-2\phi_D}$$

$$\phi_D = \phi - \log R^2/\alpha'$$

FOLLOWS SIMPLY BY DEMANDING INVARIANCE OF S_{COMP}.

S_{COMP} INVARIANT SINCE IT DOES NOT DEPEND ON $\alpha_0 \neq 0$ OR $\tilde{\alpha}_0 \neq 0$

T-DUALITY $\equiv$ FIELD REDEFINITION ON STATES WITH $\alpha_0 = \tilde{\alpha}_0 = 0$

CLOSED SUPERSTRING

2-D SUSY:

$$\hat{X}^\mu = X^\mu + \theta \psi^\mu$$

$$X_+ \to -X_+ \qquad X_- \to X_-$$

THEN $\quad \psi_-^\mu \to \psi_-^\mu \qquad \psi_+^\mu \to -\psi_+^\mu$

RECALL THAT ON RAMOND VACUUM

$$\sqrt{2}\, \psi_0^\mu = \gamma^\mu$$

$\psi_0^A \to -\psi_0^A$ IMPLEMENTED BY

$$\psi_0^\mu \to \gamma^{11} \gamma^A \psi_0^\mu \gamma^A \gamma^{11} = (-)^{\delta_{\mu, A}} \psi_0^\mu$$

RAMOND VACUUM TRANSFORMS UNDER T-DUALITY AS

$$|R\rangle \otimes |\tilde{R}\rangle \to \eta \gamma^{11} \gamma^A |R\rangle \otimes |\tilde{R}\rangle$$

$$|\eta| = 1$$

GSO PROJECTION

IIB $\quad \gamma^{11}\, |R\rangle \otimes |\tilde{R}\rangle = \tilde{\gamma}^{11}\, |R\rangle \otimes |R\rangle$

T-DUALITY $\quad |R\rangle \otimes |\tilde{R}\rangle \to \eta\gamma^{11}\gamma^{A}\, |R\rangle \otimes |\tilde{R}\rangle$

$$\gamma^{11}\, \eta\gamma^{11}\gamma^{A}\, |R\rangle \otimes |\tilde{R}\rangle = -\eta\gamma^{11}\gamma^{A}\gamma^{11}\, |R\rangle \otimes |\tilde{R}\rangle$$

$$= -\eta\gamma^{11}\gamma^{A}\, |R\rangle \otimes \tilde{\gamma}^{11}\, |\tilde{R}\rangle$$

$$\gamma^{11}\left(|R\rangle \otimes |\tilde{R}\rangle\right)_D = -\tilde{\gamma}^{11}\left(|R\rangle \otimes |\tilde{R}\rangle\right)_D$$

IIB IS MAPPED INTO IIA !

IT ACTS ON FORMS AS FOLLOWS:

$$\underbrace{C_{A\hat{\mu}_1\cdots\hat{\mu}_p}}_{p+1} \longrightarrow \underbrace{\tilde{C}_{\hat{\mu}_1\cdots\hat{\mu}_p}}_{p}$$

$$C_{\hat{\mu}_1\cdots\hat{\mu}_p} \longrightarrow \tilde{C}_{\hat{\mu}_1\cdots\hat{\mu}_p A}$$

$$|R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta \equiv C_{\alpha\beta}$$

PHYSICAL STATE CONDITION :

$$G_0 |R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta = \widetilde{G}_0 |R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta = 0$$

$$d_0^- |R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta = 0 \quad \ldots$$

$$\gamma^{11} = (d_0^+ d_0^- - d_0^- d_0^+)\, \Gamma^8$$

$$\hat{\Gamma} = \gamma^2 \cdots \gamma^9$$

ON PHYSICAL STATES

$$\gamma^{11} |R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta = -\Gamma^8 |R\rangle_\alpha \otimes |\widetilde{R}\rangle_\beta$$

I. E. $\quad \Gamma^8_{\alpha\beta} C_{\beta\gamma} = \gamma^{11}_{\alpha\beta} C_{\beta\gamma}$

G.S.O. $\quad \Gamma^8_{\alpha\beta} C_{\beta\gamma} = C_{\alpha\gamma}$

II A $\quad C_{\alpha\beta} \widetilde{\Gamma}^8_{\beta\gamma} = -C_{\alpha\gamma}$, II B $\quad C_{\alpha\beta} \widetilde{\Gamma}^8_{\beta\gamma} = C_{\alpha\gamma}$

EXPAND $C_{\alpha\beta}$ IN BASIS OF γ^i

$i = 2, \ldots 9$

$$C_{\alpha\beta} = \sum_{n=0}^{9} \gamma^{[i_1} \ldots \gamma^{i_n]} \, C^{(n)}_{i_1 \ldots i_n}$$

G.S.O. :

II A $\quad \frac{1-\Gamma^8}{2} \, C_{\alpha\beta} \, \frac{1+\Gamma^8}{2} \leftarrow$ ODD FORMS

II B $\quad \frac{1-\Gamma^8}{2} \, C_{\alpha\beta} \, \frac{1-\Gamma^8}{2} \leftarrow$ EVEN FORMS

T- DUALITY : $\quad C_{\alpha\beta} \to \eta (\gamma^{11} \gamma^A)_{\alpha\gamma} \, C_{\gamma\beta}$

$$\Rightarrow \quad C_{\alpha\beta} \to -\eta \left(\Gamma^8 \gamma^A\right)_{\alpha\gamma} C_{\gamma\beta} = -\eta (\gamma^A C)_{\alpha\beta}$$

CHANGES FORM WITH γ^A IN FORM WITHOUT & VICE VERSA