OPEN BOSONIC STRINGS

$$\delta S = -T \int_{\Sigma} d^2\xi \; \partial_\alpha \delta X^\mu \partial^\alpha X_\mu$$

$$= T \int_{\Sigma} d^2\xi \; \delta X^\mu \partial_\alpha \partial^\alpha X_\mu \; - T \int_{\partial\Sigma} d\ell \; \partial_n X_\mu \delta X^\mu$$

$$\Sigma = \mathbb{R} \times [0,\pi] \qquad\qquad T \equiv \frac{1}{4\pi\alpha'}$$

BOUNDARY CONDITIONS :

FREE (NEUMANN) $\qquad \partial_\sigma X^\mu \Big|_0 = \partial_\sigma X^\mu \Big|_\pi = 0$

FIXED (DIRICHLET) $\qquad \delta X^\mu = 0 \Rightarrow X^\mu(\tau,0) = a^\mu$

$$X^\mu(\tau,\pi) = b^\mu$$

MODE EXPANSION

NN $\quad X^\mu(\tau,\sigma) = q^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{\alpha_n^\mu}{n} e^{-in\tau} \cos(n\sigma$

DD $\quad X^\mu(\tau,\sigma) = a^\mu + \frac{b^\mu - a^\mu}{\pi}\sigma - \sqrt{2\alpha'} \sum_{n\neq 0} \frac{\alpha_n^\mu}{n} e^{-in\tau} \sin(\sigma n)$

DN $\quad X^\mu(\tau,\sigma) = c^\mu - \sqrt{2\alpha'} \sum_{r \in \mathbb{Z}+\frac{1}{2}} \frac{\alpha_r^\mu}{r} e^{-ir\tau} \sin(r\sigma)$

ND $$X^\mu(\tau,\sigma) = d^\mu + i\sqrt{2\alpha'} \sum_{r\in\mathbb{Z}+\frac{1}{2}} \frac{\alpha_r^\mu}{r} e^{-ir\tau} \cos(r\sigma)$$

LET US LOOK AT N STRINGS FIRST

$$L_n = \frac{1}{2} \sum_{m\in\mathbb{Z}} \alpha_{-m}\, \alpha_{n+m}$$

$$L_0 = \alpha' p^2 + \sum_{n=1}^{\infty} \alpha_{-n}\alpha_n$$

PHYSICAL STATE CONDITIONS

$$L_n |phys\rangle = 0 \quad n>0 \qquad (L_0 - 1)|phys\rangle = 0$$

$$M^2 = -p^2 = \frac{1}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n - 1\right)$$

LIGHTEST STATE : TACHION $|0\rangle \otimes |\bar{p}\rangle$

$\alpha_n |0\rangle = 0 \quad n>0 \quad M^2 = -\bar{p}^2 = -\frac{1}{\alpha'}$

NEXT : $\epsilon_\mu \alpha_{-1}^\mu |0\rangle \otimes |\bar{p}\rangle \qquad \bar{p}^2 = 0$

$\bar{p}_\mu \epsilon^\mu = 0$ MASSLESS VECTOR

$\epsilon^\mu = \bar{p}^\mu$ GIVES A SPURIOUS STATE

$L_{-1} |0\rangle \otimes |\bar{p}\rangle \quad \bar{p}^2 = 0$ AS FOR CLOSED STRING

VERTICES AND CFT

DEFINE $z = e^{i(\tau-\sigma)}$ $\bar{z} = e^{i(\tau+\sigma)}$ (OPEN STRING)

OR $z = e^{i2(\tau-\sigma)}$ $\bar{z} = e^{i2(\tau+\sigma)}$ (CLOSED)

$\tau \to -i\tau$ $z, \bar{z}$ HOLOMORPHIC 2-D COORDINATES

$$X^\mu(z) = q^\mu - i\sqrt{2\alpha'} \log z \, \alpha^\mu_0 + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{\alpha^\mu_n}{n} z^{-n}$$

$X^\mu(\bar{z}) = [X^\mu(z)]^\#$ WHERE $(\alpha^\mu_n)^\# \equiv \tilde{\alpha}^\mu_n$ (INDEPENDENT) $\Big\}$ CLOSED

$(\alpha^\mu_n)^\# = \alpha^\mu_n$ (OPEN, N) $(\alpha^\mu_n)^\# = -\alpha^\mu_n$ (OPEN, D)

$z^\# = \bar{z}$.

$$X^\mu = \frac{1}{2}\left[X^\mu(z) + \left(X^\mu(z)\right)^\#\right]$$ ALWAYS

$T(z) =$ STRESS ENERGY TENSOR (T_{zz} COMPONENT)

$$= \frac{1}{2} \cdot \frac{1}{2\alpha'} (\partial X)^2(z)$$

$$L_n = \frac{1}{2\pi i} \int dz \, z^{n+1} \, T(z)$$

VIRASORO ALGEBRA IS THE SAME AS

$$T(z)\,T(w) = \frac{\frac{d}{dw}T(w)}{z-w} + 2\,\frac{T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4}$$

+ REGULAR TERMS

$\Phi(w)$ PRIMARY FIELD IF

$$T(z)\,\Phi(w) = \frac{\partial_w \Phi(w)}{z-w} + h\,\frac{\Phi(w)}{(z-w)^2} + \text{REGULAR}$$

STATE-OPERATOR CORRESPONDENCE

$$|\Phi\rangle = \lim_{z\to 0} \Phi(z)|0\rangle$$

$|0\rangle$ SUCH THAT $L_1\,|0\rangle = L_{-1}\,|0\rangle = L_0\,|0\rangle$

$$L_0|\Phi\rangle = h|\Phi\rangle \qquad L_n|\Phi\rangle = 0 \qquad n>0$$

PHYSICAL STATE : PRIMARY WITH $h=1$

- CLOSED STRING : $V = (h=1, \bar{h}=1)$ STATE

- OPEN STRING $\sigma=0,\ \sigma=\pi$ $\Rightarrow$ $z=\bar{z}$

VERTEX CREATING PHYSICAL STATE

Φ WITH $h=1$ AT $z=\bar{z}$.

ADD QUARKS AT THE END OF OPEN STRING

THIS IS THE SAME AS ENLARGING THE HILBERT SPACE OF THE OPEN STRING :

$$|\psi\rangle \rightarrow |\psi\rangle \otimes |ij\rangle$$

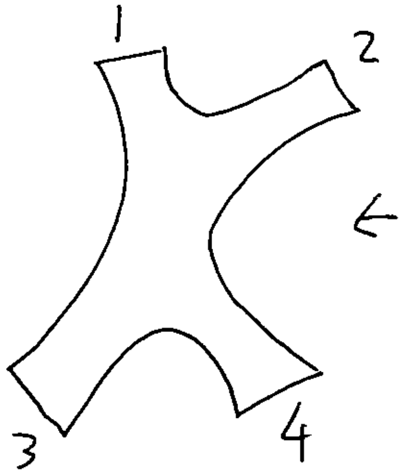
I.E. $\mathcal{H} \rightarrow \mathcal{H} \otimes H$

H SPANNED BY THE BASIS $\{ |ij\rangle \}$

$i, j = 1, .. N$

EQUIVALENTLY : $|ij\rangle = \sum_a \lambda^a_{ij} |a\rangle$

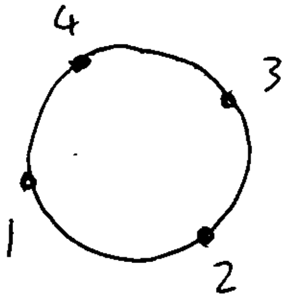
TO SEE IF THERE ARE RESTRICTIONS ON λ WE MUST LOOK AT OPEN-STRING SCATTERING AMPLITUDES.



← OPEN STRING DIAGRAM

(4 STRINGS)

$\Downarrow$ CONFORMAL MAP



CONF. MAP ⇒

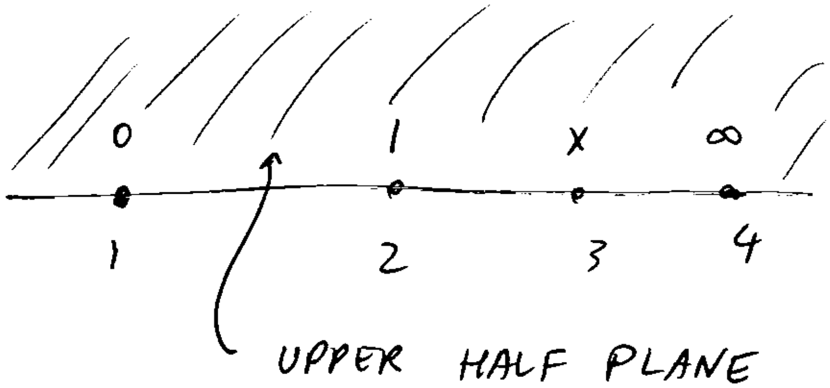


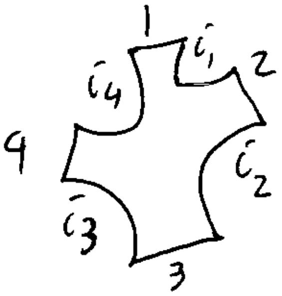
DIAGRAM GIVES AMPLITUDE

$$\int_1^\infty dx \ \langle 0| \ V_1(0) \ V_2(1) \ V_3(x) \ V_4(\infty) \ |0\rangle = A$$

ADD CHAN PATON :

$$V^a(x) \to V(x)\,\lambda^a_{ij} \qquad (\mathcal{H} \to \mathcal{H} \otimes \{|ij\rangle\})$$

IN AMPLITUDE :



$$\Rightarrow \quad A \ \mathrm{Tr}\, \lambda^1_{i_1 i_2} \lambda^2_{i_2 i_3} \lambda^3_{i_3 i_4} \lambda^4_{i_4 i_1}$$

$\mathrm{Tr}\, \lambda^{a_1}_{i_1 i_2} \cdots \lambda^{a_n}_{i_n i_1}$ INVARIANT UNDER

$\lambda^a \to U^{-1} \lambda^a U$

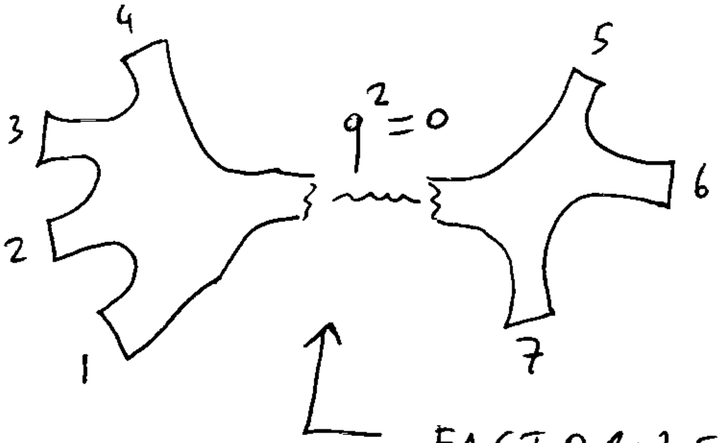
U UNITARY TO PRESERV THE NORM OF PHYSICAL STATE.

VERTEX FOR MASSLESS SPIN 1 :

$$V^a = \lambda^a_{ij} \epsilon_\mu \, \partial_t X^\mu e^{i q_\mu X^\mu} \Big|_{\sigma = 0 \text{ OR } \pi}$$

$\lambda^a \to U^{-1} \lambda^a U$ V^a TRANSFORMS IN THE ADJOINT OF U(N)

λ^a_{ij} SPAN ALL U(N) $\Rightarrow$ U(N) = GAUGE GROUP



FACTORIZE AT MASSLESS POLE

$$\mathrm{Tr}\,\lambda^1 \ldots \lambda^4 \;\; \lambda^5 \ldots \lambda^7 = \sum_a \mathrm{Tr}\,\lambda^1 \ldots \lambda^4 \lambda^a \,\mathrm{Tr}\,\lambda^a \lambda^5 \ldots \lambda^7$$

$$\sum_a \lambda^a_{ij} \lambda^a_{kl} = \delta_{il}\delta_{jk} \Rightarrow \lambda^a \text{ SPAN } U(N)$$

UNORIENTED OPEN STRINGS:

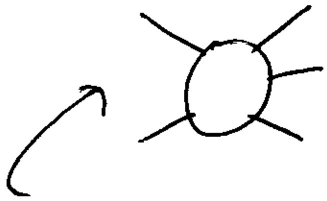
ORIENTATION REVERSAL : $\sigma \to \pi - \sigma$

ON OSCILLATORS $\quad q^\mu \to q^\mu \quad p^\mu \to p^\mu \quad \alpha^\mu_m \to (-)^m \alpha^\mu_m$

ON VACUUM : $\quad \Omega\,|0\rangle \otimes |\bar{p}\rangle = |0\rangle \otimes |\bar{p}\rangle$

$\Omega\,|0\rangle \otimes |\bar{p}\rangle = e^{i\eta}\,|0\rangle \otimes |\bar{p}\rangle$ IS NOT CONSISTENT WITH INTERACTIONS WHEN $\eta \neq 2\pi m$

EXAMPLE :



← 3 TACHIONS

$\Omega\,|3\rangle = e^{3i\eta}\,|3\rangle$

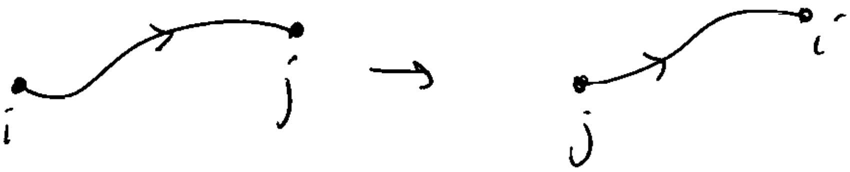
2 TACHIONS

$\Omega\,|2\rangle = e^{2i\eta}\,|2\rangle$

WHEN CHAN-PATON FACTORS ARE PRESENT, Ω MAY ACT NONTRIVIALLY ON THEM

$$\Omega\, |\bar{p}\rangle \otimes |ij\rangle = M_i{}^{l}\, (M^{-1})_j{}^{m}\, |\bar{p}\rangle \otimes |m\, l\rangle$$

ORIENTATION REVERSAL



IN TERMS OF $\lambda^a{}_{ij}$ $\quad |ij\rangle \equiv \sum_a \lambda^a_{ij}\, |a\rangle$

$$\lambda^a \to M\, (\lambda^a)^T M^{-1}$$

WHAT GAUGE VECTOR SURVIVES THE PROJECTION?

$$\Omega\, \lambda^a\, \alpha^\mu_{-1}\, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle = (-1)\, M (\lambda^a)^T M^{-1} \times \alpha^\mu_{-1}\, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle$$

$$\Omega^2 = \mathbb{1} \Rightarrow M\, [M^{-1} (\lambda^a)^T M]^T M^{-1} = \lambda^a$$

$$\Rightarrow M^T M \lambda^a = \lambda^a M^T M$$

$|i\rangle$ = IRREP OF $\lambda^a \Rightarrow M^T M = \text{CONST} \times \mathbb{1}$

UP TO A CHOICE OF BASIS TWO POSSIBILITIES

$$M = M^T = \mathbb{I}$$

$$\Omega \; \lambda^a \alpha_{-1}^{\mu} \, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle = - (\lambda^a)^T \alpha_{-1}^{\mu} \, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle$$

$(\lambda^a)^T = - \lambda^a \Rightarrow \lambda^a$ SPAN $SO(N)$

$$M = - M^T \qquad M = i \begin{bmatrix} 0 & \mathbb{I}_{N/2} \\ - \mathbb{I}_{N/2} & 0 \end{bmatrix}$$

$$\Omega \; \lambda^a \alpha_{-1}^{\mu} \, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle = - M (\lambda^a)^T M \, \alpha_{-1}^{\mu} \, |0\rangle \otimes |\bar{p}\rangle \otimes |a\rangle$$

$\lambda^a = - M (\lambda^a)^T M \Rightarrow \lambda^a$ SPAN $USp(N)$

- HOMEWORK: SHOW THAT IN THE CLOSED STRING $\Omega \, \alpha_m^{\mu} \, \Omega^{-1} = \tilde{\alpha}_m^{\mu}$ AND THAT Ω REMOVES $B_{\mu\nu} = - B_{\nu\mu}$ FROM THE PHYSICAL SPECTRUM

T-DUALITY OF OPEN STRINGS

$$X = \frac{1}{2}(X_- + X_+) \qquad NN$$

COMPACTIFY ON CIRCLE OF RADIUS R

$$X_- = q + c + \sqrt{2\alpha'}(\tau-\sigma)\alpha_0 + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-in(\tau-\sigma)}$$

$$X_+ = q - c + \sqrt{2\alpha'}(\tau+\sigma)\alpha_0 + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-in(\tau+\sigma)}$$

$$\alpha_0 = \sqrt{2\alpha'}\,\frac{n}{R}$$

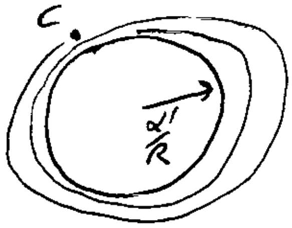
ALMOST AS CLOSED STRING EXCEPT THAT HERE $\tilde{\alpha}_n = \alpha_n$

T-DUALITY: $X_- \to X_- \qquad X_+ \to -X_+$

$X_D = \frac{1}{2}(X_- - X_+)$ IS A DD STRING ON A CIRCLE OF RADIUS $\alpha'/R \equiv R_D$:

$$X_D = c - 2\alpha'\sigma\frac{n}{R} - \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-in\tau}\sin n\sigma$$

$0 \leq \sigma \leq \pi$ STRING WINDS n TIMES AROUND A CIRCLE OF RADIUS $\frac{\alpha'}{R}$



NOTICE: X_D CONTAINS NO MOMENTUM STATES: LIGHTEST IS A TACHION

$|0\rangle \otimes |p^\mu\rangle$ μ RUNS OVER LONGITUDINAL COORDINATES *NOT* ON X_D. $-p^2 = M^2 = -1/\alpha'$

NEXT: $\alpha^D_{-1}|0\rangle \otimes |p^\mu\rangle$ ← SCALAR

$\epsilon_\mu \alpha^\mu_{-1}|0\rangle \otimes |p^\mu\rangle$ ← VECTOR

$p^{\mu 2} = 0$. $p^\mu \epsilon_\mu = 0$ $\epsilon_\mu \sim \epsilon_\mu + \alpha p_\mu$

THE MASSLESS MODES OF A Dp BRANE ARE

25 - p SCALARS + 1 (p+1)-DIM VECTOR

THEY ARE (p+1)-DIM FIELDS SINCE p^μ IS (p+1)-DIM (NO p^μ FOR DD BOUNDARY CONDITIONS).

-WITH CHAN-PATON & WILSON LINES WE HAVE NEW PHENOMENA.

$$\int_0^{2\pi R} dx\, A^a_\mu \lambda^a_{ij} \equiv \text{WILSON LINE, GAUGE INVARIANT}$$

CHOOSE $$A_D = \frac{1}{2\pi R}\begin{pmatrix}\theta_1 & & \\ & \ddots & \\ & & \theta_N\end{pmatrix}$$

COMPACTIFIED COORDINATE

A_D IS PURE GAUGE IF WE ALLOW $\Lambda(x)$ = GAUGE TRANSFORMATION TO BE NON-PERIODIC

$$A_D = -i\Lambda^{-1}\frac{\partial}{\partial x^D}\Lambda \qquad \Lambda_i^{\ j} = \begin{pmatrix} e^{i\theta_1\frac{x}{2\pi R}} & & \\ & \ddots & \\ & & e^{i\theta_N\frac{x}{2\pi R}} \end{pmatrix}$$

FUNDAMENTAL TRANSFORMS AS $|i\rangle \to \Lambda_i^{\ j}|j\rangle$

INSTEAD OF $|ij\rangle \otimes |p\rangle$ WE WRITE

$$\psi_{ij}(x) \qquad \text{(SCHRÖDINGER REPRESENTATION)}$$

UNDER A GAUGE TRANSFORMATION: $\psi_{ij}(x) \to \Lambda_i^{\ l}\Lambda^{-1\,m}_{j}\,\psi_{lm}(x)$

IN OUR CASE:

$$\begin{aligned} &\psi_{ij}(x) \to \Lambda_i(x)\,\Lambda_j^{-1}(x)\,\psi_{ij}(x) \quad \xleftarrow{\text{EQUAL}} \\ &\psi_{ij}(x+2\pi R) \to \Lambda_i(x+2\pi R)\,\Lambda_j^{-1}(x+2\pi R)\,\psi_{ij}(x+2\pi) \end{aligned}$$

PERIODICITY BECOMES CLEAR WITH $\psi \sim e^{ipx}$

$$\Lambda_i(x)\,\Lambda_j^{-1}(x)\,e^{ipx} = \Lambda_i(x+2\pi R)\,\Lambda_j^{-1}(x+2\pi R)\,e^{ip(x+2\pi R)}$$

$$\theta_i - \theta_j + 2\pi R p = 2\pi m \qquad \text{(INTEGER)}$$

$$p = \frac{m}{R} + (\theta_j - \theta_i)/2\pi R$$

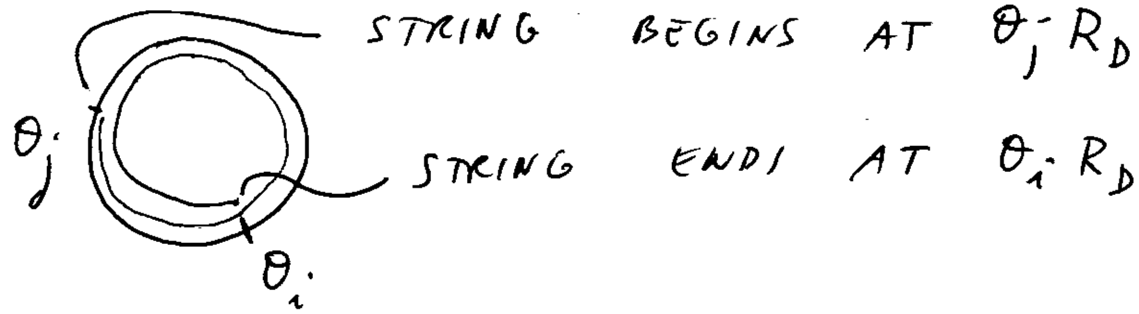
$$X_- = 2\alpha' p\,(t-\sigma) + \dots \qquad X_+ = 2\alpha' p\,(\tau+\sigma) + \dots$$

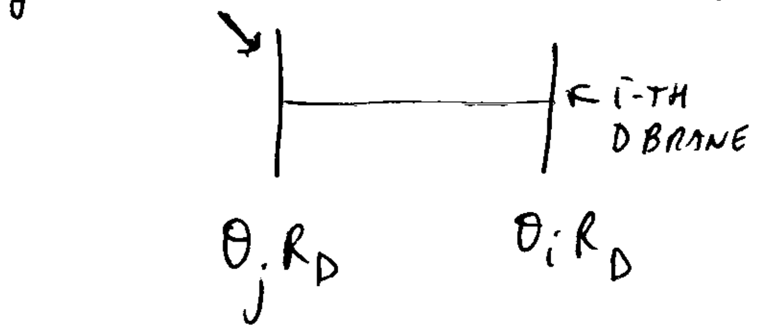
T-DUALITY $\quad X_- \to X_- \quad X_+ \to -X_+$

$$X_D = -\frac{2\alpha'}{R}\left[n + (\theta_j - \theta_i)/_{2\pi} \right]\sigma + \dots$$

IN THE DUAL PICTURE WILSON LINES BECOME POSITIONS OF THE END POINTS OF STRINGS

$$\alpha'/R = R_D$$





WILSON LINE = POSITION OF D-BRANE

MASS SPECTRUM OF STRINGS BETWEEN NON-COINCIDENT D-BRANES :

$$M^2 = -p_\mu^2 = \frac{1}{\alpha'}\left[\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n - 1\right] + p^2$$

MOMENTA OF UNCOMPACTIFIED COORDINATES

MASS INVARIANT UNDER T-DUALITY ⇒

$$M^2 = \left(\frac{1}{2\pi\alpha'}\right)^2 L^2 + \frac{1}{\alpha'}\left[\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n - 1\right]$$

L = MINIMAL LENGTH OF STRING RUNNING FROM j-TH TO i-TH BRANE

QUESTION: DOES ALL THIS MAKE SENSE?

p- BRANE CANNOT BE A RIGID HYPERPLANE IN 10-D AS WE WANT A RELATIVISTIC (GENERAL COVARIANT) THEORY. THE PLANE MUST BE ABLE TO OSCILLATE.

-1) DO WE HAVE THE DEGREES OF FREEDOM?

YES: $\alpha_{-1}^{A} | p^{\mu} \rangle$ $\quad A = p+1, .., 25$ ARE WORLD-VOL. SCALARS DESCRIBING TRANSVERSE OSCILLATIONS.

$\mu = 0, .., p$

SURPRISE: N COINCIDENT D-BRANE HAVE MUCH MORE WORLD-VOLUME FIELDS THAN GEOMETRICAL BRANES.

A) GEOMETRICAL BRANES $\quad 25-p$ SCALARS

B) D-BRANES $\quad N^2 \times (25-p)$ SCALARS + N^2 VECTORS IN SAME REPRESENTATION OF THE GAUGE GROUP (ADJOINT!)

-2) CAN WE DISCARD D-BRANES FROM SPECTRUM?

NO: WE NEED THEM TO DESCRIBE CLASSICAL FINITE ENERGY SOLUTIONS OF EFFECTIVE SUPERGRAVITY

\+

OPEN STRINGS + T-DUALITY (P-TIMES) MEANS THAT WE NEED ALL P-BRANES IN BOSONIC STRING

WE WILL STUDY THE SUPERSTRING IN A WHILE.

3) SINGE THE D-BRANE IS DYNAMICAL, WHAT IS ITS TENSION?
IT *MUST* COUPLE TO GRAVITY AS $T \int d^{p+1}x \sqrt{G}$.

THE D-BRANE ACTION

$\xi^a, a = 0, \ldots p$ = COORDINATES ON BRANE

FIELDS: $X^M(\xi)$ (EMBEDDING) $A_a(\xi)$ (GAUGE FIELD)

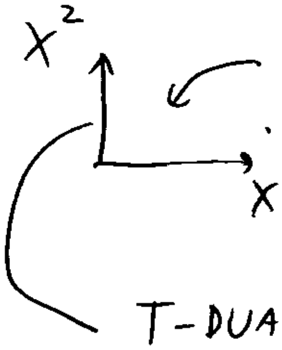
$$S_p = - T_p \int d^{p+1}\xi \sqrt{\det \left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab} \right)} e^{-\phi}$$

$$G_{ab} = \partial_a X^M \partial_b X^N G_{MN}, \quad B_{ab} = \partial_a X^M \partial_b X^N B_{MN}$$

$e^{-\phi} = \frac{1}{g_s}$ HERE BECAUSE S_p IS OPEN-STRING TREE LEVEL.

$\sqrt{\det G_{ab}}$ = WORLD-VOLUME OF D-BRANE

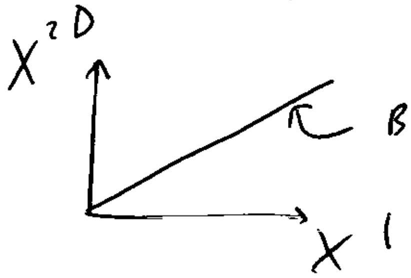
AND ACTION $\propto$ TENSION × WORLD-VOLUME. F_{ab}, B_{ab} DEPENDENCE FOLLOWS FROM T-DUALITY:



BRANE EXTENDS IN X^1-X^2

CONSTANT POTENTIAL $A_2 = X^1 F_{12}$

T-DUALIZE X^2



BRANE EXTENDS IN 1-DIM. SUBSPACE

RECALL FROM T-DUAL:

$$X^{2D}\Big|_{\text{ENDPOINT OF STRING}} = 2\pi\alpha' A_2 = 2\pi\alpha' X^1 F_{12}$$

AREA ELEMENT OF T-DUALIZED STRING:

$$\int dx^1 \sqrt{1+(\partial_1 X_D^2)^2} = \int dx^1 \sqrt{1+(2\pi\alpha' F_{12})^2}$$

BOOSTS + ROTATION REDUCE F_{ab} TO BLOCK-DIAGONAL FORM. $\Rightarrow$

$\det\left(G_{ab} + 2\pi\alpha' F_{ab}\right)$ IS ACTION OF D-BRANE AT $B_{ab} = 0$.

WHY B_{ab} APPEARS HERE?

IN WORLD-SHEET STRING ACTION:

$$S = \cdots + \frac{1}{2\pi\alpha'}\underbrace{\int_\Sigma B}_{\text{CLOSED STRING COUPLING}} + \underbrace{\int_{\partial\Sigma} A}_{\text{OPEN-STRING COUPLING}}$$

UNDER:

$B \to B + d\Lambda$ $\quad$ S NON-INVARIANT:

$$S \to S + \frac{1}{2\pi\alpha'}\int_\Sigma (B + d\Lambda) = S + \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} \Lambda$$

TO COMPENSATE $A \to A - \frac{1}{2\pi\alpha'}\Lambda$

NOW F NO LONGER GAUGE INVARIANT.

$B + 2\pi\alpha' F$ IS GAUGE INVARIANT: ONLY THIS COMBINATION CAN APPEAR IN S_p.

N D-p BRANES.

A_a IS A NON-ABELIAN GAUGE FIELD.

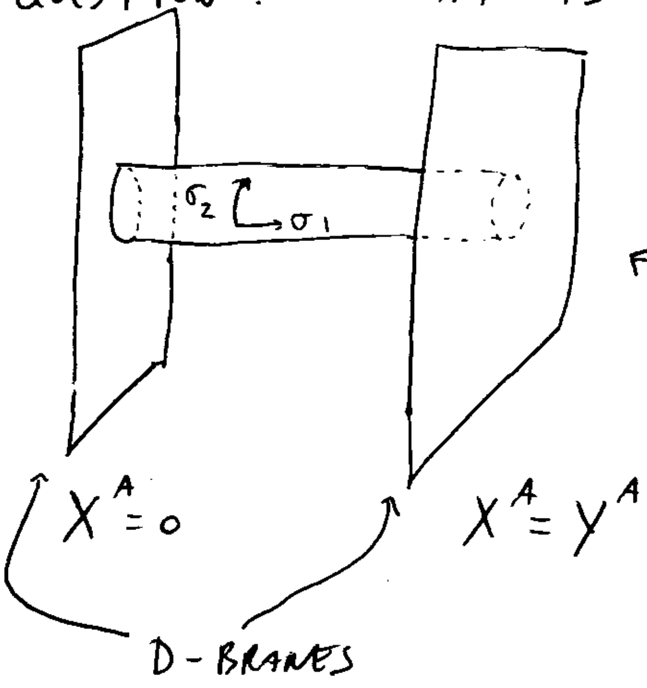
NON-DERIVATIVE TERMS : $\mathrm{Tr}\,[A_a\, A_b][A^a\, A^b]$

T-DUALITY : $A^A \to X_D^A$

$$S_p = -T_p \int d^{p+1}\xi\, e^{-\phi}\, \mathrm{Tr}\left\{ \sqrt{\det\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)} \right.$$

$$\left. + \, O[X^A X^B]^2 + \dots \right\}$$

QUESTION : WHAT IS T_p ?



$0 \le \sigma_1 \le \pi \qquad 0 \le \sigma_2 \le 2\pi t$

THIS DIAGRAM DESCRIBES

A) A ONE-LOOP OPEN STRING DIAGRAM ≡ VACUUM ENERGY

B) A CLOSED-STRING TREE-LEVEL DIAGRAM ≡ EXCHANGE OF CLOSED STRINGS BETWEEN 2 Dp BRANES.

A) AMPLITUDE $\mathcal{A} = V_{p+1}\mathrm{Tr}\int \frac{d^{p+1}k}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t}\, e^{-2\pi\alpha' t(k^2+M^2)}$

Tr = OVER STRING STATES

TRANSVERSE OSCILLATORS (LIGHT-CONE)

$$M^2 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} \left[\alpha_{-n}^{i} \alpha_{n}^{i} - 1\right] + \frac{Y^2}{4\pi^2(\alpha')^2}$$

$$\Rightarrow \quad \mathcal{A} = 2V_{p+1} \int_0^\infty \frac{dt}{2t} \left(8\pi^2\alpha' t\right)^{-\frac{p+1}{2}} e^{-Y^2 t/2\pi\alpha'} f(q)^{-24}$$

$$f(q) = q^{1/12} \prod_{n=1}^{\infty} \left(1 - q^{2n}\right) \qquad q = e^{-\pi t}$$

MODULAR PROPERTY : $f\left(e^{-\pi/s}\right) = \sqrt{s}\, f\left(e^{-\pi s}\right)$

$$\mathcal{A} = 2V_{p+1} \int_0^\infty \frac{dt}{2t} \left(8\pi^2\alpha' t\right)^{-\frac{p+1}{2}} e^{-Y^2 t/2\pi\alpha'} t^{12} \left[e^{2\pi/t} + 24 + \dots\right.$$

AT $t \to 0$ TACHION DIVERGENCE, UNINTERESTING

THIS IS INTERESTING

$$\mathcal{A} = V_{p+1} \frac{24\pi}{2^{10}} \left(4\pi^2\alpha'\right)^{11-p} G_{25-p}(Y^2)$$

GREEN'S FUNCTION OF MASSLESS SCALAR IN 25-p DIMENSIONS

COMPARE WITH FIELD THEORY

FIELD THEORY COMPUTATION = EXCHANGE OF (MASSLESS) CLOSED STRING FIELDS (G_{MN}, B_{MN}, ϕ)

ACTION : $$S = \int d^{D\,=\,26}x \sqrt{G}\, \frac{1}{2K^2} e^{-2\phi}\left[R + (4\nabla\phi)^2 + \right.$$

$$\left. - \frac{1}{12} H_{MNP} H^{MNP}\right] - T_p \int d^{p+1}\xi \sqrt{\det(G_{ab} + B_{ab})}$$

NOTICE: WE DO NOT NEED OPEN STRING FIELDS HERE ($F_{ab} = 0$).

FIELD THEORY AMPLITUDE $\propto \sum_I K^2 T_p^2 \langle 0| \frac{1}{p^2 + M_I^2} |Y\rangle$

M_I = CLOSED-STRING STATE MASS.

$$\mathcal{A} = \frac{D-2}{4} K^2 V_{p+1} T_p^2 e^{+2\phi} G_{25-p}(Y^2)$$

$\Rightarrow$ $T_p = \frac{\sqrt{\pi}}{16K}(4\pi^2\alpha')^{\frac{11-p}{2}} e^{-\phi}$ $\quad e^{\phi} = g_s$

1) O.K. DIMENSIONALLY $K \sim (\alpha')^6 \sim M_s^{-6}$

$T_p \sim (\alpha')^{-\frac{p+1}{2}} \sim M_s^{p+1}$

2) FIELD THEORY COMPUTATION STRAIGHTFORWARD BUT LONG AS ϕ AND G_{MN} MIX : THE QUADRATIC ACTION MUST BE DIAGONALIZED

3) RELATION WITH GAUGE COUPLING

EX : $p = 3$ $\quad S_3 = \int d^4\xi \underbrace{T_3(2\pi\alpha')^2}_{\text{DIMENSIONLESS}} \cdot \frac{1}{4} e^{-\phi}\, \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \dots$

$\Rightarrow$ $e^{-\phi} = \frac{1}{g_s} = \frac{1}{g_{YM}^2}$

ORIENTIFOLDS

$$X = \frac{1}{2}\left[X^{+}(\tau+\sigma) + X^{-}(\tau-\sigma)\right]$$

WORLD-SHEET PARITY (Ω) $\sigma \to \pi-\sigma$ $\Rightarrow$ $X^{+} \leftrightarrow X^{-}$

T-DUALITY (T)

$$X \to X^{D} = \frac{1}{2}\left(X^{-} - X^{+}\right)$$

WORLD-SHEET PARITY: $\sigma \to \pi - \sigma$ <u>AND</u> $X^{D} \to -X^{D}$.

ΩT ACTS ALSO ON SPACE-TIME COORDINATES FIXED POINT AT $X^{D} = 0$.

ORIENTIFOLD PROJECTION ΩT ON OPEN STRINGS:

$\Omega T\,|\psi\rangle = |\psi\rangle$ PHYSICAL STATE CONDITION

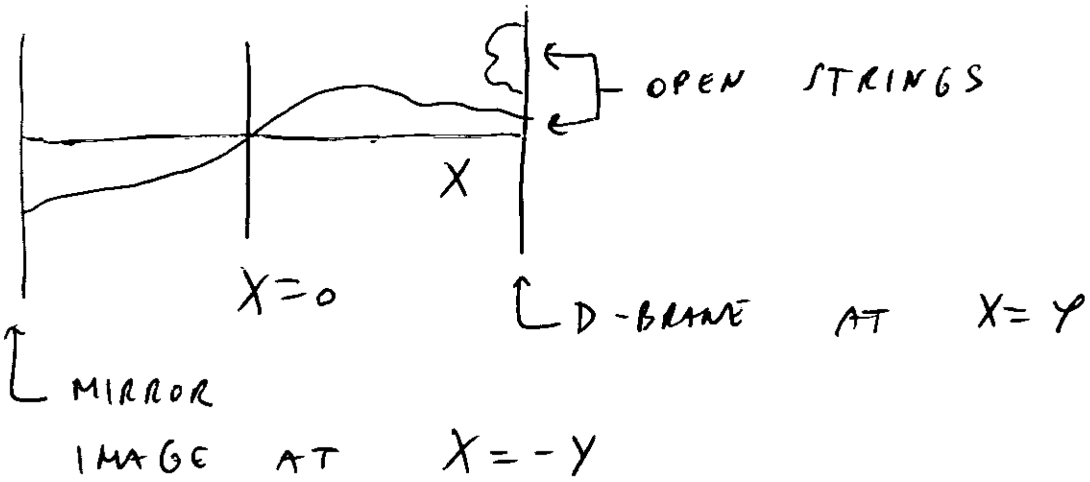
$$|\psi\rangle = \prod_i \alpha_{-m_i}|0\rangle\, \psi_{\underbrace{AB}_{\text{CHAN-PATON INDICES}}}(X^{D})$$

$$\Omega T\,|\psi\rangle = \prod_i (-)^{m_i+1}\alpha_{-m_i}|0\rangle\; M_B{}^{C}\left(M^{-1}\right)_A{}^{D}\,\psi_{CD}(-X^{D})$$

STRING AT X^{D} NEGATIVE DETERMINED BY STRING AT X^{D} POSITIVE.

X^{D} & $-X^{D}$ ARE MIRROR IMAGES (X^{D} CAN BE RESTRICTED TO $X^{D} \geq 0$).

EX: $S^{1} \to S^{1}/\mathbb{Z}_2$, $T^{D} \to T^{D}/\mathbb{Z}_2$ ETC.



OPEN STRINGS BEGIN AND END EITHER ON $X=Y$ OR $X=-Y$ D-BRANES. <u>NOT</u> AT $X=0$.

$X=0$ PLANE (ORIENTIFOLD) IS NON-DYNAMICAL (IT CANNOT OSCILLATE). IT CAN HAVE NEGATIVE TENSION.

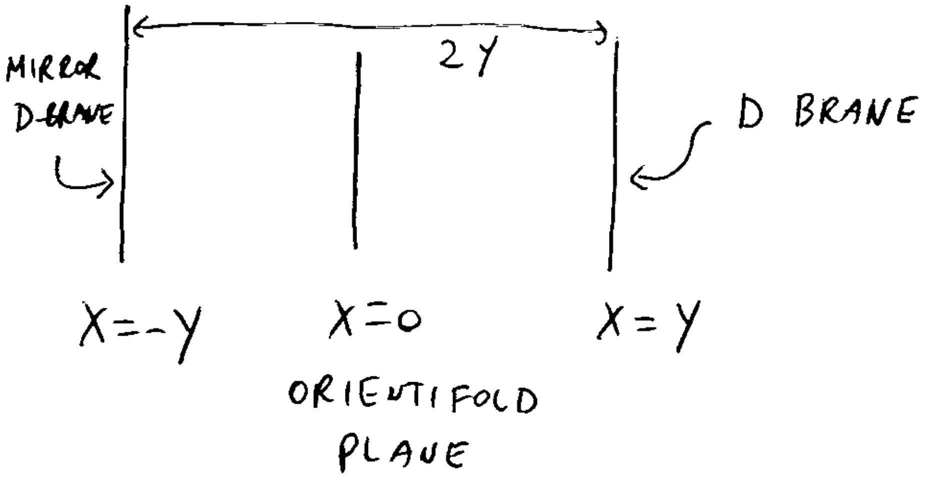
IN GENERAL 2 TENSIONS : 1) $T\int d^{p+1}\xi \sqrt{G}$

2) IF PLANE OSCILLATES, $X^M(\xi)$ $M=p+1,\ldots,25$
TRANSVERSE OSCILLATIONS

$$S \text{ CONTAINS TERM } \frac{T'}{2}\int d^{p+1}\xi \sqrt{G}\; \partial_\alpha X^M \partial^\alpha X^N G_{MN}$$

HOMEWORK : IF PLANE OSCILLATES GENERAL COVARIANCE IMPLIES $T'=T \Rightarrow T$ <u>POSITIVE</u>.

TENSION OF ORIENTIFOLD PLANE



VACUUM AMPLITUDE OF OPEN STRING BETWEEN D-BRANE AND ITS MIRROR IMAGE

$$A = V_{p+1} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t} \, \mathrm{Tr} \, \frac{1+\Omega}{2} \, e^{-2\pi\alpha' t (p^2 + M^2)}$$

$$M^2 = \frac{1}{\alpha'} \Big[\underbrace{\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n}_{N} - 1 \Big] + \frac{1}{4\pi^2\alpha'^2} (2Y)^2$$

Ω = ORIENTIFOLD PROJECTION

$$\Omega \, \alpha_{-1}^{\mu} |ab\rangle = -\alpha_{-1}^{\mu} |ba\rangle \qquad (SO(N))$$

$$-\alpha_{-1}^{\mu} M_{bl} (M^{-1})_{ma} |lm\rangle \qquad (USp(N))$$

IN TRACE $\quad$ SO(N) $\langle ab | ba \rangle = \langle aa | aa \rangle = N$ OR

USp(N) $\langle ab | M_{bl} (M^{-1})_{ma} | lm \rangle = -N$

THE CONTRIBUTION TO $\mathcal{A}$ PROPORTIONAL TO Ω IS

$$\mathcal{A}_{\Omega} = V_{p+1} \int \frac{d^{p+1}_{K}}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t} \, \mathrm{Tr} \, \frac{\Omega}{2} \, e^{-2\pi\alpha' t (p^2 + M^2)}$$

$$= \pm N \, V_{p+1} \int_0^\infty \frac{dt}{2t} \left(8\pi^2\alpha' t\right)^{-\frac{p+1}{2}} \frac{1}{2} \, e^{-4Y\cdot Y t/2\pi\alpha'} {}_{\cdot x}$$

$$\times \; \mathrm{tr} \, (-)^N \, e^{-2\pi t [N-1]}$$

COMPUTATION SIMILAR TO THAT LEADING TO T_p :

$$\mathcal{A}_{\Omega} = \mp N 2^{p-12} \, V_{p+1} \, \frac{24\pi}{2^{10}} \left(4\pi^2\alpha'\right)^{11-p} G_{25-p}(Y^2)$$

IN FIELD THEORY :

$$\begin{tikzcd} T'_p \arrow[r, leftrightarrow] & T_p \\ X=0 & X=Y \end{tikzcd}$$

$$\mathcal{A} = 2 \cdot T'_p T_p \, \frac{D-2}{4} \, V_{p+1} \, \kappa^2 \, G_{25-p}(Y^2)$$

$$T_p = N \, T_p^{(1)} \longleftarrow 1 \text{ D BRANE}$$

$$T'_p = \mp 2^{p-13} \, T_p$$