D-BRANES AND SUPERSYMMETRY

II A OR II B HAVE INDEPENDENT LEFT & RIGHT MOVERS:
2 FERMIONIC SECTORS $|R\rangle \otimes |\widetilde{NS}\rangle + |NS\rangle \otimes |\widetilde{R}\rangle$
10-D SUSY:

$$|A\rangle \in |NS\rangle \xrightarrow{Q_\alpha} |A'_\alpha\rangle \in |R\rangle$$

$Q_\alpha |A\rangle = |A'_\alpha\rangle$ $\quad Q_\alpha$ IS SUSY GENERATOR.

IN II A, II B $\quad Q_\alpha, \tilde{Q}_\alpha \quad$ 2 (MAJORANA) VECTORS

OPEN STRINGS: $\psi_\mu = \pm \tilde{\psi}_\mu \qquad \alpha_\mu = \pm \tilde{\alpha}_\mu$
ONE RAMOND & ONE Q_α
IN THE PRESENCE OF OPEN-STRING SECTOR SUSY IS $\frac{1}{2}$ OF CLOSED STRING.
SAME AS: $Q_\alpha(\alpha, \psi)$ AND $\tilde{Q}_\alpha(\tilde{\alpha}, \tilde{\psi})$ BOTH CONSERVED IN II A, II B.
IN OPEN SUPERSTRING: $Q_\alpha + \tilde{Q}_\alpha$ (OR $Q_\alpha - \tilde{Q}_\alpha$) CONSERVED.

NOTICE: SOME SUSY CONSERVED: D-BRANES ARE BPS STATES $T \propto \mu$ (CHARGE DENSITY)

DENSITY OF WHAT? $\quad$ Dp-BRANE CAN BE SOURCE OF $p+1$ FORM

NOTICE: II B EVEN FORMS p ODD, II A ODD FORMS, EVEN D BRANES

DETAILS: $\quad$ IIA $\quad A_M$, A_{MNP} + DUALS

DUALS: $\tilde{A}^{(10-2-D)}$ $\quad$ D = DEGREE OF FORM (1, 3)

$\tilde{A}^7$, $\tilde{A}^5$ $\quad$ D BRANES: 0, 2, 4, 6, 8 ← !!

8-BRANE COUPLES TO A 9-FORM POTENTIAL (NON-DYNAMICAL): A^9 HAS FIELD STRENGTH $F^{(10)}$. E.O.M.: $\quad d * F^{(10)} = 0 \Rightarrow F^{(10)} = \text{CONST.}$

IIB $\quad \psi, \; C_{MN}, \; C^+_{MNPQ} \; + \; \tilde{C}_{MNPQRS} \; + \; \tilde{C}_{M_1 \dots M_8}$

DUAL POTENTIALS

-1 BRANE = INSTANTON

1-BRANE (D-STRING)

SELF-DUAL 3-BRANE

5-BRANE

7-BRANE

ONE CAN ADD A NON-DYNAMICAL 10-FORM POTENTIAL AND CORRESPONDINGLY A 9-BRANE.

IIB + 9-BRANE IS NOTHING BUT THE OPEN SUPERSTRING THEORY (9-BRANE FILLS 10-D SPACE-TIME)

D-BRANE ACTION MUST INCLUDE A COUPLING TO $(p+1)$ FORMS C WITH G_{p+2}

$$S = \frac{1}{2}\int_{M_{(1,9)}} G_{p+2} \wedge * G_{p+2} + i\mu_p \int_{M_{(1,9)}} \delta^{9-p} \wedge C_{p+1}$$

δ^{9-p} = "DELTA FORM" FOR BRANE AT $X^{p+1} = \ldots X^9 = 0$

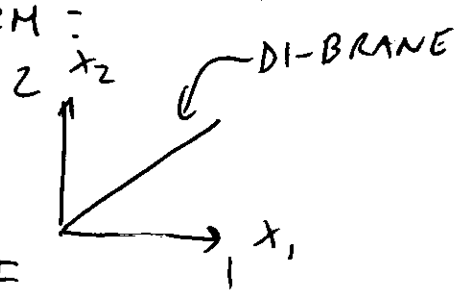
$$\delta^{9-p} = dX^{p+1}\delta(X^{p+1}) \wedge \ldots \wedge dX^9 \delta(X^9)$$

NOTICE $d\delta^{9-p} = 0$. CLOSED FORM

IN THE PRESENCE OF NS FORM B TERM $\propto \mu_p$ IS MODIFIED.

MODIFICATION FROM T-DUALITY :

D1-BRANE IN 1,2 PLANE HAS TERM :



$$\int dx^1 \left(C_1 + \partial_1 X^2 \, C_2\right)$$

T-DUALITY ALONG 2 : $\partial_1 X^2 \to 2\pi\alpha' F_{12}$

$$\underset{\substack{\uparrow \\ \text{ADD INDEX } 2}}{C_1 \to \overset{\text{2-FORM}}{C_{12}}} \qquad \underset{\substack{\uparrow \\ \text{REMOVE INDEX } 2}}{C_2 \to \overset{\text{0-FORM}}{C}}$$

WE GET : $\int dx^1 dx^2 \left(C_{12} + 2\pi\alpha' F_{12} C\right)$

RECALL THAT F APPEARS ALWAYS IN THE GAUGE-INVARIANT COMBINATION $2\pi\alpha' F + B$.
WE CAN WRITE THE COMPLETE COUPLING AS:

$$i\mu_p \int \delta^{9-p} \; \mathrm{Tr}\left(e^{2\pi\alpha' F + B}\right) \sum_p C_{p+1}$$

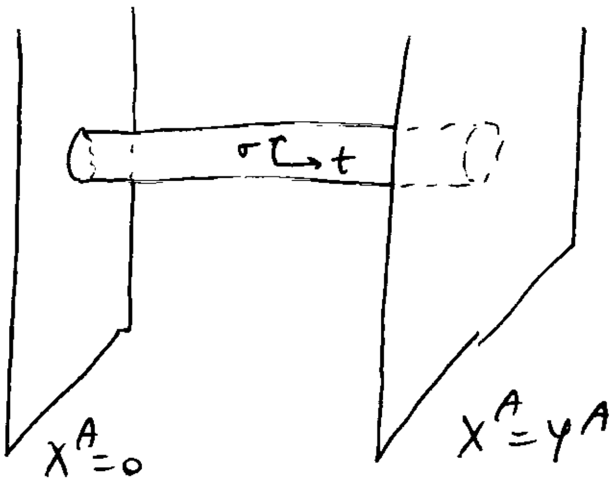
HERE ALL PRODUCTS ARE $\wedge$ PRODUCTS OF FORMS:

$$e^A = 1 + A + \frac{1}{2} A \wedge A + \frac{1}{6} A \wedge A \wedge A + \ldots$$

FINITE EXPANSION

<u>THUS</u>: Dp-BRANE COUPLES NOT ONLY TO C_{p+1} BUT ALSO TO C_{p-1}, C_{p-3} ...

μ_p CAN BE COMPUTED FROM STRING DIAGRAM:



AS IN THE BOSONIC CASE DIAGRAM DESCRIBES:

A) OPEN-STRING VACUUM ENERGY

B) ONE-PARTICLE EXCHANGE OF CLOSED (SUPER) STRING STATES.

NOTICE: WE WANT TO FIND CONTRIBUTION TO VACUUM ENERGY OF R-R STATES (ψ & $\tilde{\psi}$ BOTH PERIODIC IN σ).

IN TERMS OF THE <u>OPEN</u> STRING DIAGRAM WE HAVE TO COMPUTE THEN:

$$\mathcal{A} = V_{p+1} \int \frac{d^{p+1}_K}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t} \left[\mathrm{Tr}_{NS} (-)^F e^{-2\pi\alpha' t (K^2+M_{NS}^2)} + \mathrm{Tr}_R (-)^F e^{-2\pi\alpha' t (K^2+M_R^2)} \right]$$

ENSURE $\psi(\sigma+\pi) = \psi(\sigma)$ (R-R OF <u>CLOSED</u> STRING STATES)

$$\mathcal{A} = \frac{1}{2} V_{p+1} \int \frac{dt}{t} (2\pi t)^{-(p+1)/2} \left(t/_{2\pi\alpha'} \right)^4 e^{-t\, Y^2/8\pi\alpha'^2}$$

$$= V_{p+1}\, 2\pi\, (4\pi^2\alpha')^{3-p}\, G_{9-p}(Y)$$

FIELD THEORY: EXCHANGE OF R-R FORMS

$$S = \frac{1}{2} \int G_{p+2} \wedge * G_{p+2} + i\mu_p \int \delta^{9-p} \wedge C_{p+1}$$

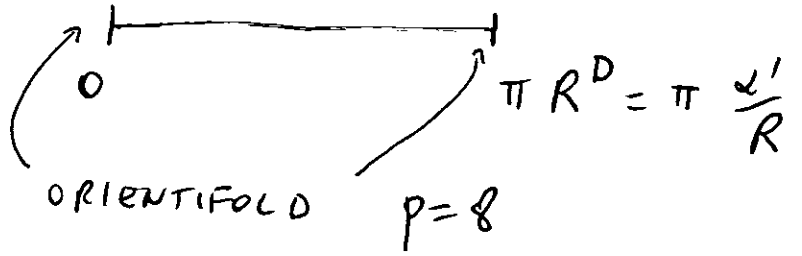
$$\mathcal{A} = V_{p+1} \mu_p^2\, G_{9-p}(Y)$$

$$\mu_p^2 = 2\pi\, (4\pi^2\alpha')^{3-p} \left(\equiv 2\kappa^2 e^{2\phi} T_p^2 \right)$$

ORIENTIFOLD ALSO BREAKS $\frac{1}{2}$ SUSY. COMPUTATION ANALOGOUS TO THE BOSONIC CASE GIVES:

$$\mu_p^{\text{ORIENTIFOLD}} = \pm 2^{p-5} \mu_p \quad \left(T_p^{\text{ORIENT.}} = \mp 2^{p-5} T_p \right)$$

EXAMPLE: OPEN UNORIENTED SUPERSTRING:
GAUGE GROUP $SO(N)$. COMPACTIFY X^9.
T-DUALITY ALONG X^9: $S^1/\mathbb{Z}_2$



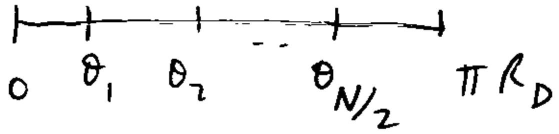
$SO(N)$ WITH GENERIC WILSON LINE, $SO(N)$ VECTOR ALONG X^9:

$$A_9 = \frac{1}{2\pi R}\begin{pmatrix} \theta_1 & & & & & & \\ & -\theta_1 & & & & & \\ & & \theta_2 & & & & \\ & & & -\theta_2 & & & \\ & & & & \ddots & & \\ & & & & & \theta_{N/2} & \\ & & & & & & -\theta_{N/2} \end{pmatrix}$$

T-DUALITY: 9-BRANE $\rightarrow$ 8 BRANE AT POSITION

$$(X^9)^{DUAL} \simeq 2\pi R_D \left(\overbrace{\theta_1, \underbrace{-\theta_1}_{}, \theta_2, \underbrace{-\theta_2}_{}, \ldots, \theta_{N/2}, \underbrace{-\theta_{N/2}}_{}}^{\text{BRANES}} \right)$$

MIRROR BRANES



$$\mu_8^{ORIENT.} = -2^3 \mu_8 \times 2 \leftarrow \text{NUMBER OF ORIENTIFOLDS}$$

TO ABSORB THE FLUX OF THE ORIENTFOLD POINTS AT $X^9_{DUAL} = 0$, $X^9_{DUAL} = \pi R^D$ WE NEED $N/2 = 16$ 8-BRANES

ZD GAUGE GROUP $SO(32)$

$Dp - Dp'$ BRANE SYSTEM

$p' > p$

Dp BRANE ALONG $X^1 \dots X^p$

Dp' BRANE ALONG $X^1 \dots X^i,\ X^{p+1} \dots X^{p+p'-i} \qquad i \le p$

T-DUALITY TRANSFORMS N BOUNDARY CONDITIONS INTO D & D → N

T-DUALIZE $X^{i+1} \dots X^p \quad X^{p+1} \dots X^{p+p'-i} \Rightarrow$ MAP Dp INTO Dp'

ON Q_α, $\tilde{Q}_\alpha$ T-DUALITY ACTS AS :

$$Q_\alpha \to Q_\alpha \,, \quad \tilde{Q}_\alpha \to \prod_A \gamma^{11}\gamma^A \ \tilde{Q}_\alpha \qquad A = i+1, \dots p + p' - i$$

Dp LEAVES ONE SUSY UNBROKEN: $Q_\alpha + \tilde{Q}_\alpha$

Dp' " UNBROKEN THE LINEAR SUPERPOSITION:

$$Q_\alpha + \Big(\prod_A \gamma^{11}\gamma^A\Big)_{\alpha\beta} \tilde{Q}_\beta$$

$Dp + Dp'$ LEAVES UNBROKEN ALL SUSY η^i_α SUCH THAT

$$(Q_\alpha + \tilde{Q}_\alpha)\,\eta^\alpha_i = \Big(Q_\alpha + \prod_A \gamma^{11}\gamma^A \tilde{Q}_\alpha\Big)\,\eta^\alpha_i$$

$$\gamma^{11}\gamma^A\gamma^{11}\gamma^A = -(\gamma^{11})^2(\gamma^A)^2 = -1 \qquad \text{THUS ON } \eta^\alpha_i :$$

$$(\gamma^{11}\gamma^A)\,\eta_i = \begin{pmatrix} i & \\ & -i \end{pmatrix}\eta_i$$

$\prod_A \gamma^{11}\gamma^A \eta_i$ MUST BE $\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}\eta_i \Rightarrow$ ND + DN

DIRECTIONS $\nu = p + p' - 2i$ MUST BE MULTIPLE OF 4

FOR A Dp BRANE $\mu_p = \text{CONSTANT} \times T_p$ AND SUSY UNBROKEN
THESE TWO RESULTS ARE RELATED BY THE BPS BOUND.

$\mathcal{N} = 2$ ALGEBRA (MAJORANA BASIS)

CENTRAL CHARGES

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^0\gamma^\mu)_{\alpha\beta}\left(P_\mu + \frac{1}{2\pi\alpha'} Q_\mu\right)$$

$$\{\tilde{Q}_\alpha, \tilde{Q}_\beta\} = 2(\gamma^0\gamma^\mu)_{\alpha\beta}\left(P_\mu - \frac{1}{2\pi\alpha'} Q_\mu\right)$$

$$\{Q_\alpha, \tilde{Q}_\beta\} = 2\sum_p (\gamma^0\gamma^{\mu_1 \cdots \mu_p})_{\alpha\beta}\, \hat{Q}_{\mu_1 \cdots \mu_p}$$

p EVEN IN IIA, ODD IN IIB.

BRANE WRAPPED AROUND TORUS OF VOLUME V_p, SPANNING COORDINATES $X^1 \cdots X^p$.

$$P_\mu = (T_p V_p, 0, \ldots, 0) \qquad P_0 = T_p V_p \quad P_i = 0$$

D-BRANE: $Q_\mu = 0$, $\hat{Q}_{1 \cdots p} = \sqrt{2}\, K\, V_p \mu_p$

$$\frac{1}{2}\left\{(Q_\alpha, \tilde{Q}_\alpha), \begin{pmatrix} Q_\beta \\ \tilde{Q}_\beta \end{pmatrix}\right\} = \begin{pmatrix} V_p T_p & \\ & V_p T_p \end{pmatrix} + \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \sqrt{2}\, K V_p \mu_p \gamma^0 \gamma^1 \cdots \gamma^p$$

UNBROKEN SUSY MEANS THAT A LINEAR COMBINATION OF Q_α's AND $\tilde{Q}_\alpha$'s VANISH, I.E. THAT THE RIGHT-HAND SIDE HAS A ZERO EIGENVALUE

$$\Rightarrow \quad T_p = \pm\sqrt{2}\, K\, \mu_p$$

L.H.S. POSITIVE $\Rightarrow$ $T_p \geq |\sqrt{2}\, K \mu_p|$

OTHER EXAMPLE : q_2 D1 , q_1 FUNDAMENTAL STRINGS
WRAP THEM ALONG X^1, CIRCLE OF LENGHT L

$$P_0 = L\,T_{(q_1,q_2)} \qquad Q_1 = q_1 \frac{1}{2\pi\alpha'} L \,,\; \hat{Q}_1 = q_2 \frac{e^{-\phi}}{2\pi\alpha'} L$$

$$\frac{1}{2}\left\{ (Q_\alpha, \tilde{Q}_\alpha), \begin{pmatrix} Q_\beta \\ \tilde{Q}_\beta \end{pmatrix} \right\} = \begin{pmatrix} L\,T_{(q_1,q_2)} & \\ & L\,T_{(q_1,q_2)} \end{pmatrix} + \begin{pmatrix} q_1 & q_2 e^{-\phi} \\ q_2 e^{-\phi} & -q_1 \end{pmatrix} \frac{L}{2\pi\alpha'} \gamma^0\gamma^1$$

EIGENVALUES OF $\gamma^0\gamma^1$ ARE ± 1. R.H.S :

$$\begin{pmatrix} T_{(q_1,q_2)} \pm q_1/2\pi\alpha' & \pm q_2 \frac{e^{-\phi}}{2\pi\alpha'} \\ \pm q_2 \frac{e^{-\phi}}{2\pi\alpha'} & T_{(q_1,q_2)} \mp \frac{q_1}{2\pi\alpha'} \end{pmatrix}$$

DETERMINANT : $$T^2_{(q_1,q_2)} - \left(q_1/2\pi\alpha'\right)^2 - e^{-2\phi}\left(\frac{q_2}{2\pi\alpha'}\right)^2 \geq 0$$

BECAUSE OF POSITIVITY OF L.H.S.

$$T_{(q_1,q_2)} \geq \frac{1}{2\pi\alpha'}\sqrt{q_1^2 + e^{-2\phi} q_2^2}$$

BPS BOUND $$T_{(q_1,q_2)} = \frac{1}{2\pi\alpha'}\sqrt{q_1^2 + e^{-2\phi} q_2^2} < \frac{|q_1|}{2\pi\alpha'} + e^{-\phi}\frac{|q_2|}{2\pi\alpha'}$$

FOR q_1, q_2 COPRIME
THEN :
q D1 AND q_1 FUNDAMENTAL STRINGS FORM A BOUND STATE
(LOWER ENERGY)

THE NS5 BRANE.

PROPERTIES:

1) IT IS A CLASSICAL SOLITON OF IIA OR IIB SUPERGRAVITY. ITS TENSION MUST BE

$$T \sim e^{-2\phi} M_s^6 \equiv (\alpha')^{-3}$$ BY DIMENSIONAL ANALYSIS

TENSION IS A TREE-LEVEL EFFECT

RECALL THAT Dp BRANES, INSTEAD, HAVE TENSION $T_p \sim e^{-\phi} M_s^{p+1}$.
AT WEAK COUPLING, $e^\phi = g_s \ll 1$, THEY ARE PARAMETRICALLY LIGHTER THAN THE NS5.

2) THE NS5 METRIC IS:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \left(1 + \frac{Q}{y^2}\right)\left[dy^2 + y^2 d\Omega_3^2\right]$$

$$e^{2\phi} = 1 + Q/y^2$$

$$H = Q\,\epsilon_3$$

NO SINGULARITY AT $y \to 0$, INSTEAD THE METRIC DEVELOPS A "THROAT"

$$y^2 \ll Q \implies ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + Q\, d\psi^2 + Q\, d\Omega_3^2$$

$y \equiv e^{\psi}$ $\qquad$ $\phi = -\psi + \frac{1}{2}\log Q$

DILATON LINEAR IN ψ

$\psi \to +\infty \qquad e^{\phi} = g_s \to 0 \qquad$ (WEAK COUPLING)

$\psi \to -\infty \qquad e^{\phi} = g_s \to +\infty \qquad$ (STRONG COUPLING)

THE METRIC BECOMES A DIRECT PRODUCT

$$\mathbb{R}^{(5,1)} \times \mathbb{R} \times S^3$$

WITH A CONSTANT 3-FORM ON THE 3-SPHERE.
THE THREE SPHERE HAS RADIUS $\sqrt{Q}$.
THIS METRIC CAN BE DESCRIBED BY AN <u>EXACT</u> CFT:
THE CHS BACKGROUND
(C.G. CALLAN, J.A. HARVEY AND A. STROMINGER, HEP-TH/9112030)

$\mathbb{R}^{(5,1)}$ IS A FREE SUPER-CONFORMAL FIELD THEORY WITH FIELDS $X^{\mu}(z,\bar{z})$, $\psi^{\mu}(z)$, $\bar{\psi}^{\mu}(\bar{z})$ AND STRESS-ENERGY TENSOR

$$T_{\mathbb{R}^{(5,1)}} = -\frac{1}{2}\,\partial X^{\mu}\,\partial X^{\nu}\,\eta_{\mu\nu} - \frac{1}{2}\,\psi^{\mu}\,\partial\,\psi^{\nu}\,\eta_{\mu\nu}$$

$$\bar{T}_{\mathbb{R}^{(5,1)}} = -\frac{1}{2}\,\bar{\partial} X^{\mu}\,\bar{\partial} X^{\nu}\,\eta_{\mu\nu} - \frac{1}{2}\,\bar{\psi}^{\mu}\,\bar{\partial}\,\bar{\psi}^{\nu}\,\eta_{\mu\nu}$$

$\mu,\nu = 0,\dots 5$

$\mathbb{R}$ HAS A LIOUVILLE COORDINATE $\hat{\varphi}(z,\bar{z})$ + WORLD-SHEET FERMIONS $\chi(z)$, $\bar{\chi}(\bar{z})$

THE LINEAR DILATON CORRESPONDS TO AN "IMPROVEMENT" TERM IN $T, \bar{T}$:

$$T = -\frac{1}{2}\,\partial\hat{\varphi}\,\partial\hat{\varphi} - \frac{1}{2}\,\chi\partial\chi - \frac{\alpha}{2}\partial^2\hat{\varphi}$$

$$\bar{T} = -\frac{1}{2}\,\bar{\partial}\hat{\varphi}\,\bar{\partial}\hat{\varphi} - \frac{1}{2}\,\bar{\chi}\bar{\partial}\bar{\chi} - \frac{\alpha}{2}\bar{\partial}^2\hat{\varphi}$$

$$\alpha = \sqrt{\frac{2}{Q}}\,.$$

S_3 IS DESCRIBED BY A SUPERSYMMETRIC WZW $SU(2)$ MODEL WITH LEVEL Q

THIS MEANS:

1) THERE EXIST HOLOMORPHIC (ANTIHOLOMORPHIC) CURRENTS

$$J_B^i(z)\,, \qquad J_F^i(z) = \epsilon^{ijk}\chi^j\chi^k$$

$$\chi^i = \text{FREE 2-D FERMIONS IN ADJOINT OF } SU(2)$$

O.P.E OF $J_B^i(z)$: $\quad J_B^i(z)\,J_B^j(0) = \delta^{ij}\frac{K}{z^2} + \frac{\epsilon^{ijk}J^k(0)}{z}$

THIS IS THE LEVEL OF AFFINE-LIE ALGEBRA

$$J_F^i(z)\,J_F^j(0) = 2\delta^{ij}/z^2 + \frac{\epsilon^{ijk}}{z}J_F^k(0)$$

LEVEL 2

TOTAL LEVEL: $Q = K+2 \Rightarrow K = Q-2$.

NOTICE THAT $\psi \to -\infty$ IS A REGION WHERE THE STRING COUPLING IS LARGE: STRING LOOPS ARE RELEVANT THERE AND THE CFT DESCRIPTION OF THE THROAT BECOMES INSUFFICIENT (CFT ≡ STRING TREE-LEVEL).

3) WHAT ARE THE MASSLESS EXCITATIONS ON THE NS5?

SUSY ALGEBRA WITH NS5 CHARGE:

$$\{Q_\alpha Q_\beta\} = 2\gamma^0\gamma^\mu P_\mu + 2\gamma^0\gamma^{\mu_1\cdots\mu_5} Z_{\mu_1\cdots\mu_5}$$

$$\{\tilde{Q}_\alpha \tilde{Q}_\beta\} = 2\gamma^0\gamma^\mu P_\mu + 2a\gamma^0\gamma^{\mu_1\cdots\mu_5} Z_{\mu_1\cdots\mu_5}, \quad a = \pm 1$$

5-BRANE ALONG $1,2,\ldots 5$. EXTREMAL: $P_\mu = (E,0,\ldots,0)$

$|Z_{12345}| = Z, \quad E = Z$

UNBROKEN SUSY: $Q_\alpha \epsilon^\alpha$ WITH ϵ^α SUCH THAT

$$\left(2\gamma^0\gamma^0 P_0 + 2\gamma^0\gamma^{12345} Z_{12345}\right)\epsilon = 0$$

<u>SO</u>:
$$\left(1 + \gamma^{012345}\right)\epsilon = 0$$

$$\left(1 + a\gamma^{012345}\right)\tilde{\epsilon} = 0$$

$a = +1$ FOR IIA (PARITY $x^6 \to -x^6$ $Q_\alpha \to \tilde{Q}_\alpha$, $Z_{\mu_1\cdots\mu_5} \to Z_{\mu_1\cdots\mu_5}$

IN IIA SURVIVING SUSY HAVE SAME 6-D CHIRALITY

ON THE WORLD-VOLUME OF NS5 BRANE THERE IS A CHIRAL (2,0) SUPERSYMMETRY

IN IIB $a=1$: WRITE ALGEBRA IN TERMS OF COMPLEX SUPERCHARGE $\Sigma_\alpha = \frac{1}{\sqrt{2}}[Q_\alpha + i\,\widehat{Q}_\alpha]$

$$\{\Sigma_\alpha, \Sigma_\alpha\} = 2\,\gamma^0\gamma^{\mu_1\ldots\mu_5}\,(Z_{\mu_1\ldots\mu_5} + i\,\widehat{Q}_{\mu_1\ldots\mu_5})$$

MAKES MANIFEST THE ISOMORPHISM OF THE IIB ALGEBRA:

$$\Sigma_\alpha \to e^{i\theta}\Sigma_\alpha\ ,\quad Z_{\mu_1\ldots\mu_5} + i\,\widehat{Q}_{\mu_1\ldots\mu_5} \to e^{2i\theta}(Z_{\mu_1\ldots\mu_5} + i\,\widehat{Q}_{\mu_1\ldots\mu_5})$$

IN IIB THE WORLDVOLUME SUSY IN 6D IS NON-CHIRAL: (1,1)

- FROM THE ALGEBRA WE GET THE MASSLESS EXCITATIONS.

LITTLE GROUP OF $SO(1,5)$ IS $SO(4) \simeq SU(2)\times SU(2)$

(1,0) SUPERCHARGE IS IN $(\frac{1}{2}, 0)$ OF $SO(4)$

(0,1) " " " $(0, \frac{1}{2})$ "

IIB : Q_α , $Q_{\dot\alpha}$ MULTIPLET :

$$|0\rangle,\quad Q_\alpha Q_{\dot\beta}|0\rangle,\ \epsilon^{\alpha\beta}Q_\alpha Q_\beta|0\rangle,\ \epsilon^{\dot\alpha\dot\beta}Q_{\dot\alpha}Q_{\dot\beta}|0\rangle,\ \epsilon^{\alpha\beta}Q_\alpha Q_\beta\,\epsilon^{\dot\alpha\dot\beta}Q_{\dot\alpha}Q_{\dot\beta}|0\rangle$$

4 SCALARS + 1 VECTOR

$$Q_\alpha|0\rangle,\quad Q_\alpha Q_{\dot\alpha} Q_{\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}|0\rangle,\quad Q_{\dot\alpha}|0\rangle,\ \epsilon^{\alpha\beta}Q_\alpha Q_\beta Q_{\dot\beta}|0\rangle \ =$$

λ^i_α , $\lambda^I_{\dot\alpha}$ $i, I = 1,2$ $\equiv$ 2 SPINORS WITH

6-D CHIRALITY + IN $(\frac{1}{2}, 0)$ OF $SU(2)\times SU(2)$ +
2 SPINORS WITH CHIRALITY − IN $(0, \frac{1}{2})$ OF
$SU(2)\times SU(2)$;
THE $SU(2)\times SU(2)$ R-SYMMETRY IS SIMPLY THE
ISOMETRY OF TRANSVERSE SPACE
5-BRANES BREAK $SO(1,9) \to SO(1,5)\times SO(4)$
$\llcorner$ R-SYMMETRY

II A SUPERCHARGES ARE Q_α $\tilde{Q}_\alpha$

MULTIPLET:

$|0\rangle$, $Q_\alpha \tilde{Q}_\beta |0\rangle$, $\epsilon^{\alpha\beta} Q_\alpha Q_\beta |0\rangle$, $\epsilon^{\alpha\beta}\tilde{Q}_\alpha \tilde{Q}_\beta |0\rangle$,
$\epsilon^{\alpha\beta} Q_\alpha Q_\beta \epsilon^{\alpha\beta}\tilde{Q}_\alpha \tilde{Q}_\beta |0\rangle$

5 SCALARS + 1 ANTISYMMETRIC TENSOR

SPINORS HAVE ALL THE SAME 6-D CHIRALITY:

λ^i_α $\hat{\lambda}^I_\alpha$.

SOME APPLICATIONS OF BRANE TECHNIQUES.

I) SELF-DUALITY OF TYPE IIB SUPERSTRING

LOW-ENERGY EFFECTIVE ACTION

$$S = \int d^{10}x \sqrt{-g} \Big\{ e^{-2\phi} \Big[(1+\tilde{\phi}^2 e^{2\phi}) H_{\mu\nu\rho} H^{\mu\nu\rho} + \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi \Big]$$

$$+ \Big[\tfrac{1}{2} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \tilde{H}_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + e^{-\phi} H_{\mu\nu\rho} \tilde{H}^{\mu\nu\rho} + G_{\mu\nu\rho\sigma\tau} G^{\mu\nu\rho\sigma\tau} \Big] \Big\}$$

$g_s = e^{\phi}$ WHAT HAPPENS IN THE LIMIT $g_s \to \infty$?

GO TO EINSTEIN FRAME $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$

$e^{-2\phi} \sqrt{-g}\, R \to \Omega^8 \sqrt{-g}\, R\, e^{-2\phi}$. CHOOSE $\Omega^8 e^{-2\phi} = 1$

$\Rightarrow \Omega = e^{\phi/4}$ EINSTEIN TERM IN CANONICAL FORM, INDEPENDENT OF ϕ.

$$(H_{\mu\nu\rho})^2 \sqrt{-g}\, e^{-2\phi} \to \Omega^{10-6} e^{-2\phi} \sqrt{-g} (H_{\mu\nu\rho})^2 = e^{-\phi} (H_{\mu\nu\rho})^2$$

$$(\tilde{H}_{\mu\nu\rho})^2 \sqrt{-g}\, e^{-2\phi} \to \Omega^4 \sqrt{-g}\, (\tilde{H}_{\mu\nu\rho})^2 = e^{\phi} (\tilde{H}_{\mu\nu\rho})^2 \sqrt{-g}$$

$$\sqrt{-g}\, (G_{\mu\nu\rho\sigma\tau})^2 \to \sqrt{-g}\, (G_{\mu\nu\rho\sigma\tau})^2$$

$$\sqrt{-g} \Big[e^{-2\phi} \tfrac{1}{2} (\partial\phi)^2 + \tfrac{1}{2} (\partial\tilde{\phi})^2 \Big] \to \Omega^8 \sqrt{-g} \Big[e^{-2\phi} \tfrac{1}{2} (\partial\phi)^2 + (\tfrac{1}{2} \partial\tilde{\phi})^2 \Big]$$

$$= \tfrac{1}{2} \sqrt{-g} \Big[(\partial\phi)^2 + e^{2\phi} (\partial\tilde{\phi})^2 \Big]$$

- DEFINE THE COMPLEX FIELD $S = i e^{-\phi} + \tilde{\phi}$

- THE KINETIC TERM BECOMES: $-\dfrac{\partial S \, \partial S^*}{(S - S^*)^2}$

THE B, $\tilde{B}$ KINETIC TERM IS:

$$\left(H_{\mu\nu\rho}, \; \tilde{H}_{\mu\nu\rho} \right) \frac{i}{S - S^*} \begin{pmatrix} SS^* & S^* \\ S & 1 \end{pmatrix} \begin{pmatrix} H_{\mu\nu\rho} \\ \tilde{H}_{\mu\nu\rho} \end{pmatrix}$$

THE ACTION IS INVARIANT UNDER $SL(2, \mathbb{R})$:

$$S \to \frac{a S + b}{c S + d} \qquad a, b, c, d \in \mathbb{R}, \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$$

$$B_{\mu\nu} \to a B_{\mu\nu} + c \tilde{B}_{\mu\nu}$$

$$\tilde{B}_{\mu\nu} \to b B_{\mu\nu} + d \tilde{B}_{\mu\nu}$$

IN PARTICULAR $S \to -\dfrac{1}{S}$ $\qquad e^{\phi} = g_s \to e^{-\phi} = \dfrac{1}{g_s}$
THIS IS A NON-PERTURBATIVE SYMMETRY SINCE IT EXCHANGES WEAK COUPLING WITH STRONG COUPLING.

THIS S-DUALITY IS NOT A SYMMETRY OF PERTURBATIVE IIB SUPERSTRING THEORY SINCE IN IT THERE EXIST STATES CHARGED UNDER B (F-STRINGS) BUT NO STATES CHARGED UNDER $\tilde{B}$.

CONJECTURE: IIB IS SELF-DUAL, S-DUALITY EXCHANGES F-STRING WITH D-STRING.

CHECK #1.

ACTION OF FUNDAMENTAL STRING $\sim M_s^2 \int \sqrt{-g_{IND}}\, d^2\xi$

$(g_{IND})_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$ INDUCED METRIC

UNDER SCALING $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$

$$M_s^2 \int \sqrt{-g_{IND}}\, d^2\xi \to M_s^2 \int \Omega^2 \sqrt{-g_{IND}}\, d^2\xi \qquad \Omega = (g_s)^{1/4}$$

$$M_s^2 \to M_s^2 \sqrt{g_s}$$

ACTION OF D-STRING $\sim M_s^2/g_s \int \sqrt{-g_{IND}}\, d^2\xi$

RESCALED TENSION: $M_s^2/\sqrt{g_s}$

F-STRING COUPLES TO B; ITS ACTION CONTAINS THE TERM

$$\int_\Sigma B$$

D-STRING COUPLES TO $\tilde{B}$: ITS ACTION CONTAINS THE TERM

$$\int_\Sigma \tilde{B}$$

$g_s \to \frac{1}{g_s}$ F-1 $\longrightarrow$ D-1 OK.

IN REALITY $SL(2,\mathbb{Z})$ ACTS AS A SYMMETRY OF THE SPECTRUM:

TENSION OF $(\underbrace{p}_{F-1}, \underbrace{q}_{D-1})$ STRING:

$$T^2 \propto (p,q)\ \frac{i M_s^2}{S-S^*} \begin{pmatrix} SS^* & S^* \\ S & 1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

UNDER $S \to \dfrac{aS+b}{cS+d}$ $\quad \begin{pmatrix} p \\ q \end{pmatrix} \to \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$

THIS SYMMETRY MAPS INTEGERS INTO INTEGERS WHEN $a,b,c,d \in \mathbb{Z}$.

TENSION OF D3: M_s^4/g_s UNDER RESCALING

$$\frac{M_s^4}{g_s} \to \frac{M_s^4}{g_s}\Omega^4 = M_s^4$$

D3 COUPLES TO SELF-DUAL 4-FORM $C^+_{\mu\nu\rho\sigma}$.

UNDER $S \to -\frac{1}{S}$

D3 $\to$ D3 (TENSION INVARIANT & $C^+ \to C^+$).

NS5 COUPLES TO 6-FORM DUAL OF $B_{\mu\nu}$

D5 COUPLES TO " " " $\tilde{B}_{\mu\nu}$

UNDER $S \to -\frac{1}{S}$ $B \to \tilde{B}$ SO DO THEIR 6-FORM DUALS

TENSION OF NS5 $\sim M_s^6/g_s^2$. RESCALING: $M_s^6/g_s^2\, \Omega^6$

$T_{NS5} = M_s^6/\sqrt{g_s}$. RESCALED TENSION OF D5 IS $\sqrt{g_s}\, M_s^6$

$g_s \to \frac{1}{g_s}$ $\quad T_{NS5} \to T_{D5}$ $\quad$ O.K.

CHECK # 2

- DEGREES OF FREEDOM OF D-1 : 2-D VECTOR (NO DEGREES OF FREEDOM) + 8 SCALARS + SPINOR.

SPINOR OBTAINED FROM DIMENSIONAL REDUCTION UNDER $SO(1,9) \to SO(1,1) \times SO(8)$ OF A MAJORANA-WEYL OF $SO(1,9)$

$$\lambda \to \lambda_+^{8c} + \lambda_-^{8s}$$

EXACTLY THE SAME D.O.F. OF F-1 IN IIB

IN F-1 IN GREEN-SCHWARZ FORMULATION D.O.F. ARE

$X^\mu, \quad \Theta^\alpha(z), \quad \Theta^\alpha(\bar z)$

SAME $SO(1,9)$ CHIRALITY.

GAUGE FIXING :

$$p^+\gamma^+ \theta^\alpha(z) = 0 \; \hookrightarrow \; \Theta_+^{8c}, \qquad p^-\gamma^- \Theta^\alpha(\bar z) = 0 \; \hookrightarrow \; \Theta_-^{8s}$$

- DEGREES OF FREEDOM OF D5 : 4 SCALARS + VECTOR + $\lambda^i_\alpha, \; \lambda^I_{\dot\alpha}$

SAME AS D.O.F. OF NS5

TYPE IIA AT LARGE COUPLING

FIELDS : $g_{\mu\nu}$, ϕ, $B_{\mu\nu}$, A_μ, $A_{\mu\nu\rho}$ + FERMIONS

$$S = \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi + H_{\mu\nu\rho} H^{\mu\nu\rho} \right) + F_{\mu\nu}^2 + G_{\mu\nu\rho\sigma}^2 \right]$$

$e^\phi = g_S$, WHAT HAPPENS AT $g_S \to \infty$?

- OLD OBSERVATION : ACTION OF IIA CAN BE OBTAINED BY DIMENSIONAL REDUCTION OF $D=11$ SUPERGRAVITY

$D=11$ SUGRA BOSONIC FIELDS : g_{MN}, A_{MNP}

DIMENSIONAL REDUCTION ON $\mathbb{R}^{(9,1)} \otimes \underbrace{S^1}_{\text{RADIUS } r}$

FIELDS : $g_{\mu\nu}$ $g_{\mu 10} = A_\mu r$, $g_{10,10} = r^2$

$A_{\mu\nu\rho}$, $A_{\mu\nu 10} = B_{\mu\nu}$

ALL THE FIELDS OF IIA.

- MOREOVER :

$$\psi_M \to \underbrace{\psi_\mu}_{\text{NON-CHIRAL 10-D GRAVITINO}} + \psi_{10} \leftarrow \psi$$

ACTION OF $D=11$ SUGRA :

$$S_{D=11} = \int d^{11}x \sqrt{-g} \left(R + G_{MNPQ}^2 \right)$$

DIMENSIONAL REDUCTION:

$$S = \int d^{10}x \; 2\pi r \sqrt{-g}\,\Big[\, R + r^2 F_{\mu\nu}^2 + G_{\mu\nu\rho\sigma}^2$$
$$+ \frac{1}{r^2} G_{\mu\nu\rho 10}^2 \,\Big]$$

REDEFINE 10-D METRIC AS $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$

$$\sqrt{-g}\, R \to \Omega^8 \sqrt{-g}\, R$$

$$\sqrt{-g}\; G_{\mu\nu\rho\sigma}^2 \to \Omega^{10-8} \sqrt{-g}\; G_{\mu\nu\rho\sigma}^2$$

$$\frac{1}{r^2} \sqrt{-g}\; G_{\mu\nu\rho 10}^2 \longrightarrow \frac{1}{r^2}\, \Omega^{10-6} \sqrt{-g}\; G_{\mu\nu\rho 10}^2$$

CHOOSE SCALING SO THAT THE EINSTEIN TERM AND THE KINETIC TERM $G_{\mu\nu\rho 10}^2 = H_{\mu\nu\rho}^2$ HAVE THE SAME FUNCTION OF r IN FRONT:

$$\Omega^4 / r^2 = \Omega^8 \qquad\qquad \Omega^2 = \frac{1}{r}$$

SET $\frac{1}{r^3} = e^{-2\phi}$: THE ACTION BECOMES IDENTICAL TO THE IIA

COEFFICIENT IN FRONT OF R & $H_{\mu\nu\rho}^2$: $\Omega^8 r = \frac{1}{r^3}$
$= e^{-2\phi}$

COEFFICIENT IN FRONT OF $F_{\mu\nu}^2$: $r^3 \Omega^6 = 1$ O.K.

" " " " $G_{\mu\nu\rho\sigma}^2$: $r\Omega^2 = 1$ O.K.

CONJECTURE: $g_s \to \infty$ MEANS $r \to \infty$ SO

IIA $\longrightarrow$ 11-D SUPERGRAVITY.
AT STRONG COUPLING IIA "GROWS" AN EXTRA DIMENSION.

CHECK: NONZERO KALUZA-KLEIN MODES.
MOMENTUM ALONG THE 11-TH DIMENSION IS $p_{10} = \frac{n}{r}$

ACTION OF KK MODE ϕ_n:

$$S = \int d^{10}x \; 2\pi r \sqrt{-g} \Big[\partial_\mu \phi_n \partial_\nu \phi_{-n} g^{\mu\nu}$$
$$+ \partial_\mu \phi_n \partial_{10} \phi_{-n} g^{\mu 10} + \partial_\mu \phi_{-n} \partial_{10} \phi_n g^{\mu 10}$$
$$+ {n^2}/{r^2} \, \phi_n \phi_{-n} + \dots \Big]$$

$$g_{\mu 10} \sim \frac{1}{r} A_\nu g^{\mu\nu}$$

RESCALE:

$$S = \int d^{10}x \, 2\pi r \, \Omega^8 \Big[|\partial_\mu \phi_n|^2 + \Big(\partial_\mu \phi_n \partial_{10} \phi_n^* \frac{1}{r} g^{\mu\nu} A_\nu$$
$$+ \text{c.c.} \Big) + \Omega^2 \frac{n^2}{r^2} |\phi_n|^2 + \dots \Big]$$

MASS OF KK MODE $M_n = \frac{1}{r} \Omega |n| = \frac{1}{r^{3/2}} |n|$
$$= e^{-\phi} |n|$$

ϕ_M CHARGED UNDER A_μ : $\partial_{10}\phi_M = iM\phi_M$
CHARGE $\propto$ MASS.

- IN TYPE IIA STATES WITH MASS $\propto$ CHARGE AND MASS $\sim \frac{1}{g_s}$ EXIST: THEY ARE THE D0 BRANES!
- IIA GIVES NOT ONLY THE ZERO MODES OF D=11 SUGRA BUT ALSO ALL THE K-K SPECTRUM.

D=11 SUGRA INCOMPLETE (NON-RENORMALIZABLE)
IT NEEDS EXTRA DEGREES OF FREEDOM BESIDES THE GRAVITON MULTIPLET.
- IT HAS A_{MNP}. THE SOURCE COUPLING TO A_{MNP} IS A 2-BRANE
DUAL $\tilde{A}_{M_1 \ldots M_6}$. SOURCE: 5-BRANE.

IN D=11 NO DILATON (NO COUPLING CONSTANTS).

TENSION OF M2 $\propto (M_{\text{PLANCK } 11\text{-D}})^2 \equiv (M_{11})^2$
TENSION OF M5 $\propto (M_{11})^6$

M2 AND M5 GIVE ALL THE DYNAMICAL BRANES OF IIA.

r = RADIUS OF 11-TH DIM $\Rightarrow (M_{11})^{-1}\, r$
DIMENSIONLESS.

HERE WE SET, AS WE DID ALREADY IMPLICITLY, $M_{11} = 1$

ACTION OF M2 $S_2 \sim \int \sqrt{-g_{IND}^{(11-D)}}\, d^3\sigma$.

WRAP M2 ALONG S_1. IN 10-D WE SEE A STRING WITH ACTION

$$S \sim 2\pi r \int \sqrt{-g_{IND}^{(10-D)}}\, d^2\sigma \xrightarrow[\text{RESCALE}]{} 2\pi r \Omega^2 \int \sqrt{-g_{IND}}\, d^2\sigma$$

$2\pi r \Omega^2 = 1$ O.K. TENSION OF F-STRING INDEPENDENT OF $g_s = r^{3/2}$.

M2 IN $\mathbb{R}^{(9,1)}$: $S \sim \int \sqrt{-g_{IND}^{(10-D)}}\, d^3\sigma$

RESCALE: $S \sim \Omega^3 \int \sqrt{-g_{IND}}\, d^3\sigma$

$$\underbrace{\Omega^3}_{= \left(\frac{1}{r}\right)^{3/2} = \frac{1}{g_s}}$$

O.K. AGREES WITH TENSION OF D2

M5. ACTION $S_5 \sim \int \sqrt{-g_{IND}^{(11-D)}}\, d^6\sigma$

WRAP ON S_1. ACTION IS THAT OF A 10-D 4-BRANE

$$S \sim 2\pi r \int \sqrt{-g_{IND}^{(10-D)}}\, d^5\sigma \xrightarrow[\text{RESCALE}]{} \underbrace{2\pi r \Omega^5}_{2\pi r \left(\frac{1}{r}\right)^{5/2} \propto \frac{1}{g_s}} \int \sqrt{-g_{IND}}\, d^5\sigma$$

M5 IN $\mathbb{R}^{(9,1)}$: $S \sim \int \sqrt{-g_{IND}^{(10-D)}}\, d^6\sigma$

RESCALE $S \sim \Omega^6 \int \sqrt{-g_{IND}}\, d^6\sigma$

$\Omega^6 = \frac{1}{r^3} \sim \frac{1}{g_s^2}$ O.K. WITH TENSION OF NS5.

WHERE IS THE D6 ?

IN 4-D EUCLIDEAN GRAVITY COMPACTIFIED ON A CIRCLE OF RADIUS r THERE EXISTS A (FINITE-ACTION) INSTANTON.

ACTION $S \sim M_{Pl}^2 \int d^4x \sqrt{-g}\, R$

BY DIMENSIONAL ANALYSIS $S_{INST} \propto M_{Pl}^2 r^2$

IN 11-D THIS IS A 6-BRANE WITH TENSION

$T_6 \propto r^2$ RESCALE $T_6 \to \Omega^7 T_6 \propto \left(\frac{1}{r}\right)^{7/2} r^2$

$= \left(\frac{1}{r}\right)^{3/2} = \frac{1}{g_s}$ O.K. WITH TENSION OF D6

IIA AT STRONG COUPLING = 11-D THEORY WITH 32 SUPERSYMMETRIES (1 MAJORANA IN D=11) + 2 BRANES + 5 BRANES. = M(YSTERY) THEORY

OR MATRIX, SEE DAN KABAT'S LECTURE

NOTICE : <u>NO STRINGS</u>

WHY ONLY 2 AND 5-BRANES?

LOOK AT SUSY ALGEBRA WITH CENTRAL CHARGES IN 11-D:

$$\{Q_\alpha , Q_\beta\} = (\Gamma_0 \Gamma_M)_{\alpha\beta} P^M + \sum_{p=1}^{5} (\Gamma_0 \Gamma_{M_1 \dots M_p})_{\alpha\beta} Z^{M_1 \dots M_p}$$

32-DIM REAL SPINOR. L.H.S HAS $\frac{33 \times 32}{2} = 528$ (SYMMETRIC MATRIX IN α, β)

ONLY $(\Gamma_0 \Gamma_M)_{\alpha\beta}$, $(\Gamma_0 \Gamma_{M_1 M_2})_{\alpha\beta}$, $(\Gamma_0 \Gamma_{M_1 M_2 M_3 M_4 M_5})_{\alpha\beta}$

SYMMETRIC IN α, β.

IN THE Γ-MATRIX BASIS USED HERE Γ_M ARE REAL, Γ_0 IS ANTI-SYMMETRIC $\Gamma_1 \dots \Gamma_{10}$ ARE SYMMETRIC.

R.H.S. : $11 + \frac{11 \times 10}{2} + \frac{11 \times 10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4 \times 5} =$

$= 11 + 55 + 462 = 528$ CHARGES.

DIMENSION OF L.H.S = DIMENSION OF R.H.S.

$\Rightarrow$ WE HAVE WRITTEN HERE THE

<u>MAXIMALLY EXTENDED</u> SUPERSYMMETRY ALGEBRA IN 11-D