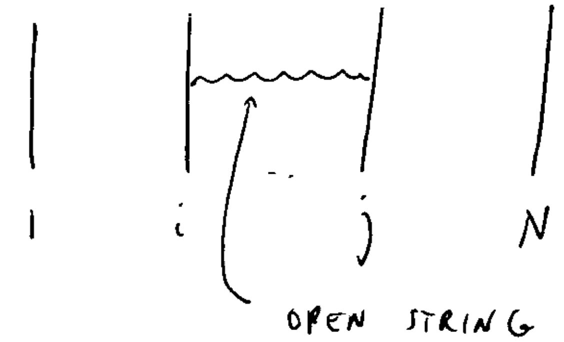
AdS/CFT AND HOLOGRAPHY

A) Dp BRANES DESCRIBE GAUGE THEORIES.

WE WILL CONSIDER MOSTLY THE EXAMPLE D3 (4-D GAUGE THEORIES)



LOWEST LYING STATES HAVE MASS

$$M \sim d_{ij} M_s^2$$

THEY FORM AN $\mathcal{N}=4$ $D=4$ MULTIPLET

$$\underbrace{\left(1(1),\ 4\left(\tfrac{1}{2}\right),\ 5(0)\right)}_{\text{MASSIVE}}$$

$$\left(1(1),\ 4\left(\tfrac{1}{2}\right),\ 6(0)\right) \leftarrow \text{MASSLESS.}$$

FIELDS : A_μ , λ^I , Φ^{IJ} $\quad I, J = 1, .., 4$

$$\left(\Phi^{IJ}\right)^* = \frac{1}{2} \epsilon^{IJKL} \Phi^{KL}$$

- ALL FIELDS IN ADJOINT OF $U(N)$
- WHEN BRANES ARE AT DIFFERENT POSITIONS

$$U(N) \to U(1)^N$$

SPECIAL POINT : ALL BRANES AT SAME POSITION I.E.

$$\langle \Phi^{IJ} \rangle = 0$$

IN THIS CASE $U(N)$ IS UNBROKEN.

ALSO: LAGRANGIAN IS $\mathcal{N}=4$ SUPER YANG-MILLS

$$S = \frac{1}{g_s} \int d^4x \left(\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \dots \right) e^{-\phi(x,0)}$$

$\phi(x, y)$ IS THE DILATON. WE DEFINED IT SO THAT

x^μ TRANSVERSE COORDINATES

$$\phi(x,y) \xrightarrow[|y|\to\infty]{} 0.$$

I.E. g_s IS THE STRING COUPLING AWAY FROM THE BRANE.

$\mathcal{N}=4$ $D=4$ HAS ALL β FUNCTIONS $=0$ AND $\langle \phi^{IJ} \rangle = 0$ IMPLIES THAT THE THEORY IS CONFORMAL.

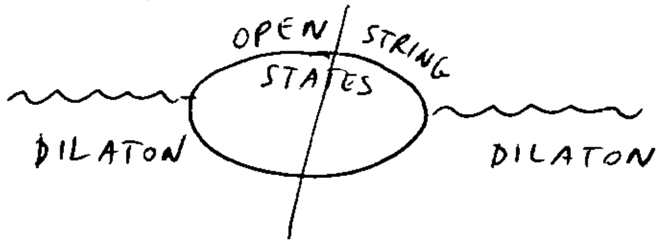
LARGE N LIMIT: $g_s = g_{YM}^2 \to 0$, $N \to \infty$ | $g_s N$ = CONSTANT $\equiv$ 't HOOFT PARAMETER

IN THIS LIMIT CLOSED STRINGS DO NOT INTERACT.
AT ENERGIES $E \ll M_s$ THE N D3 BRANE SYSTEM DESCRIBES JUST A $U(N)$ GAUGE THEORY WITHOUT GRAVITY

EXAMPLE: DILATON ABSORPTION.

$$S = S_{IIB} + S_{D3} = \frac{1}{g_s^2} \int d^4x\, d^6y \sqrt{-g}\, e^{-2\phi} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu}$$
$$+ \frac{1}{g_s N} \int d^4x\, e^{-\phi(x,0)} N \left[\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \dots \right]$$

WHEN $g_s \to 0$ ϕ INTERACTS ONLY WITH THE OPEN STRING FIELDS (A_μ + SUSY PARTNERS)

ABSORPTION $\propto$ 

$$A \sim \mathrm{Im} \int d^4x \, e^{i K_\mu X^\mu} \langle \mathrm{Tr}\, F_{\mu\nu}^2(x) \; \mathrm{Tr}\, F_{\mu\nu}^2(0) \rangle$$

NOTICE : DILATON MOMENTUM K_μ, K_m ($m = 4, \ldots, 9$)

OBEYS $K_\mu K^\mu + K_m K^m = 0$. THE 4 COMPONENTS OF THE MOMENTUM ALONG THE D3 BRANE DIRECTIONS ARE UNCONSTRAINED $\Rightarrow$ $A(K_\mu)$ IS PROPORTIONAL TO AN <u>OFF-SHELL</u> GREEN'S FUNCTION OF THE GAUGE THEORY. <u>PERTURBATIVE WHEN $g_s N \ll 1$</u>

OTHER DESCRIPTION.

D3 BRANES HAVE NONZERO TENSION AND CHARGE: THEY CURVE SPACE-TIME.

$$ds^2 = f^{-\frac{1}{2}} \underbrace{dx_\mu^2}_{0,1,2,3} + f^{\frac{1}{2}} \left(dr^2 + r^2 d\Omega_5^2 \right)$$

r: RADIAL COORD.

$d\Omega_5^2$: LINE-ELEMENT OF SPHERE OF UNIT RADIUS IN 5-D, S_5

$$A_{0123} = -\frac{1}{2}\left(f^{-1} - 1\right) \quad , \quad f = 1 + \frac{4\pi g_s N}{(r M_s)^4}$$

FOR $r \to \infty$ WE RECOVER FLAT SPACE. FOR $r \to 0$ THE METRIC IS REGULAR

$$r \to 0 \qquad f \approx \frac{4\pi g_s N}{(r M_s)^4}$$

DEFINE T COORDINATE AS $r = R\, e^{T/R}$

$$R \equiv (4\pi g_s N)^{1/4} \frac{1}{M_s}$$

METRIC: $$ds^2 = dT^2 + e^{2T/R} dx_\mu^2 + R^2 d\Omega_5^2$$

THIS IS THE METRIC OF THE HOMOGENEOUS SPACE

$AdS_5 \times S_5$

ANTI-DE-SITTER SPACE.

BOTH AdS_5 AND S_5 HAVE CONSTANT CURVATURE. CURVATURE RADIUS FOR BOTH IS R.

$$g_s N = g_{YM}^2 N \equiv x \quad (\text{'t HOOFT COUPLING})$$

$x \gg 1 \Rightarrow R \gg \frac{1}{M_s} = L_s$

THIS ENSURES THAT THE SUPERGRAVITY ACTION RECEIVES ONLY SMALL HIGHER-CURVATURE CORRECTIONS.

EQUIVALENTLY: σ-MODEL CORRECTIONS TO THE METRIC ARE NEGLIGIBLE WHEN $x \gg 1$.

CLOSED STRING LOOP CORRECTIONS NEGLIGIBLE WHEN $g_s \to 0$.

IN SUMMARY: IIB SUPERGRAVITY METRIC ACCURATE EVERYWHERE IN THE LARGE-N LIMIT FOR

$$X = g_s N \gg 1.$$

NOTICE THIS REGIME IS THE OPPOSITE OF THE PERTURBATIVE REGIME FOR GAUGE THEORY ($X \ll 1$).

FUNDAMENTAL CONSEQUENCE: WHEN $X \gg 1$ WE CAN COMPUTE THE ABSORPTION CROSS-SECTION ALSO USING CLASSICAL (SUPER) GRAVITY, SINCE THAT DESCRIPTION IS EVERYWHERE ACCURATE.

THUS:

WHEN $X \ll 1$ $\quad A$ IS $\propto$ PERTURBATIVE OFF-SHELL CORRELATOR IN SYM.

WHEN $X \gg 1$ $\quad A \propto$ CLASSICAL (GEOMETRICAL) SCATTERING AMPLITUDE

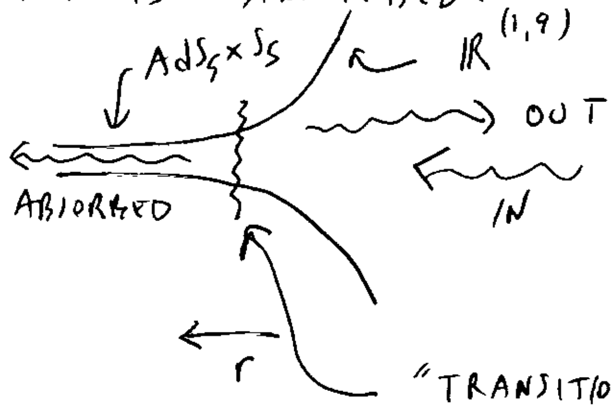
NOTICE: BY COMPUTING A AT $X \gg 1$ WE MAY BE ABLE TO SAY SOMETHING ABOUT NON-PERTURBATIVE, STRONGLY COUPLED SYM.

- LET US COMPUTE NEXT A USING CLASSICAL GEOMETRY.

DILATON IN D3 BACKGROUND OBEYS E.O.M.

$$\partial_M g^{MN}\sqrt{-g}\,\partial_N \phi = 0.$$

CLASSICALLY: A PART OF INCOMING WAVE IS REFLECTED AND A PART IS ABSORBED.



"TRANSITION POINT" BETWEEN ~ $AdS_5 \times S_5$ GEOMETRY AND FLAT $\mathbb{R}^{(1,9)}$ GEOMETRY

$$r_0 \equiv R e^{T_0/R}$$

SIMPLEST CASE: ϕ INDEPENDENT OF ANGULAR VARIABLES (CONSTANT ON S_5)

$$\phi = e^{iKX}\,\hat{\phi}(T)$$

$$\hat{\phi}(T_0) = 1 \qquad \text{FLUX IN}: \quad F_{IN} = K$$

F_{IN} − FLUX OUT: $\quad K - F_{OUT} = i\, e^{4T/R}\,\hat{\phi}^*(T)\,\partial_T \hat{\phi}(T)\Big|_{T_0}$

$$A = 1 - \frac{F_{OUT}}{F_{IN}} = \frac{i}{K}\, e^{4T/R}\,\hat{\phi}^*\partial_T\hat{\phi}\Big|_{T_0}$$

ADS/CFT CONJECTURE (MALDACENA)

IIB STRINGS ON $AdS_5 \times S_5$ $\equiv$ $SU(N)$ SYM $\mathcal{N}=4$

AT $g_s \to 0$ AT $g_{YM}^2 N = X$

$g_s N =$ CONSTANT $= X$ $g_{YM} \to 0$

FOR ANY X

0) IT IS A DUALITY : $X \ll 1$ WEAKLY COUPLED DESCRIPTION IS PERTURBATIVE SYM ; IIB ON $ADS_5 \times S_5$ IS STRONGLY COUPLED SINCE CURVATURE RADIUS $R \propto (X)^{1/4} L_s \ll L_s$

$X \gg 1$ $R \gg L_s$, IIB SUPERGRAVITY DESCRIPTION IS WEAKLY COUPLED. SYM IS STRONGLY COUPLED.

1) (SUPER) SYMMETRIES IN BOTH DESCRIPTIONS ARE THE SAME

- BOSONIC SYMMETRIES OF SYM: $SO(2,4) \otimes SU(4)$

CONFORMAL GROUP IN 4-D R-SYMMETRY

= BOSONIC SYMMETRIES OF IIB ON $AdS_5 \times S_5$ $\equiv$ ISOMETRIES.

S_5 HAS ISOMETRY GROUP $SU(4)$ (COVER OF $SO(6)$; COVER NEEDED BECAUSE IIB HAS FERMIONS)

- ISOMETRY OF AdS_5 IS $SO(2,4)$.

TO SEE THIS DEFINE AdS_5 AS HYPERBOLOID

IN $\mathbb{R}^{(2,4)}$:

$$UV + X^\mu \eta_{\mu\nu} X^\nu = -R^2 \qquad \eta_{\mu\nu} = (-1, 1, 1, 1)$$

HYPERBOLOID HAS ISOMETRY $SO(2,4)$.

PARAMETRIZE HYPERBOLOID AS :

$$X^\mu = U x^\mu \qquad V = -\frac{R^2}{U} - U x^\mu \eta_{\mu\nu} x^\nu$$

$$dV = R^2 \frac{dU}{U^2} - dU\, x^\mu \eta_{\mu\nu} x^\nu - 2U\, x^\mu \eta_{\mu\nu} dx^\nu$$

$$dU\, dV = R^2 \frac{dU^2}{U^2} - dU^2\, x^\mu \eta_{\mu\nu} x^\nu - 2\, U dU\, x^\mu \eta_{\mu\nu} dx^\nu$$

$$dX^\mu = dU\, x^\mu + U\, dx^\mu$$

$$\eta_{\mu\nu}\, dX^\mu dX^\nu = dU^2\, x^\mu \eta_{\mu\nu} x^\nu + U^2\, dx^\mu dx^\nu \eta_{\mu\nu} + 2 U dU\, x^\mu \eta_{\mu\nu} dx^\nu$$

INDUCED METRIC :

$$dU\, dV - dX^\mu dX^\nu \eta_{\mu\nu} = R^2 \frac{dU^2}{U^2} + U^2\, dx^2$$

SET $U = e^{T/R}$. METRIC $ds^2 = dT^2 + e^{2T/R} dx^2$.

= SUPERSYMMETRIES.

IN IIB ON $AdS_5 \times S_5$ 32 SUSY : $Q_\alpha{}^I \quad S_{\alpha I}$

IN SYM 32 SUSY $Q_\alpha{}^I$ + CONFORMAL SUSY $S_{\alpha I}$

SUPERGROUP IS

$$U(2,2/4)$$

2) TO MAKE FURTHER CHECKS OF THE AdS / CFT DUALITY WE MUST DEFINE IT MORE PRECISELY:

CONJECTURE: $\phi(T, x, \theta) =$ FIELD ON $AdS_5 \times S_5$
(SCALAR, FOR SIMPLICITY)

$$\Phi(T, x, \theta) = \sum_M \phi_M(T, x)\, Y^M(\theta)$$

$Y^M(\theta)$ — HARMONIC ON S_5

$\phi_M(T,x)$ — 5-D FIELD ON AdS_5

$\phi_M(T, x)$ IS A 5-D FIELD OF MASS m.

E.O.M. $$\left[-\partial_T\, e^{4T/R}\, \partial_T - e^{2T/R}\, \partial_\mu \partial^\mu + (mR)^2 e^{4T/R}\right]\phi_M = 0.$$

IN THE LIMIT $T \to +\infty$ $\quad \phi_M(T, x) \simeq e^{-\Delta T/R}\, \varphi_M(x)$

SUBSTITUTE IN E.O.M.: $\quad \Delta(\Delta - 4) = (mR)^2$
TWO ROOTS $\Delta_+ > \Delta_-$

CHOOSE THE (GENERIC) SOLUTION $e^{-\Delta_- T/R}\, \varphi_M(x)$
SINCE AdS_5 HAS A BOUNDARY, THE PARTITION FUNCTION OF IIB SUPERSTRING ON $AdS_5 \times S_5$ DEPENDS ON 4-D FIELDS $\varphi_M(x)$

$$Z_{IIB}[\varphi_M(x)]$$

NOTICE: AT $x \gg 1$ SUPERGRAVITY O.K., SEMICLASSICAL

APPROXIMATION O.K.

$$Z_{IIB}[\varphi_M(x)] \underset{X\gg 1}{\simeq} e^{-S_{IIB}[\varphi]}\Big|_{\text{ON-SHELL.}}$$

ON-SHELL MEANS S_{IIB} EVALUATED AT THE STATIONARY POINT:

$$\frac{\delta S_{IIB}}{\delta \varphi} = 0 \quad , \quad \phi_M(T,x) \underset{T\to+\infty}{\longrightarrow} e^{-\Delta_- T/R}\varphi_M(x)$$

ALSO AT $X \ll 1$

$$Z_{IIB}[\varphi_M(x)] = \langle e^{\int d^4x\, \varphi_M(x)\, O_M(x)} \rangle$$

VEV OVER SYM $\mathcal{N}=4$

(OPEN-STRING AT $E \ll M_s$)

COMPOSITE OPERATOR IN $SU(N)$ $\mathcal{N}=4$ SYM

EX: WHEN ϕ IS THE DILATON $O_0(x) = \mathrm{Tr}\, F_{\mu\nu}^2$ AS WE SAW EARLIER

CONJECTURE: THE EQUALITY HOLDS FOR ANY X.

IN PARTICULAR, AT $X \gg 1$ THE GENERATING FUNCTIONAL OF THE SYM $SU(N)$ $\mathcal{N}=4$ GAUGE THEORY IS GIVEN BY THE CLASSICAL ON SHELL ACTION OF IIB SUPERGRAVITY.

QUESTION : WHAT IS THE CONFORMAL DIMENSION OF O_M ?

ANSWER: $$ds^2 = dT^2 + R^2 e^{2T/R} dx^\mu dx^\nu \eta_{\mu\nu}$$

ISOMETRY $x^\mu \to \lambda x^\mu$ $T \to T - R\log\lambda$ ACTS AS A DILATION ON THE BOUNDARY.

THUS : $$\phi^M_\lambda (T - R\lg\lambda, \lambda x) = \phi^M (T, x)$$

$$\Rightarrow \lambda^{\Delta_-} \varphi^M_\lambda (\lambda x) = \varphi^M(x)$$

$$\varphi_\lambda{}^M(\lambda x) = \lambda^{-\Delta_-} \varphi^M(x)$$

φ^M HAS CONFORMAL DIMENSION Δ_-.

$\int d^4x\, \varphi_M(x)\, O_M(x)$ HAS CONFORMAL DIMENSION 0,

THUS O_M HAS DIMENSION $4 - \Delta_- = \Delta_+$.

- EXCEPTION TO THIS RULE :
WHEN $1 \leq \Delta_- \leq 2$ WE CAN DEFINE A SOURCE THAT BEHAVES AS

$$\phi^M (T, x) \to e^{-\Delta_+ T/R} \varphi^M(x)$$

AND STILL GET A MEANINGFUL CORRESPONDENCE WITH SYM. IN THAT CASE, THE DIMENSION OF THE OPERATOR O_M IS $\Delta_- \geq 1$; ALLOWED BY UNITARITY

SINCE UNITARY REPRESENTATIONS OF $SO(2,4)$ HAVE CONFORMAL DIMENSION ≥ 1.

EXAMPLE: BREAK $SU(N) \to U(1)^N$ (I.E. CHOOSE $\langle \Phi^{IJ} \rangle \neq 0$) THEN THE OPERATOR

$$\mathrm{Tr}\ \langle \phi^{IJ} \rangle (\phi^{IJ})^* \quad \text{HAS DIMENSION } 1.$$

WHEN $SU(N)$ UNBROKEN, $\langle \phi^{IJ} \rangle = 0$ AND THE MINIMUM CONFORMAL DIMENSION OF ANY OPERATOR IN SYM IS $\Delta = 2$.

ANOTHER CHECK OF ADS/CFT.

IN $\mathcal{N}=4$ $D=4$ THE FIELDS A_μ, λ^I, Φ^{IJ} FIT INTO A CHIRAL SUPERFIELD:

$$\Phi^{IJ} + \theta^{[I} \lambda^{J]} + \theta^I \sigma^{\mu\nu} \theta^J F_{\mu\nu} + \dots$$

SUPERSPACE (GRASSMANN) COORDINATE

NO $\bar{\theta}_I$ IN THIS EXPANSION.

$[IJ] = 6_R$ OF $SU(4)$ = VECTOR REP. OF $SO(6)$

LABEL 6 OF $SO(6)$ WITH INDEX A.

SUPERFIELD: $\quad \mathbf{\Phi}^A = \Phi^A + \dots$

CONFORMAL DIMENSION $\Delta = 1$

IN $\square$ OF $SO(6)$

$\mathbf{\Phi}^A$ IS THE SIMPLEST EXAMPLE OF CHIRAL (SHORT)

MULTIPLET OF $U(2,2/4)$. IN GENERAL THESE SHORT MULTIPLETS BELONG TO THE n-TIMES SYMMETRIC IRREP OF $SO(6)$: $\underbrace{\boxed{}\boxed{}\cdots\boxed{}}_{n}$
AND HAVE CONFORMAL DIMENSION $\Delta = n$.

$$\Phi_{A_1 \ldots A_n} = \phi_{A_1 \ldots A_n} + \ldots$$

$$\mathrm{Tr}\, \phi^{(A_1} \cdots \phi^{A_n)\,\mathrm{TRACELESS}} \quad \text{IS CHIRAL.}$$

<u>IMPORTANT CONSEQUENCE</u>:

AT $x \equiv g_{YM}^2 N = 0$ CONFORMAL DIMENSION OF THIS MULTIPLET IS $\Delta = n$
BUT SINCE Δ IS PROTECTED, BECAUSE THE MULTIPLET IS CHIRAL, $\Delta = n$ ALSO AT $x \gg 1$.

ADS/CFT CONJECTURES THAT AT $x \gg 1$ $\mathcal{N}=4$ SYM IS THE SAME AS SEMICLASSICAL IIB SUPERGRAVITY ON $AdS_5 \times S_5$.
WE MUST FIND IN IIB SUGRA ALL CHIRAL MULTIPLETS OF $\mathcal{N}=4$ SYM.
IN PARTICULAR SCALARS WITH MASS m SUCH THAT:

$$n(n-4) = (Rm)^2 \qquad \text{FOR } n = 1, 2, 3, \ldots$$

+ SUPERPARTNERS.

THEY EXIST! THEY ARE THE KALUZA-KLEIN MODES OF IIB ON $AdS_5 \times S_5$ COMPUTED IN PHYS. REV D32 389 (1985)

EXAMPLE: $\mathrm{Tr}\, \Phi^{(A} \Phi^{B)}{}_{T}$

THE SCALAR COMPONENTS OF THE MULTIPLET ARE

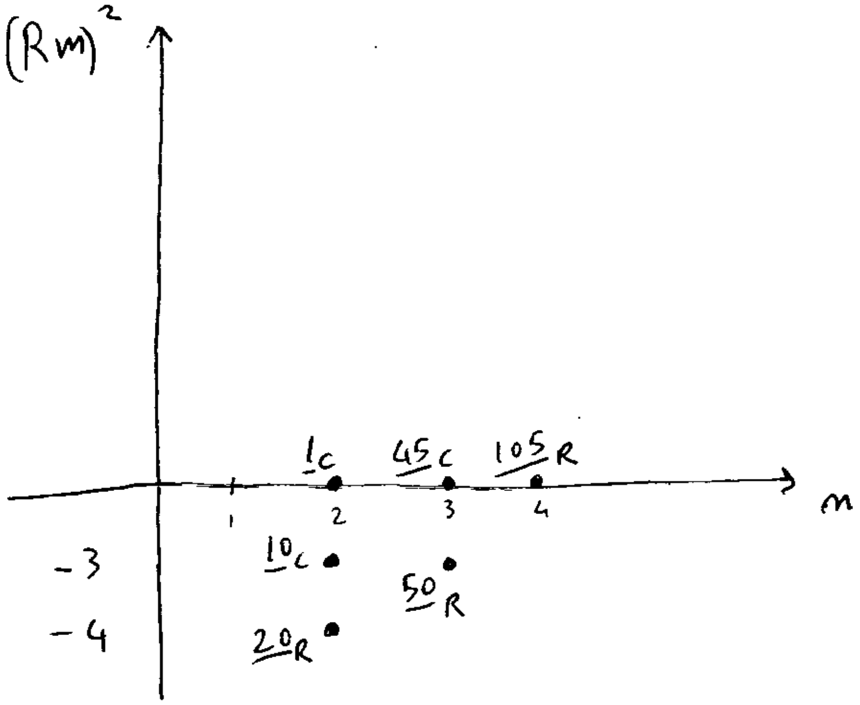
$\mathrm{Tr}\, \phi^{(A} \phi^{B)}{}_{T}$, $\Delta = 2$, IN $\underline{20}_R$ OF $SU(4) \times SO(6)$

$\mathrm{Tr}\, \lambda^I \lambda^J$, $\Delta = 3$, IN $\underline{10}_C$ OF $SU(4)$

$\mathcal{L}_{\mathcal{N}=4} + i\, \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu}$, $\Delta = 4$, IN $\underline{1}_C$ OF $SU(4)$

$\uparrow$ $\mathcal{N}=4$ LAGRANGIAN

SPECTRUM OF KK MODES



$$\left.\begin{array}{lll} \Delta(\Delta-4) = -4 & \Rightarrow & \Delta_+ = 2 \\ \Delta(\Delta-4) = -3 & \Rightarrow & \Delta_+ = 3 \\ \Delta(\Delta-4) = 0 & \Rightarrow & \Delta_+ = 4 \end{array}\right\} \text{O.K.}$$

OTHER CONSISTENCY CHECKS

1) R-SYMMETRY $SU(4)$ IS ANOMALOUS IN SYM SINCE λ_α^I ARE CHIRAL FERMIONS IN THE 4 OF $SU(4)$.

- IN SUPERGRAVITY ON $AdS_5 \times S_5$ THERE EXIST GAUGE VECTORS OF $SO(6) \sim SU(4)$. THEY ARE SOURCES FOR J_μ^{IJ} ($SU(4)$ R-CURRENT).

$$\langle e^{-\int d^4x \, A_\mu^{IJ} J^\mu_{IJ}} \rangle \equiv e^{-W[A]}$$

UNDER $SU(4)$ $\quad \delta W = \frac{N}{16\pi^2} \int d^4x \, \mathrm{Tr}_{SU(4)} \, \omega \, F \wedge F$

ADS/CFT : $W[A] = S_{IIB}$ ON $AdS_5 \times S_5$

$$S_{IIB} = \dots + \frac{N}{16\pi^2} \int d^5x \; C$$

$C =$ CHERN-SIMONS TERM $= \mathrm{Tr}_{SU(4)} \, A \wedge dA \wedge dA + \dots$

$$\delta S_{IIB} = \frac{N}{16\pi^2} \int d^5x \; d\left[\mathrm{Tr}_{SU(4)} \, \omega \wedge F \wedge F \right] = \frac{N}{16\pi^2} \int d^4x \, \mathrm{Tr} \, \omega \wedge F \wedge F$$

2) SCALE TRANSFORMATION IS ALSO ANOMALOUS

$$\int d^4x \sqrt{g} \, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} W = \text{CONST} \cdot E_4 + \text{CONST}' \, A$$

$E_4 =$ EULER DENSITY $= \int d^4x \sqrt{g} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right)$

$A = \int d^4x \sqrt{g} \; C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$ $\qquad C_{\mu\nu\rho\sigma} =$ WEYL TENSOR

HOW THE ANOMALY WORKS IN S_{IIB}? ON $AdS_5 \times S_5$:

$$S_{IIB} = \frac{M_S^8}{g_s^2} V_{S_5} \int d^5x \sqrt{-g}\,(R - 2\Lambda) + \ldots$$

$V_{S_5} \sim R^5$, EINSTEIN EQS.: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -g_{\mu\nu}\Lambda$

$$\Downarrow$$

$$\left(1 - \tfrac{5}{2}\right) R = -5\Lambda$$

$$R = \tfrac{10}{3}\Lambda$$

$$S_{IIB} = \frac{4}{3} \frac{M_S^8}{g_s^2} R^5 \int d^5x \sqrt{-g}\,\Lambda \quad \text{ON SHELL}$$

$$\int d^5x \sqrt{-g} = \int_{-\infty}^{+\infty} e^{4T/R}\, dT \int d^4x$$

THUS THE INTEGRAL DIVERGES MUST BE REGOLARIZED BY CUTTING OFF THE INTEGRAL IN T I.E. INTEGRATING ONLY UP TO T_0

$$S_{IIB}^{T_0} = \int_{-\infty}^{T_0} dT \int d^4x \sqrt{-g} \left(-\frac{4}{3} \frac{M_S^8 R^5}{g_s^2} \frac{6}{R^2} \right) \qquad \Lambda = -\frac{6}{R^2}$$

NEAR AdS_5 BOUNDARY AT $T = +\infty$ ANY METRIC CAN BE CAST IN THE FORM

$$ds^2 = dT^2 + g_{\mu\nu}(T, x)\, dx^\mu dx^\nu$$

$$g_{\mu\nu}(T, x) = e^{2T/R} g_{\mu\nu}^{(0)}(x) + g_{\mu\nu}^{(1)}(x) + \frac{T}{R} e^{-2T/R} g_{\mu\nu}^{(2)}(x) + e^{-2T/R} h_{\mu\nu}(x)$$

$g_{\mu\nu}^{(0)}(x)$ = BOUNDARY VALUE OF THE METRIC

$g^{(1)}_{\mu\nu}(x)$, $g^{(2)}_{\mu\nu}(x)$ = LOCAL FUNCTIONS OF $g^{(0)}_{\mu\nu}(x)$ ONLY.

$h_{\mu\nu}$ DEPENDS ON THE METRIC INSIDE AdS_5

$$S^{T_0}_{IIB} = CR\, e^{4T_0/R}\, a_0 + CR\, e^{2T_0} a_1 + C T_0\, a_2 + S_{FINITE}$$

S_{FINITE} INDEPENDENT OF T_0 IN THE LIMIT $T_0 \to \infty$

$$a_0 = \int d^4x \sqrt{-g^0}\,, \qquad a_1 = R^2 \int d^4x \sqrt{-g^0}\, R(g^0)$$

$$a_2 = R^4 \int d^4x \sqrt{-g^0} \left(\frac{1}{3} R^2 - R_{\mu\nu} R^{\mu\nu} \right)$$

$$C = -8 M_s^8 R^3 / g_s^2$$

ASYMPTOTIC FORM OF THE METRIC $\sim e^{2T_0/R}\, g^{(0)}_{\mu\nu}(x)$

$\Rightarrow$

$$R \frac{\partial}{\partial T^0} = \int d^4x \sqrt{-g}\, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}}$$

$$R \frac{\partial}{\partial T^0} S_{IIB} = CR\, a_2 + 2CR\, e^{2T_0/R} a_1 + 4CR\, e^{4T_0/R} a_0$$

$$\int d^4x \sqrt{-g}\, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} S_{IIB} = 2CR\, e^{2T_0/R} a_1 + 4CR\, e^{4T_0/R} a_0 + \int d^4x \sqrt{g}\, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} S_{FINITE}$$

THUS:

$$\int d^4x \sqrt{-g}\, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} S_{FINITE} = \underbrace{-8 M_s^8 R^8 / g_s^2}_{-128\pi^2 N^2}\; a_2$$

$$R = \frac{1}{M_s} (4\pi g_s N)^{1/4}$$

CENTRAL CHARGE

PROPORTIONAL TO # DEGREES OF FREEDOM OF SYM

TWO SAMPLE DYNAMICAL CALCULATIONS

I) TWO POINT FUNCTION OF A SCALAR OPERATOR

$O(x)$ = OPERATOR OF DIMENSION Δ.

SOURCE $\phi(x)$ OF DIMENSION $4-\Delta$.

IN ADS/CFT $\exists$ 5-D FIELD $\phi(T,x)$ SUCH THAT

$$\phi(T,x) \xrightarrow[T\to\infty]{} e^{-(4-\Delta)T}\phi(x).$$

SIMPLEST CASE: $\phi(T,x)$ FREE SCALAR OF MASS

$$(mR)^2 = \Delta(\Delta-4)$$

ACTION:

$$S = \int_{AdS_5} d^5x\sqrt{-g}\,[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2]$$

E.O.M.:

$$\left[-\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\,g^{\mu\nu}\partial_\nu + m^2\right]\phi = 0$$

$$S\Big|_{\text{ON-SHELL}} = \int_{\partial AdS_5} d^4x\sqrt{g}\,g^{TT}\partial_T\phi\,\phi$$

SET: $\phi(T,x) = \int\frac{d^4k}{(2\pi)^4}e^{ikx}\,\tilde{\phi}_K(x)\,\varphi_K$

TO SOLVE THE E.O.M. DEFINE NEW COORDINATE z:

$$e^{T/R} = R/z$$

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + dx^\mu dx^\nu \eta_{\mu\nu}\right)$$

IN TERMS OF NEW COORDINATES E.O.M. READ

$$\left[-\partial_z \left(\frac{R}{z}\right)^3 \partial_z + K^2 \frac{R^3}{z^3} + m^2\left(\frac{R}{z}\right)^5\right]\tilde{\phi}_K(z) = 0$$

SET : $\tilde{\phi}_K(z) = z^2 \psi_K(z)$

$$\partial_z \frac{1}{z^3}\partial_z\left(z^2\psi_K\right) = \partial_z \frac{1}{z^3}\left(2z\psi_K + z^2\partial_z\psi_K\right)$$

$$= \partial_z\left(\frac{2}{z^2}\psi_K + \frac{1}{z}\partial_z\psi_K\right) = -\frac{4}{z^3}\psi_K + \frac{1}{z^2}\partial_z\psi_K + \frac{1}{z}\partial_z^2\psi_K$$

EQUATION FOR ψ_K : $z^2\partial_z^2\psi_K + z\partial_z\psi_K - 4\psi_K - K^2z^2\psi_K$

$$-(mR)^2\psi_K = 0$$

EUCLIDEAN MOMENTA : $K^2 \geq 0$ DEFINE $Kz = y$

$$\left\{y^2\partial_y^2 + y\partial_y - y^2 - \left[(mR)^2 + 4\right]\right\}\psi_K = 0$$

MODIFIED BESSEL EQ.

THE SOLUTION THAT VANISHES AT $y \to \infty$ IS $\psi_K = K_\nu(y)$

$$\nu = \sqrt{4 + (mR)^2} = \Delta - 2$$

$$\tilde{\phi}_K = z^2 K_\nu(Kz) \underset{z\to 0}{\simeq} z^{4-\Delta} K^{2-\Delta}$$

NORMALIZE TO $z^{4-\Delta}$

$$\tilde{\phi}_K(z) = \frac{2^{3-\Delta}}{\Gamma(\Delta-2)} K^{\Delta-2} z^2 K_\nu(kz)$$

FOR z SMALL THE EXPANSION OF READS

$$K_\nu(kz) = z^{4-\Delta} + \sum_{n<\Delta-2} \alpha_n K^{2n} z^{4-\Delta+2n}$$

$$+ \frac{2^{-2\Delta+4}\pi}{\sin\nu\pi} \frac{1}{\Gamma(\nu)\Gamma(\nu+1)} K^{2\Delta-4} z^{\Delta}$$

$$+ O(z^{\Delta+2})$$

FROM AdS/CFT

$$\frac{\delta^2}{\delta\varphi_K\,\delta\phi(0)} S\Big|_{\text{ON-SHELL}} = \langle \tilde{O}(k)\ O(0)\rangle$$

$$= \lim_{z\to 0} \left(\frac{R}{z}\right)^3 \tilde{\phi}^*_K(z)\, \partial_z\, \tilde{\phi}_K(z)$$

$$= \lim_{z\to 0} \left[\left(\frac{R}{z}\right)^3 z^{4-\Delta} \frac{2^{4-2\Delta}\pi}{\sin\nu\pi} \frac{1}{\Gamma(\nu)\Gamma(\nu+1)} K^{2\Delta-4} \partial_z z^{\Delta} \right.$$

$$\left. + P(K^2) \right]$$

POLYNOMIAL IN K^2 <u>LOCAL</u> DIVERGENT TERM = HARMLESS CONTACT TERM

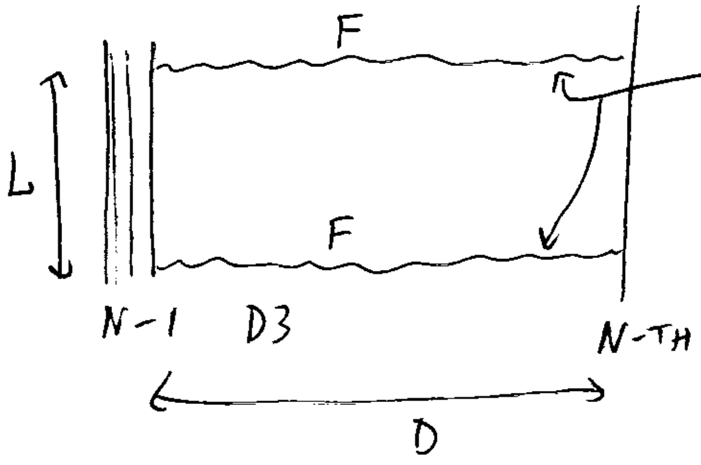
$$\langle \tilde{O}(k)\, O(0) \rangle = \text{CONST} \cdot K^{2\Delta - 4} + \text{CONTACT TERM}$$

O.K. WITH CONFORMAL INVARIANCE

DYNAMICAL INFORMATION.

II) WILSON LOOP

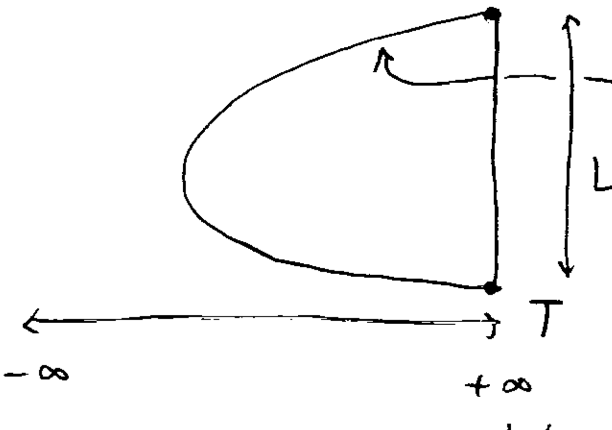
AT $g_s = 0$



F-STRINGS : ELECTRICALLY CHARGED STATES OF MASS $M_s^2 D$ $\equiv$ MASSIVE "W's"

ENERGY OF THIS CONFIGURATION : $E = g N/L$ (COULOMB)

AdS DUAL PICTURE AT $g_s \neq 0$ $\quad g_s N \gg 1$



MINIMAL ENERGY STRING.

$$E_{\text{STRING}} \approx M_s^2 \int_{-L/2}^{L/2} dx \sqrt{e^{2T/R}\left[(\partial_x T)^2 + e^{2T/R}\right]}$$

CHANGE VARIABLE : $U = R e^{T/R}$

$$E_{STRING} \approx 2 M_S^2 \int_0^{L/2} dx \sqrt{(\partial_x U)^2 + U^4/R^4}$$

IT DIVERGES. WE MUST SUBTRACT THE MASS OF THE 2 W'S

$$E - 2M_W = 2 M_S^2 \int_0^{L/2} dx \left[\sqrt{(\partial_x U)^2 + U^4/R^4} - \partial_x U \right]$$

DEFINE DIMENSIONLESS VARIABLES $U = \frac{R^2}{L} \hat{U}$, $x = L \hat{x}$

$$E - 2M_W = 2 M_S^2 L \frac{R^2}{L^2} \underbrace{\int_0^{\frac{1}{2}} d\hat{x} \left[\sqrt{(\partial_{\hat{x}} \hat{U})^2 + \hat{U}^4} - \partial_{\hat{x}} \hat{U} \right]}_{\text{DIMENSIONLESS CONSTANT, } \underline{\text{INDEPENDENT}} \text{ OF } R \,\&\, L\,!}$$

$$R^2 = (4\pi g_s N)^{1/2} \frac{1}{M_s^2}$$

$$E - 2M_W = \text{CONST} \cdot (g_s N)^{1/2} \frac{1}{L}$$

COULOMB

$= \sqrt{g_{YM}^2 N}$ <u>VERY</u> NON-PERTURBATIVE !

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A GLIMPSE OF NON-CONFORMAL HOLOGRAPHY

DEFORMATIONS OF $\mathcal{N}=4$.

IF WE WANT $\mathcal{N}=4$ IN THE UV WE MUST DEFORM WITH OPERATORS WITH UV DIMENSION < 4 (SCALAR TO PRESERVE LORENTZ)

$$\mathcal{L}_{\mathcal{N}=4} \to \mathcal{L}_{\mathcal{N}=4} + \lambda \mathcal{O}$$

$$\Delta = 2 \quad : \quad \mathcal{O} = \mathrm{Tr}\, \phi^{(I} \phi^{J)_T} \qquad I,J = 1,..6 \quad \underline{20}_R$$

$$\Delta = 3 \qquad \mathcal{O} = \mathrm{Tr}\, \lambda^i \lambda^j + O(\phi^3) \quad i,j=1,..,4 \quad \underline{10}_C$$

IIB SUGRA ON $AdS_5 \times S_5$ IS COMPLEX, BUT FOR THESE DEFORMATIONS WE CAN STUDY INSTEAD $d=5$ GAUGED SUPERGRAVITY.

FIELDS : $g_{\mu\nu}$ + 42 SCALARS + $SU(4)$ GAUGE FIELDS + ...

$$\mathcal{L}_{d=4 \text{ SUGRA}} = R(g) + G_{\Sigma\Pi}(\lambda) D_\mu \lambda^\Sigma D^\mu \lambda^\Pi + V(\lambda) + ..$$

SCALARS

4-d POINCARÈ INVARIANT METRIC

$$ds^2 = dT^2 + e^{2\phi(T)} dx^\mu dx^\nu \eta_{\mu\nu} \qquad \lambda = \lambda(T)$$

E.O.M. : $\frac{d}{dT}\left[G_{\Sigma\Pi} \dot{\lambda}^\Pi\right] + 4\dot{\phi}\, G_{\Sigma\Pi} \dot{\lambda}^\Pi = \frac{\partial V}{\partial \lambda^\Sigma}$

5-D NEWTON CONSTANT

$$G_{\mu\nu} = 8\pi\, G_5\, T_{\mu\nu}$$

$$\Rightarrow \quad G_{TT} = 6\dot{\phi}^2 = G_{\Sigma\Pi}\,\dot{\lambda}^\Sigma \dot{\lambda}^\Pi - 2V \leftarrow T_{TT}$$

$$G_{00} = -3\left(2\dot{\phi}^2 + \ddot{\phi}\right) e^{2\phi} = G_{\Sigma\Pi}\,\dot{\lambda}^\Sigma \dot{\lambda}^\Pi + 2V \leftarrow T_{00}$$

SOLUTIONS : $\dot{\lambda}^\Pi = 0$, $\dfrac{\partial V}{\partial \lambda^\Pi} = 0$ $\quad \phi = T/R \quad (AdS_5)$

CONFORMALLY INVARIANT SOLUTION

$$R^3 \sim \dot{\phi}^{-3} \sim G_{TT}^{-3} = \text{CENTRAL CHARGE}$$

C-THEOREM: CENTRAL CHARGE IS MONOTONIC IN T

LIGHT-LIKE VECTOR: $V^\mu = (\, e^{-\phi},\ 1,\ 0,\ 0,\ 0)$

$$g_{\mu\nu} V^\mu V^\nu = -\, e^{-2\phi} e^{2\phi} + 1 \cdot 1 = 0. \qquad \underset{V^0}{\uparrow} \quad \underset{V^T}{\uparrow}$$

NULL POSITIVITY (<u>WEAKER</u> THAN WEAK ENERGY CONDITION)

$$V^\mu G_{\mu\nu} V^\nu \geq 0 \quad \Rightarrow \quad V^0 G_{00} V^0 + V^T G_{TT} V^T =$$

$$= e^{-2\phi} G_{00} + G_{TT} = -3\ddot{\phi} \geq 0$$

$$\ddot{\phi} \leq 0 \quad \Rightarrow \quad \frac{d}{dT}\left(\dot{\phi}^{-3}\right) = -3\ddot{\phi}\left(\dot{\phi}\right)^{-4} \geq 0$$

CENTRAL CHARGE INCREASES MONOTONICALLY FROM IR ($T = -\infty$) TO UV ($T = +\infty$)

HOLOGRAPHIC C EXISTS . DOES AN R.G. EQUATION EXISTS.

DEFINE $\mu = \frac{1}{R} e^{\phi(T)}$. CUT OFF SUGRA ACTION BY IMPOSING $\mu \le \Lambda$

$$ds^2 = \left(\mu^{-1} \frac{dT}{d\phi}\right)^2 d\mu^2 + \mu^2 g_{\mu\nu}(\mu, x)\, dx^\mu dx^\nu$$

$$g_{\mu\nu}(\mu, x) \underset{|x|\to\infty}{\longrightarrow} \eta_{\mu\nu}$$

DEFINE "BARE" λ^Σ_B, $g_{\mu\nu}^B(x)$ AS FOLLOWS

$$\lambda^\Sigma_B = \lambda^\Sigma(\mu = \Lambda, x) \qquad g_{\mu\nu}^B(x) = g_{\mu\nu}(\mu = \Lambda, x)$$

$S = \int d\mu\, \mathcal{L}[g_{\mu\nu}, \lambda]$ IS INDEPENDENT OF μ:

$$0 = \mu \frac{d}{d\mu} S = \mu \frac{\partial}{\partial\mu} S + \int d^4x \left[\mu \frac{d\lambda^\Sigma}{d\mu} \frac{\delta S}{\delta \lambda^\Sigma} + \mu \frac{d g_{\mu\nu}}{d\mu} \frac{\delta S}{\delta g_{\mu\nu}}\right]$$

DEFINE $\lambda^\Sigma_R = \lambda^\Sigma(\mu, x)$ "RENORMALIZED FIELD".
THE EQUATION FOR S IS NOT A TAUTOLOGY
ONCE THE EQUATIONS FOR λ^Σ_R AND $g^R_{\mu\nu} \equiv g_{\mu\nu}(\mu, x)$.

$\dot\lambda^\Sigma_R = \mu \frac{d}{d\mu} \lambda^\Sigma_R$ BY DEFINITION ARE THE β FUNCTIONS

WE OBTAIN THEM AS FOLLOWS $S = S_{UV}(\mu) + S_{IR}(\mu)$

$$S_{UV}[\mu] = \int d^4x \int_\mu^\Lambda d\hat\mu\, \mathcal{L}(\hat\mu, x), \quad S_{IR}[\mu] = \int d^4x \int_{\mu_{MIN}}^{\mu} d\hat\mu\, \mathcal{L}(\hat\mu, x)$$

$$\underbrace{G_{\Sigma\pi} \dot\lambda^\Sigma_R = -\frac{\delta S_{UV}}{\delta \lambda^\pi} \qquad \dot g^R_{\mu\nu} = -\frac{\delta S_{UV}}{\delta g_R^{\mu\nu}}}_{\beta\text{-FUNCTIONS}}$$

HOW DO WE FIND CONDENSATES $\langle O \rangle \neq 0$?

$\mathcal{L} \to \mathcal{L} + \hat{\lambda} O$, $\hat{\lambda}$ IS THE BOUNDARY VALUE OF SCALAR λ :

$$\lambda \underset{z \to 0}{\longrightarrow} \hat{\lambda} z^{4-\Delta} + \dots$$

E.O.M. FOR λ : $\left[-\partial_z \left(\frac{R}{z}\right)^3 \partial_z + m^2 \left(\frac{R}{z}\right)^5 \right] \lambda = 0$

$$\Delta(\Delta - 4) = (Rm)^2$$

$$S \propto \int_{z_0}^{\epsilon} dz \left(\frac{R}{z}\right)^5 \left[\left(\frac{z}{R}\right)^2 (\partial_z \lambda)^2 + m^2 \lambda^2 + \dots \right]$$

z_0 = IR BOUNDARY. THERE $\partial_z \lambda = 0$

$$S \Big|_{\text{ON-SHELL}} = \lim_{z \to 0} \left(\frac{R}{z}\right)^3 \lambda \partial_z \lambda \Big|_{z=\epsilon}$$

HOLOGRAPHY :

$$S[\lambda] \Big|_{\text{ON-SHELL}} = W[\hat{\lambda}]$$

AND: $\left. \frac{\partial W}{\partial \hat{\lambda}} \right|_{\hat{\lambda}=0} = \langle O \rangle$

$$S \Big|_{\text{ON-SHELL}} = \lim_{\epsilon \to 0} \left(\frac{R}{\epsilon}\right)^3 \left[\hat{\lambda} \epsilon^{4-\Delta} + B \epsilon^{\Delta} \right] \partial_\epsilon \left[\hat{\lambda} \epsilon^{4-\Delta} + B \epsilon^{\Delta} \right]$$

$$= R^3 \left[\hat{\lambda}^2 \epsilon^{4-2\Delta} + (4 - \Delta + \Delta) \hat{\lambda} B + \Delta B^2 \epsilon^{2\Delta - 4} \right]$$

$\Delta < 4$ (UV SOFT OPERATOR)

$$\left. \frac{\partial S}{\partial \hat{\lambda}} \right|_{\hat{\lambda}=0} = \left. \frac{\partial W}{\partial \hat{\lambda}} \right|_{\hat{\lambda}=0} = \langle O \rangle = 4 R^3 B \quad \text{CONDENSATE}$$

GOLDSTONE THEOREM.

ADD 5-D GAUGE FIELD $\hat{\lambda} = 0$.

$$\lambda(z) = z^{\Delta}\left[B + \int d^4k \, \tilde{A}^*_\mu(k)\, C^\mu(k) \right]$$

$$\langle O \rangle_{\tilde{A}_\mu} = 4\left[B + \int d^4k \, \tilde{A}^*_\mu(k)\, C^\mu(k) \right]$$

BY CONSTRUCTION $\dfrac{\delta}{\delta \tilde{A}^*_\mu(k)} \langle O \rangle = \langle O \; \tilde{J}^\mu(k) \rangle$

CHOOSE:

$$\tilde{A}_\mu(k) = i k_\mu \, \tilde{\Lambda}(k)$$

CANCEL A_μ WITH A GAUGE TRANSFORMATION

$$A_\mu \to A_\mu - \partial_\mu \Lambda = 0$$

NOTE 5-D GAUGE TRANSFORMATION ACTS AS 4-D GLOBAL SYMMETRY ($SU(4)$ IS R-SYMMETRY IN $\mathcal{N}=4$ CASE)

$$B \to B + \int d^4x \; \delta B(k)\, \tilde{\Lambda}^*(k) + O(\tilde{\Lambda}^2)$$

$$\frac{\delta}{\delta \tilde{\Lambda}^*} \langle O \rangle = -i k_\mu \langle O \; \tilde{J}^\mu(k) \rangle = \delta B(k)$$

$$\lim_{k \to 0} \delta B(k) = \delta B \neq 0$$ (SPONTANEOUS SYMMETRY BREAKING)

$$\langle O \; \tilde{J}(k) \rangle = C(k) k^\mu$$ (LORENTZ)

$\lim -i k^\mu k_\mu C(k) = \delta B \neq 0$ ⇒ $C(k)$ HAS A POLE AT $k^2 = 0$

⇒ MASSLESS PARTICLE (GOLDSTONE THEOREM)

MASS GAP

λ = 5-D MASSLESS FIELD $\quad ds^2 = e^{-2\phi(z)}(dz^2 + dx^\mu dx^\nu \eta_{\mu\nu})$

$$S = \int d^4x \int^{z_0} dz\, e^{-3\phi(z)} \left(\partial_z \lambda\, \partial_z \lambda + \partial_\mu \lambda\, \partial_\nu \lambda\, \eta^{\mu\nu}\right)$$

$$\tilde{\lambda}(z, k) = \hat{\lambda}(k)\Big[\,1 + \alpha k^2 z^2 + \beta k^4 z^4 \lg z/R + z^4 A(k)\Big]$$

$$S\Big|_{\text{ON-SHELL}} = \lim_{z\to 0} e^{-3\phi(z)}\, \tilde{\lambda}^*(z)\, \partial_z \tilde{\lambda}(z)$$

$$e^{\phi(z)} \xrightarrow[z\to 0]{} z/R \qquad = R^3\, 4\, |\hat{\lambda}(k)|^2 A(k) + P(k^2)$$

CONTACT TERMS

$A(k) = \langle \tilde{O}(k)\, O(0)\rangle$ $\Rightarrow$ POLES OF $A(k)$ IMPLY THAT 2-POINT FUNCTION HAS A POLE I.E. A PARTICLE OF MASS $m^2 = -k^2\big|_{\text{POLE}}$.

POLE : $A(k) \xrightarrow[-k^2\to m^2]{} \infty \qquad \dfrac{1}{A(k)}\, \tilde{\lambda}(z,k) \xrightarrow[k^2\to -m^2]{} z^4 \hat{\lambda}(k)$.

$$\exists\ \tilde{\lambda} \underset{z\to 0}{\sim} z^4$$

AT z_0 (IR) $\quad \partial_z \tilde{\lambda} = 0 \quad \Rightarrow \quad e^{-3\phi(z)} \tilde{\lambda}^* \partial_z \tilde{\lambda}\Big|_{z_0} = 0$.

THUS :

$$S = \int dz\, e^{-3\phi(z)}\left[\partial_z \tilde{\lambda}^* \partial_z \tilde{\lambda} - k^2 |\tilde{\lambda}|^2\right] =$$

$$= e^{-3A(z)}\, \tilde{\lambda}^*\, \partial_z \tilde{\lambda}\,\Big|_0^{z_0} = 0$$

$\Rightarrow$ $k^2 < 0$ STRICTLY : MASS GAP

AN EXACT $\mathcal{N}=1$ R.G. FLOW

$\mathcal{N}=4 \longrightarrow \mathcal{N}=1$

WRITE $\mathcal{N}=4$ IN $\mathcal{N}=1$ SUPERFIELDS.

$$S_{\mathcal{N}=4} = \mathrm{Tr}\Big[\int d^4\theta\,(S+\bar{S})\,\Phi^+ e^V \Phi + S\int d^2\theta\, W_\alpha W^\alpha$$
$$+ S\int d^2\theta\; \Phi^I \Phi^J \phi^K \epsilon_{IJK}\Big] \qquad I,J,K=1,2,3.$$

$$S \equiv \frac{4\pi}{g_{YM}^2} + i\,\theta/2\pi$$

$V =$ VECTOR MULTIPLET $\qquad \Phi^I = 3$ CHIRAL MULTIPLETS.

BREAK $\mathcal{N}=4$ TO $\mathcal{N}=1$ BY ADDING $m^{IJ}\int d^2\theta\, \mathrm{Tr}\,\Phi^I\Phi^J$

EXAMPLE: $m^{IJ} = \delta^{IJ} m$.

PURE $\mathcal{N}=1$ SYM AT $E \ll m$.

$\mathcal{N}=4$ INTERPRETATION: $\int d^2\theta\, \mathrm{Tr}\,\phi^I\phi^J \sim \mathrm{Tr}\,\lambda^I\lambda^J \in \underline{10}_c$ OF $SU(4)$

IT BELONGS TO THE $\underline{6}$ OF $SU(3)$ INSIDE $\underline{10}$ OF $SU(4)$

$$\underline{10} \longrightarrow \underline{6} + \underline{3} + 1$$

$m^{IJ} = \delta^{IJ} m \quad = 1$ OF $SO(3) \subset SU(3) \qquad \underline{6} \to \underline{1} + \underline{5}$

HOLOGRAPHIC $\mathcal{N}=1$ FLOW: IN 5-D GAUGED SUGRA WE MUST PRESERVE $\mathcal{N}=1$. WE NEED:

$$\delta\psi_\mu = 0 \qquad \delta\psi = 0$$

5-D FERMIONS

WE CAN SET TO ZERO ALL SCALARS EXCEPT 2 : m, σ

$$\underline{10} \to \underbrace{\underline{6}}_{\to\, 1 + \underline{5}} + \underline{3} + \underline{1} \leftarrow \sigma \text{ SINGLET}$$

$$\underline{6} \to 1 + \underline{5}$$

$1 \leftarrow m$ SINGLET

σ = SOURCE FOR $\mathrm{Tr}\, \lambda^4 \lambda^4 = \mathrm{Tr}\, \lambda\lambda$ (GAUGINO CONDENSATE)

$$S \to S + m \int d^2\theta\, \mathrm{Tr}\, \lambda^I \lambda^I + \sigma\, \mathrm{Tr}\, W_\alpha W^\alpha \Big|_{\theta=0}$$

$\mathcal{N}=1$ FLOW : $\delta\psi_\mu = \delta\psi = 0$

METRIC: $ds^2 = dT^2 + R^2 e^{-2\phi(T)} dx^\mu dx^\nu \eta_{\mu\nu}$

POTENTIAL:

$$V = \frac{1}{8} \sum_\Sigma \left| \frac{\partial W}{\partial \lambda^\Sigma} \right|^2 - \frac{1}{3} |W|^2$$

$\mathcal{N}=1$ FLOW EQUATIONS: $\dfrac{d\lambda^\Sigma}{dT} = \dfrac{1}{2} \dfrac{\partial W}{\partial \lambda^\Sigma}$, $\dot{\phi} = -\dfrac{1}{3} W$

KEEP ONLY m & σ : $W = \frac{3}{4}\left(\cosh \frac{2m}{\sqrt{3}} + \cosh 2\sigma\right)$

SOLUTION:

$$m(T) = \frac{\sqrt{3}}{2} \log \frac{1 + e^{-(T-C_1)/R}}{1 - e^{-(T-C_1)/R}}, \quad \sigma(T) = \frac{1}{2} \log \frac{1 + e^{-3(T-C_2)/R}}{1 - e^{-3(T-C_2)/R}}$$

$$m(T) \xrightarrow[T \to +\infty]{} \sqrt{3}\, e^{-T/R}\, e^{C_1/R}$$

COEFFICIENT OF DEFORMATION

$4 - \Delta = 1$, $\Delta = 3$ O.K.

$$\sigma(T) \xrightarrow[T \to +\infty]{} \underbrace{e^{-3T/R}}_{\approx z^3}\, e^{3C_2/R} \longleftarrow$$

GAUGINO CONDENSATE $\langle \mathrm{Tr}\, \lambda^4 \lambda^4 \rangle \neq 0$

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