S-DUALITY IN $N=4$ $D=4$ SYM

WORLD-VOLUME ACTION OF N D3 BRANE IN IIB

$$S_{D3} = \frac{M_s^4}{g_s}\int d^4\sigma\sqrt{-g_{\mu\nu}^{IND}} + \frac{1}{g_s}\int_{\mathbb{R}^4} d^4\sigma \, \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \ldots$$

U(N) YANG-MILLS IN 4-D

$\ldots$ = SUPERSYMMETRIC COMPLETION.

D3-BRANE IS SELF-DUAL UNDER $g_s \to \frac{1}{g_s}$

FROM D3 ACTION : $g_s = g_{YM}^2$

$g_s \to \frac{1}{g_s}$ $\quad g_{YM} \to \frac{1}{g_{YM}}$ (STRONG-WEAK COUPLING TRANSFORMATION I.E. DUALITY).

HOW DUALITY ACTS ON D3 FIELDS A_μ^{ij} ETC.?

- SIMPLE CASE 1 D3

$$S_{D3} = \frac{1}{g_s}\int_{\mathbb{R}^4} F\wedge *F + \int F\wedge d\tilde{A}$$

EQUATIONS OF MOTION :

$$F : \frac{1}{g_s} * F + d\tilde{A} = 0, \quad \tilde{A} : \quad dF = 0 \Rightarrow F = dA$$

1) SUBSTITUTE $F = dA$ IN ACTION

$$S_{D3} = \frac{1}{g_s} \int dA \wedge * dA$$

USUAL YANG MILLS $U(1)$ LAGRANGIAN.

2) SOLVE F EQUATION : $-\frac{1}{g_s} * F = d\tilde{A}$

$$S_{D3} = g_s \int d\tilde{A} \wedge * d\tilde{A}$$

A = POTENTIAL , $\tilde{A}$ = DUAL POTENTIAL

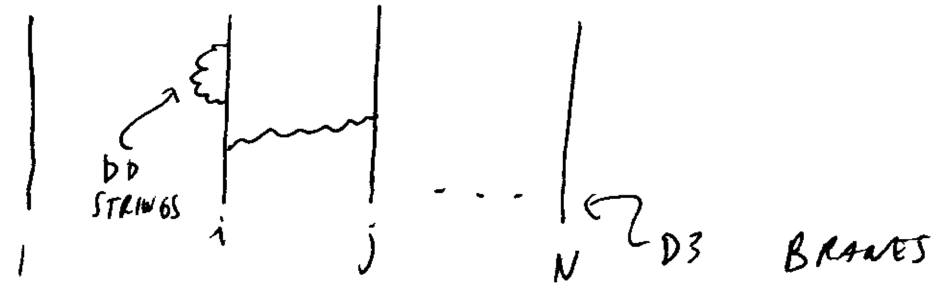
SINCE $d\tilde{A} = \underbrace{* F}_{\text{DUAL FIELD-STRENGTH}}$

NOTICE : THE POINCARÉ DUALITY

$$F \to \tilde{F} = -\frac{1}{g_s} * F \qquad \text{SENDS} \quad g_s \to \frac{1}{g_s}$$

FOR A SINGLE D3, S-DUALITY OF IIB INDUCES AN ELECTRIC-MAGNETIC DUALITY.

– WHAT HAPPENS FOR N D3 BRANES?



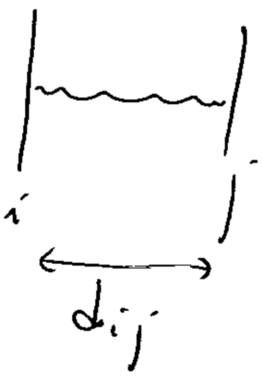
MASSLESS STATES : A_μ^{ii} λ_α^{Iii} X^{Aii} $i=1,\dots,N$

SPINOR INDEX IN 4-D

$I=1,\dots,4$ IN $\underline{4}$ OF $SU(4)$ $A = 6$ OF $SO(6)$ $\equiv [IJ]$ OF $SU(4)$.

THEY COME FROM OPEN STRINGS

STRINGS FROM i TO j BRANE (DISTANCE d_{ij})



STATES A_μ^{ij} , λ_α^{Iij} , X^{Aij} HAVE MASS $M_s^2 d_{ij}$.

ALL THIS IS IN PERFECT AGREEMENT WITH FIELD THEORY.

$\mathcal{N}=4$ 4D LAGRANGIAN:

$$\mathcal{L} = \frac{1}{g_{YM}^2} \mathrm{Tr}\Big[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + M_s^4 D_\mu X D^\mu X + M_s^8 V(X) +$$

$$+ \text{FERMIONS} \Big]$$

$$V(X) = [X^A, X^B][X^B, X^A] : \quad V=0 \Leftrightarrow [X^A, X^B]=0$$

THUS $X^A \in$ CARTAN OF $U(N) \equiv U(1)^N$

X^A CAN BE DIAGONALIZED SIMULTANEOUSLY

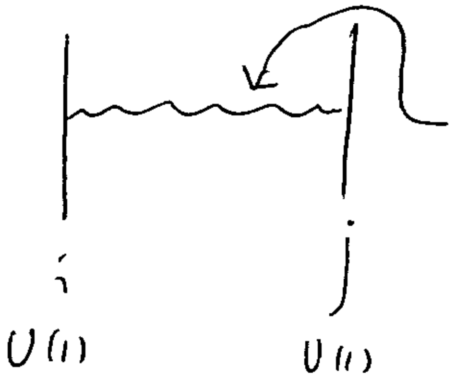
$$X^A = \begin{pmatrix} X^{A11} & & \\ & \ddots & \\ & & X^{ANN} \end{pmatrix}$$

X^A IN ADJOINT OF $SU(N)$. MASS OF A_μ^{ij} IS

$$M_{ij} = M_s^2 \sqrt{\sum_{A=1}^{6} \left(X_A^{ii} - X_A^{jj} \right)} = M_s^2 d_{ij}$$

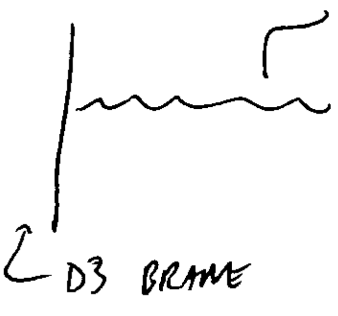
$M_s^2 X^A = \Phi^A$ = CANONICALLY NORMALIZED HIGGS.

MASS OF VECTOR $= \sqrt{\sum_A \langle \Phi^A \rangle \langle \phi^A \rangle}$!



THIS STATE IS CHARGED UNDER BOTH i-TH AND j-TH U(1)

ΣD



FUNDAMENTAL STRING:

IT IS A STATE CHARGED UNDER THE U(1) OF D3 BRANE.

CAN A F-STRING END ON D3?

D3 SPANS COORDINATES 0 1 2 3 ($1 = x^1$, $2 = x^2$, $3 = x^3$)

F " " 0 $4 = x^4$

COORDINATES 5, 6, 7, 8, 9 : $y^1 \ldots y^5$.

ACTION OF IIB WITH F-STRING

$$S_{IIB} + S_F = \frac{1}{2}\int H \wedge * H + \int \theta(x^4)\, \delta^5(y) \wedge \delta^3(x) \wedge B$$

$$H = dB$$

B EQ. OF MOTION $\quad d * H + \theta(x^4) \wedge \delta^5(y) \wedge \delta^3(x) = 0$

INCONSISTENT! : $dd = 0 \Rightarrow \delta(x^4) \wedge \delta^5(y) \wedge \delta^3(x) = 0$!

SOLUTION : S_{D3} BRANE ALSO CONTAINS B.

$$S = S_{IIB} + S_F + S_{D3} \qquad S_{D3} = \frac{1}{2}\int \delta(x^4) \wedge \delta^5(y) \wedge \mathcal{F} \wedge * \mathcal{F}$$

$$\mathcal{F} = dA - B$$

B EQ. OF MOTION $\quad d * H - \theta(x^4) \wedge \delta^3(\vec{x}) \wedge \delta^5(y) +$

$$+ \delta(x^4) \wedge \delta^5(y) \wedge * \mathcal{F} = 0$$

d OF E.O.M. : $\quad d * \mathcal{F} = \delta^3(x)$

ON THE WORLD-VOLUME OF THE D3 BRANE THE F-STRING APPEARS AS AN ELECTRICALLY CHARGED STATE.

- TO HAVE ELECTRIC-MAGNETIC DUALITY WE NEED ALSO MAGNETICALLY CHARGED STATES

- D3 BRANE COUPLES TO 2-FORM RR FIELD C_2:

$$\int \delta(x^4) \wedge \delta^5(y) \wedge \left[C_4 + C_2 \wedge \mathcal{F} + \frac{1}{2} C_0 \, \mathcal{F} \wedge \mathcal{F} \right]$$

- COLLECT ALL TERMS THAT DEPEND ON C_2:

IN S_{IIB} : $\frac{1}{2} \int \tilde{H} \wedge * \tilde{H} \qquad \tilde{H} = dC_2$

IN S_{D1} : $\int \theta(x^4) \, \delta^5(y) \wedge \delta^3(x) \wedge C_2$

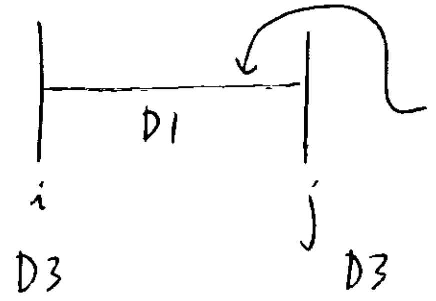
IN S_{D3} : $\int \delta(x^4) \wedge \delta^5(y) \wedge C_2 \wedge \mathcal{F}$

C_2 E.O.M. :

$$d * \tilde{H} + \theta(x^4) \wedge \delta^5(y) \wedge \delta^3(x) + \delta(x^4) \wedge \delta^5(y) \wedge \mathcal{F} = 0$$

d OF E.O.M. $\Rightarrow$ $\qquad d\mathcal{F} = \delta^3(x)$

D1 STRING APPEARS AS MAGNETICALLY CHARGED STATE ON WORLD-VOLUME OF D3.



MASS $= \dfrac{M_s^2}{g_s}\, d_{ij} = \dfrac{1}{g_{YM}^2}\underbrace{\sqrt{\sum_A |\langle \phi^A \rangle|^2}}_{g_{YM} v}$

$M_{\text{ELECTRICALLY-CHARGED STATE}} = g_{YM}\, v$

$M_{\text{MAGNETICALLY-CHARGED STATE}} = \dfrac{1}{g_{YM}}\, v$

E-M DUALITY (ALSO CALLED S) : $g_{YM} \to \dfrac{1}{g_{YM}}, \; v \to v$.

MORE GENERALLY $SL(2,\mathbb{Z})$ OF TYPE IIB INDUCES $SL(2,\mathbb{Z})$ OF $\mathcal{N}=4$ $D=4$ SYM THAT SENDS STATE WITH ELECTRIC CHARGE p AND MAGNETIC CHARGE q INTO $(ap + cq, bp + dq)$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1 \quad (a, b, c, d) \in \mathbb{Z}.$

S-DUALITY OF $\mathcal{N}=4$ SYM IS A <u>CONSEQUENCE</u> OF S-DUALITY OF IIB.

LAST THING TO CHECK : ELECTRIC MULTIPLET IS A VECTOR MULTIPLET OF $\mathcal{N}=4$:

MULTIPLICITY

$\left(1\ (1),\ 4\ (\tfrac{1}{2}),\ 5\ (0)\right)$

SPIN

WHAT IS THE MAGNETIC MULTIPLET

D3 PRESERVES $\frac{1}{2}$ SUSY:

$$Q_\alpha = \Gamma^{11}\Gamma^4\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9\tilde{Q}_\alpha \quad (1)$$

D1 PRESERVES $\frac{1}{2}$ SUSY:

$$Q_\alpha = \Gamma^{11}\Gamma^1\Gamma^2\Gamma^3\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9\tilde{Q}_\alpha \quad (2)$$

D1 + D3 PRESERVE $\frac{1}{4}$ SUSY

FROM (2)

$$\tilde{Q}_\alpha = \Gamma^9\Gamma^8\Gamma^7\Gamma^6\Gamma^5\Gamma^3\Gamma^2\Gamma^1\Gamma^{11}Q_\alpha$$

MULTIPLY BY $\Gamma^{11}\Gamma^4\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9$

FROM (1) $$Q_\alpha = \Gamma^{11}\Gamma^4\Gamma^3\Gamma^2\Gamma^1\Gamma^{11}Q_\alpha$$

8 REAL SUSY = 4 COMPLEX SUSY SURVIVE
($\Gamma^4\Gamma^3\Gamma^2\Gamma^1$ EIGENVALUE +1)

WRITE THESE SUSYS AS $q^1_\alpha \quad q^2_\alpha$

$\alpha = 1, 2 =$ WEYL SPINOR INDEX IN 4-D

MULTIPLET: $|0\rangle$, $\epsilon_{\alpha\beta}\, q^{(i}_\alpha q^{j)}_\beta|0\rangle$, $\epsilon_{\alpha\beta}\, q^1_\alpha q^1_\beta$ $\epsilon_{\alpha\beta}\, q^2_\alpha q^2_\beta|0\rangle$

5 SPIN 0

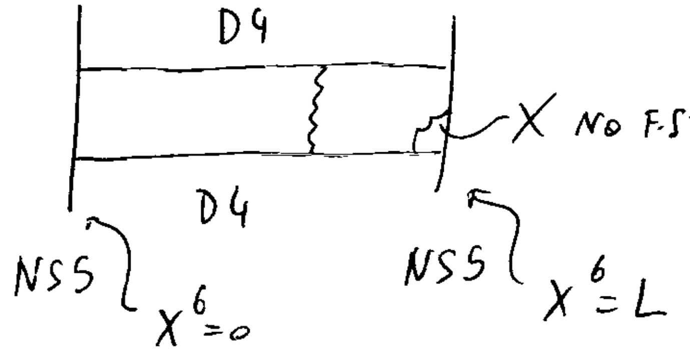
$q^1_{(\alpha} q^2_{\beta)}|0\rangle \leftarrow$ 1 SPIN 1

$\epsilon_{\alpha\beta}\, q^2_\alpha q^2_\beta q^1_\gamma|0\rangle$, $q^1_\alpha|0\rangle$, $q^2_\alpha|0\rangle$, $\epsilon_{\alpha\beta}\, q^1_\alpha q^1_\beta q^2_\gamma|0\rangle$

4 SPIN $\frac{1}{2}$

SEIBERG-WITTEN DUALITY FROM IIA ↔ M THEORY DUALITY

A) PERTURBATIVE CONSTRUCTION ($g_s \ll 1$)

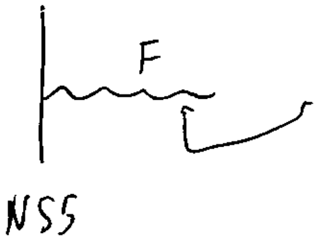


X NO F-STRINGS BETWEEN D4 AND NS5

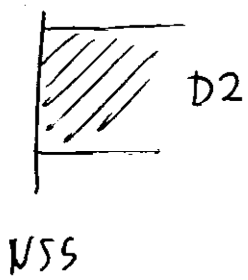
NS5	SPANS	0	1	2	3	4	5	
D4	SPANS	0	1	2	3			6

WORLD-VOLUME FIELDS OF NS5 : $b^+_{\mu\nu}$, ϕ, $X^6 \ldots X^9$

↳ SELF-DUAL $h = db$ IN 6-D



IMPOSSIBLE : F STRING ENDING HAS A CHARGE, BUT ON NS5 WORLD-VOLUME THERE IS NO VECTOR TO CARRY IT.



D2 ENDING ON NS5 IS A STRING ON THE WORLD-VOLUME OF NS5. IT COUPLES TO $b^+_{\mu\nu}$.

D4 CAN END ON NS5. ON WORLD-VOLUME OF NS5 IT IS A 3-BRANE. IT MUST COUPLE TO A

4- FORM A_4. A_4 EXISTS AS IT IS THE 6-D DUAL OF ϕ:

$$* d\phi = d A_4 .$$

FIELDS ON WORLD-VOLUME OF D4 SUSPENDED IN BETWEEN 2 NS5.

$$\underset{\uparrow 0,1,2,3,6}{A_\mu} \;,\quad X^4,\ X^5,\ \underbrace{X^7 \ldots X^9}$$

BOUNDARY CONDITIONS ON THESE FIELDS ARE: $X^7(0) = X^7(L)$ ETC.

D4 HAS FINITE LENGTH IN X^6: $0 \le X^6 \le L$

EXPAND IN FOURIER SERIES IN X^6:

$$X^{7,..9}(\underset{\uparrow 0,1,2,3}{x}, X^6) = \sum_{n=1}^{\infty} X_n^{7,..9}(x)\ \sin(X^6/L\ \pi n)$$

$\uparrow$ NO ZERO MODES.

AT ENERGY SCALES $E \ll \frac{1}{L}$ ONLY ZERO MODES SURVIVE: NO $X^7, .. X^9$.

NOTICE: COORDINATE X^6 IS LONGITUDINAL FOR D4, TRANSVERSE FOR NS5. BOTH $\partial_+ X^6 = 0$, $\partial_\sigma X^6 = 0$

VERTEX FOR A^6 IS $\partial_\sigma X^6 \exp(iKX) = 0$.

$$A^6(x,0) = A^6(x,L) = 0 \qquad \text{NO ZERO MODES FOR } A^6.$$

- DEGREES OF FREEDOM AT $E \ll \frac{1}{L}$ $A_\mu^{(0)}(x)$, $X_{(0)}^4(x)$, $X_{(0)}^5(x)$.

FOR $E \ll \frac{1}{L}$ $\quad S_{D4} = \frac{M_s L}{g_s} \int d^4x \, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \ldots$

$$\hookrightarrow \quad \frac{M_s L}{g_s} = \frac{1}{g_{YM}^2} \quad \left(\text{AT SCALE } \mu = \frac{1}{L}\right)$$

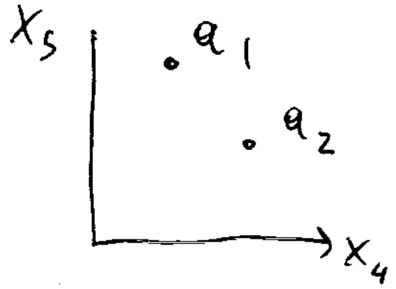
$Q_{\alpha 1} \tilde{Q}_{\alpha} = 32$ SUSY $\quad$ NS5 BREAK THEM TO 16 SUSY

D4 BREAK 16 SUSY TO 8 $(\equiv \mathcal{N}=2 \;\; D=4$ SUSY$)$

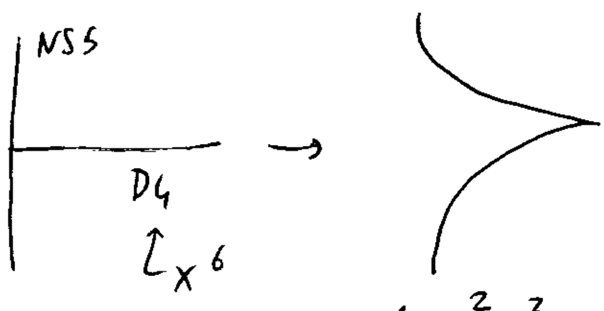
MULTIPLET : $\underbrace{A_\mu, \quad v \equiv X^4 + i X^5, \quad \Omega^{i \leftarrow 1,2}_{\alpha}}_{\text{VECTOR MULTIPLET OF } \mathcal{N}=2}$

4-D GAUGE GROUP IS $SU(2)$, <u>NOT</u> $U(2)$

$U(2)$ HAS 2 COMPLEX MODULI - THEY ARE HERE THE POSITION OF THE D4 INSIDE NS5



D4 IS MASSIVE : IT BENDS NS5 :



EQUATION OF MOTION OF X^6: $\quad \Delta X^6 = \sum_{i=1}^{2} \delta^2 (v - a_i)$

SOLUTION : $X^6 = \sum_i \log |v - a_i|$

KINETIC TERM FOR NS5 CONTAINS:

$$\int d^4x\, d^2v\ \partial_\mu X^6 \partial^\mu X^6 = \int d^4x\, d^2v\ \left|\sum_i \frac{1}{v-a_i}\partial_\mu a_i\right|^2$$

$|v| \to \infty$

$$\left|\sum_i \frac{1}{v-a_i}\partial_\mu a_i\right|^2 \longrightarrow \frac{1}{|v|^2}\left|\partial_\mu \sum a_i\right|^2$$

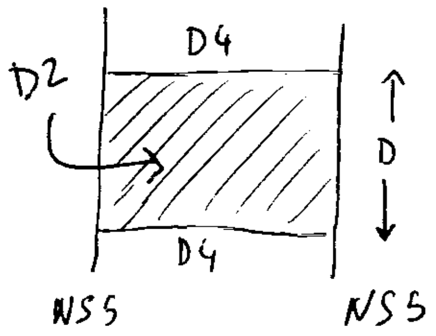
FINITE KINETIC TERM ONLY IF $\partial_\mu \sum a_i = 0$

$\Rightarrow \sum a_i =$ CONSTANT.

CENTER OF MASS OF SYSTEM OF 2-D4 BRANES IS NON-DYNAMICAL. $a_1 - a_2$ IS THE ONLY DYNAMICAL MODULUS

1 MODULUS $\Rightarrow$ GAUGE GROUP IS $SU(2)$

- QUESTION: WHERE ARE THE MONOPOLES?



NS5	SPANS	COORDINATES	$0\ \underbrace{123}_{X}\ 45$
D4	"	"	$0\ 123\ \ \ 6$
D2	"	"	$0\ \ \ \ \ 4\ 6$

$$\text{MASS OF D2} = \frac{1}{g_s} M_s^3 L D = \frac{M_s L}{g_s}\langle\phi\rangle$$

$$\frac{M_s L}{g_s} \equiv \frac{1}{g_{YM}^2}$$

O.K. WITH MASS OF MONOPOLE

IS THIS D2 MAGNETICALLY CHARGED UNDER THE U(1) OF THE D4?

ACTION: $$S_{IIA} + S_{D2} + S_{D4} = \int \overset{IIA}{G_4 \wedge * G_4}$$

$$+ \underbrace{\int \theta(x^4)\, \delta^3(x) \wedge \delta^4(y) \wedge C_3}_{D2} + \underbrace{\int \delta(x^4) \wedge \delta^4(y) \wedge \mathcal{F} \wedge C_3}_{D4}$$

C_3 E.O.M.:

$$d * G_4 - \theta(x^4)\, \delta^3(x) \wedge \delta^4(y) - \delta(x^4) \wedge \delta^4(y) \wedge \mathcal{F} = 0$$

d OF E.O.M.: $\quad d\mathcal{F} = + \delta^3(x)$ ← MAGNETIC CHARGE

SUSY: $\quad 32 \xrightarrow{NS5} 16 \xrightarrow{D4} 8 \xrightarrow{D2} 4 = \mathcal{N}=1$ 4-D SUSY

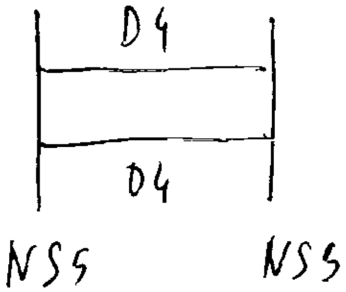
SURVIVING SUSY: $\quad Q_{\underset{1,2}{\alpha}}$ (WEYL SPINOR)

MULTIPLET:

$|q\rangle$, $\quad Q_\alpha |q\rangle$, $\quad Q_1 Q_2 |q\rangle$ $\qquad q =$ CHARGE OF MONOPOLE

$+$ C.P.T. $\quad |-q\rangle$, $\quad Q_\alpha |-q\rangle$, $\quad Q_1 Q_2 |-q\rangle$

4 SPIN ZERO + 2 SPIN $\frac{1}{2}$ (HYPERMULTIPLET OF $\mathcal{N}=2$, $D=4$)



DESCRIBES A $SU(2)$ $\mathcal{N}=2$ SUPER YANG MILLS.

WE GAVE THE PERTURBATIVE DESCRIPTION HERE. WHAT IS THE NON-PERTURBATIVE ONE?

B) NON-PERTURBATIVE DESCRIPTION

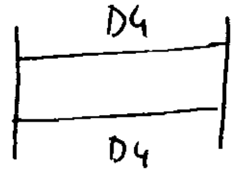
$g_S \to \infty \qquad M_S L \to \infty \qquad$ WITH $\qquad \dfrac{M_S L}{g_S} \equiv \dfrac{1}{g_{YM}^2} =$ CONSTANT

$g_S \to \infty \qquad$ IIA $\longrightarrow$ M THEORY

RELATION $\dfrac{M_S L}{g_S} = \dfrac{1}{g_{YM}^2}$ IS PERTURBATIVE

NON-PERTURBATIVELY: $(g_S, M_S L) = 2$ PARAMETERS
BUT SYM DEPENDS ON g_{YM} ONLY.
ONE CAN SEND $g_S \to \infty$ $M_S L \to \infty$ KEEPING $g_{YM} =$ CONST.

- HOW DOES ONE LIFT



TO $D=11$ SUPERGRAVITY?

RECALL: IN M-THEORY DESCRIPTION OF IIA 11-TH COORDINATE X^{10} COMPACTIFIED $X^{10} \propto X^{10} + 2\pi R$

BOTH D4 AND NS5 COME FROM THE SAME M-THEORY OBJECT: A 5-BRANE

DEFINE $s = (X^6 + i X^{10})/R \qquad s \approx s + 2\pi i$
BETTER TO USE

$$t = e^{-s}$$

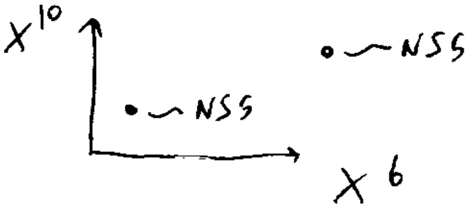
M5-BRANE CONFIGURATION IS $\mathbb{R}^{(3,1)} \times \Sigma$

Σ IS A RIEMANN SURFACE IN $\mathbb{C}^2\ (t, v)$

M5 HAS POSITIVE TENSION: Σ IS A MINIMAL SURFACE IN $\mathbb{C}^2$.

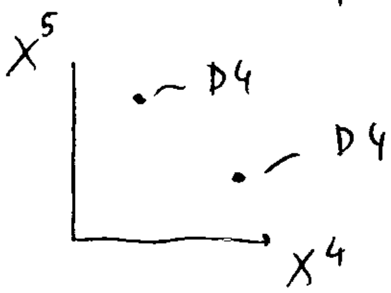
DESCRIBED BY HOLOMORPHIC CURVE $F(t,v) = 0$

AT $v =$ CONST IT MUST HAVE 2 ROOTS (THE POSITION OF THE NS5 BRANES)



$$F = A(v)\,t^2 + B(v)\,t + C(v)$$

AT $t =$ CONST IT MUST ALSO HAVE 2 ROOTS (THE POSITION OF THE D4 BRANES)

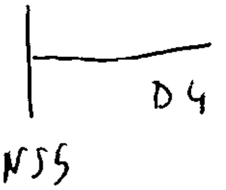


A, B, C QUADRATIC IN v.

IF $C(v)$ HAS ROOT $C(\bar{v}) = 0$ THEN $t = 0$ BELONGS TO Σ : $F(0, \bar{v}) = 0$

THIS MEANS THAT THERE IS A D4 AT $t = 0$

I.E. AT $e^{-(x^6 + i x^{10})/R} = 0$ $\quad x^6 = +\infty$

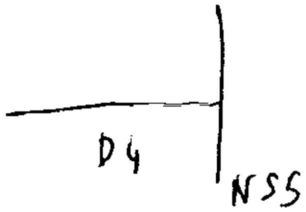


NOT OUR CONFIGURATION.

THUS $C =$ CONST ($= 1$ BY RESCALING).

IF $A(\bar{v}) = 0$ THEN $t = \infty$ $(x^6 = -\infty)$ IS A ROOT.

CONFIGURATION:



SO: $A(\bar{v}) = 1$ WITH A TRANSLATION IN v

$$F(t,v) = t^2 + (v^2+u)\,t + 1$$

u ← CONST.

$t^2 + (v^2+u)\,t + 1 = 0$ IS THE "AUXILIARY" SURFACE IN THE SEIBERG-WITTEN SOLUTION OF $\mathcal{N}=2$ SYM. HERE IT IS A REAL SURFACE: PART OF THE WORLD VOLUME OF THE M5 BRANE.

BRANCH POINTS OF F: $(v^2+u)^2 - 4 = 0$

$$v^2 = \pm 2 - u$$

THE SURFACE BECOMES SINGULAR WHEN THE BRANCH POINTS COINCIDE:

1) $v^2 = 2 - u = 0 \Rightarrow u = 2$

$v^2 = -2 - u = 0$ $\quad u = -2$

AND $v^2 = \infty$. $\quad u = \infty$.

u PARAMETRIZES A FAMILY OF SURFACES. u IS $S^2 - 3$ POINTS (AS IN SEIBERG-WITTEN)

IN M THEORY ELECTRICALLY & MAGNETICALLY CHARGED STATES COME BOTH FROM THE SAME OBJECT: THE M2 BRANE ENDING ON Σ.

AREA OF M2 $\propto \int_D \omega$

$D \leftarrow$ WORLD-VOLUME OF M2

$\omega =$ HOLOMORPHIC 2-FORM $\frac{dt}{t} \wedge dv = d\alpha$ (LOCALLY)

$$\int_D \omega = \int_D \frac{dt}{t} \wedge dv = \int_{\partial D \subset \Sigma} \lg t(v)\, dv = \int_{\partial D} v \frac{dt}{t}$$

AREA DEPENDS ONLY ON ∂D.

- CHOOSE A BASIS OF HOMOLOGY CYCLES IN Σ:

γ_a, γ_b $\quad \gamma_a \cap \gamma_b = 1$ (INTERSECTION NUMBER 1)

CALL $\int_{\gamma_a} \lg t\, dv = a(u)$ $\quad \int_{\gamma_b} \lg t\, dv = a_D(u)$

STATE WITH $\partial D = p\gamma_a + q\gamma_b$ $\quad p, q$ INTEGERS HAS MASS

$$\left| p \int_{\gamma_a} \alpha + q \int_{\gamma_b} \alpha \right| = \left| p\, a(u) + q\, a_D(u) \right| = \text{MASS}$$

OF BPS STATES OF $\mathcal{N} = 2$!

WE CAN ALSO GET THE LOW-ENERGY ACTION OF THE MODULUS OF $\mathcal{N}=2$.

RECALL THAT M5 HAS WORLD-VOLUME 2-FORM b^+ : $db = H = *H$ (SELF-DUAL)

b^+ GIVES A 4-D VECTOR:

$$H = F\wedge(\Lambda+\bar{\Lambda}) + *F\wedge *(\Lambda+\bar{\Lambda})$$

$$dF = d*F = 0 \qquad \text{(EQ. OF MOTION)}$$

$$d\Lambda = d\bar{\Lambda} = 0.$$

Λ IS A NORMALIZABLE 1-FORM ON Σ.

$$\Lambda = \frac{\partial\alpha}{\partial u} \qquad \alpha = \frac{v}{t}dt$$

FUNCTION OF t ONLY ON Σ.

OR $\qquad t \neq 0$ IN Σ

$$\alpha = \lg t\, dv$$

FUNCTION OF v ONLY ON Σ

COMPUTE $\frac{\partial\alpha}{\partial u}$.

FROM $\quad t^2 + (v^2+u)t + 1 = 0 \qquad 2\frac{\partial t}{\partial u}t + (v^2+u)\frac{\partial t}{\partial u} = 1$

WHEN $t = t(v,u)$

OR $\quad 2v\frac{\partial v}{\partial u}t + t = 0 \qquad$ WHEN $v = v(t,u)$

SO $\frac{\partial t}{\partial u} = \frac{-t}{2t+v^2+u}$ OR $\frac{\partial v}{\partial u} = -\frac{1}{2v}$

TWO FORMS FOR $\frac{\partial \alpha}{\partial u}$:

1) $\frac{\partial \alpha}{\partial u} = -\frac{1}{2vt}\, dt$ 2) $\frac{\partial \alpha}{\partial u} = -\frac{1}{2t+v^2+u}\, dv$

$F(t,v)=0$ REGULAR $(u \neq \pm 2, \infty)$

$$\left(\frac{\partial F}{\partial t}, \frac{\partial F}{\partial v}\right) = (2t+v^2+u, 2vt) \neq (0,0)$$

SO WHENEVER F IS REGULAR EITHER $2vt$ OR $2t+v^2+u$ ARE NONZERO. $\frac{\partial \alpha}{\partial u}$ IS WELL DEFINED EVERYWHERE ON Σ.

MOREOVER $|\Lambda|^2 \sim \frac{dv \wedge d\bar{v}}{|v|^4}$ FOR $|v| \to \infty$ $\Rightarrow$ NORMALIZABLE

Λ HOLOMORPHIC : $*\Lambda = i\Lambda$

KINETIC TERM OF F : $\int_{\mathbb{R}^{(3,1)}} F \wedge * F \; i \int_{\Sigma} \Lambda \wedge \bar{\Lambda}$

$d\Lambda = 0$ SO $\Lambda = \frac{\partial a}{\partial u} h_1 + \frac{\partial a_D}{\partial u} h_2 + d\omega$

HARMONIC 1-FORMS ON Σ

$$\int_\Sigma \Lambda \wedge \bar{\Lambda} = i \int_\Sigma \left(\frac{\partial a}{\partial u} h_1 + \frac{\partial a_D}{\partial u} h_2 + d\omega\right) \wedge \left(\frac{\partial \bar{a}}{\partial u} h_1 + \frac{\partial \bar{a}_D}{\partial u} h_2 + d\bar{\omega}\right)$$

$$= i \frac{\partial a}{\partial u} \overline{\left(\frac{\partial a_D}{\partial u}\right)} - i \overline{\left(\frac{\partial a}{\partial u}\right)} \frac{\partial a_D}{\partial u}$$

SINCE : $\int_\Sigma h_1 \wedge d\omega = \int_\Sigma h_2 \wedge d\omega = 0$

$\int h_1 \wedge h_2 = 1$

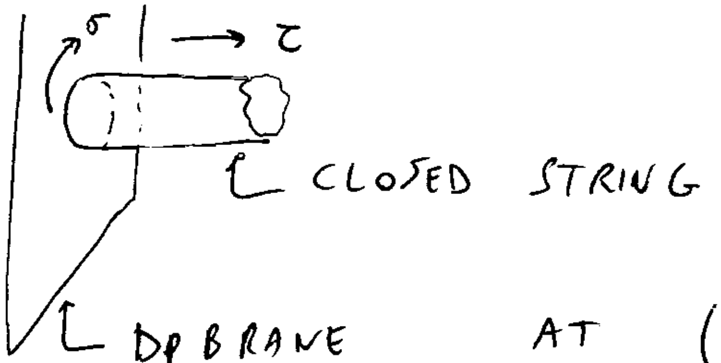
$i \frac{\partial a}{\partial u} \overline{\left(\frac{\partial a_D}{\partial u}\right)} - i \overline{\left(\frac{\partial a}{\partial u}\right)} \frac{\partial a_D}{\partial u}$ IS THE KINETIC TERM OF THE LOW-ENERGY EFFECTIVE THEORY AS IN SEIBERG-WITTEN.

REFS. :

E. WITTEN HEP-TH/9503124 , HEP-TH/9703166

J. DE BOER, K. HORI, H. OOGURI AND Y. OZ HEP-TH/9711143

INTERLUDE: BOUNDARY STATES.



AT $(X^{p+1}, \ldots X^{D}) = (y^{p+1}, \ldots y^{D})$

$X^{\mu}(z,\bar{z})$ = CLOSED STRING COORDINATE

Dp = CLOSED STRING STATE $|B\rangle$ (BOUNDARY STATE)

$|B\rangle$ DEFINED BY

$$\partial_\tau X^{\mu}\Big|_{\tau=0} |B\rangle = 0 \qquad \mu = 0, \ldots, p$$

HOMEWORK: FIND THESE BOUNDARY CONDITIONS BY INTERPRETING THE CLOSED-STRING DIAGRAM AS AN OPEN-STRING 1-LOOP DIAGRAM.

ON DIRICHLET COORDINATES:

$$X^{A} |B\rangle = y^{A} |B\rangle \qquad A = p+1, \ldots, D.$$

MODE EXPANSION: $\partial_\tau X^{\mu}\Big|_{\tau=0} = 2\alpha' p^{\mu} + \sqrt{2\alpha'} \sum_{n\neq 0}^{\infty} (\alpha_n + \tilde{\alpha}_{-n}) e^{2in\sigma}$

$$X^A\Big|_{\tau=0} = q^A + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^A - \tilde{\alpha}_{-n}^A\right) e^{2in\sigma}$$

EQUATIONS FOR BOUNDARY STATE BECOME:

N-COORDINATES: $$(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)|B\rangle = 0 \qquad n \neq 0$$

$$p^\mu |B\rangle = 0$$

D-COORDINATES: $$(\alpha_n^A - \tilde{\alpha}_{-n}^A)|B\rangle = 0 \qquad n \neq 0$$

$$q^A |B\rangle = y^A |B\rangle$$

SOLUTION:

$$|B\rangle = N \, \delta^{D-p}(q^A - y^A) \prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n} S \tilde{\alpha}_{-n}} |0\rangle \otimes |\tilde{0}\rangle \otimes |p^\mu = 0\rangle$$

NORMALIZATION CONSTANT

$$S = \mathrm{diag}\,(\eta^{\mu\nu}, -\delta^{AB})$$

THIS IS THE SOLUTION TO THE (LINEARIZED) CLOSED-STRING EQUATIONS OF MOTION IN THE PRESENCE OF A Dp-BRANE SOURCE.
AT LARGE DISTANCE FROM THE Dp, $|B\rangle$ MUST APPROXIMATE THE SUPERGRAVITY SOLUTIONS WE FOUND EARLIER.

MORE PRECISELY: $|\Phi(x)\rangle$ = MASSLESS STRING STATE

$$D = \text{CLOSED-STRING PROPAGATOR} = \int_0^1 \frac{d^2z}{|z|^2}\, z^{L_0-1}\, \bar{z}^{\tilde{L}_0-1}$$

THEN: $\langle \phi(x)|\, D\, |B\rangle \approx \phi_{S.G.}(x)$

$\Phi_{S.G.}(x)$ = SUPERGRAVITY BACKGROUND $\approx$

$$\approx \frac{\text{CONST}}{|x^A - y^A|^{D-p-2}} \quad \text{FAR FROM THE BRANE}$$

$$|\tilde{\Phi}(p^A)\rangle \approx C_{ij}\, \alpha^i_{-1}\, \tilde{\alpha}^j_{-1}\, |0\rangle \otimes |\tilde{0}\rangle \otimes |p^A\rangle \otimes |p^\mu = 0\rangle$$

$$i, j = (\mu, A)$$

$$\langle \tilde{\Phi}(p^A)|\, D\, |B\rangle \sim \langle p^A| \int_0^1 \frac{d^2z}{|z|^2}\, |z|^{\frac{\alpha'}{2} p_A^2}\, \underbrace{\delta^{D-p}(q^A - y^A)\,|q^A\rangle \times}_{\times \text{ STUFF}}$$

APPLY D TO $\langle \tilde{\phi}(p^A)|$

$$= \text{CONSTANT} \int_0^1 \frac{d\rho}{\rho} \int \frac{d^{D-p} q}{(2\pi)^{D-p}}\, e^{i\hat{p} q^A}\, \delta^{D-p}(q^A - y^A)\, \rho^{\frac{\alpha'}{2} p_A^2}$$

$$= \text{CONSTANT} \times \frac{1}{(p_A)^2}$$

O.K.: FOURIER TRANSFORM IN D-p IS $\frac{1}{|x-y|^{D-p-2}}$

CORRESPONDENCE BETWEEN STRINGY BOUNDARY STATE AND SUPERGRAVITY SOLUTIONS BECOMES MORE PRECISE IN $D=10$ SUPERSTRINGS REF: P. DI VECCHIA, A. LICCARDO, HEP-TH/9912161.

HERE WE PERFORMED A SEMI-QUANTITATIVE CHECK OF THE DOUBLE ROLE OF D_p BRANES:

A) THEY DESCRIBE OPEN-STRING SECTORS OF (SUPER)-STRING THEORY, CORRESPONDING TO THEIR EXCITATIONS.

<u>IMPORTANT</u>: AT LOW ENERGY THE OPEN-STRING SECTOR IS A GAUGE THEORY

B) THEY ARE SOURCES FOR CLOSED-STRING STATES. THEY ARE SOLUTIONS OF THE CLOSED-STRING EQUATIONS OF MOTION.
IN STRING THEORY, THESE SOLUTIONS ARE KNOWN AT LINEAR ORDER (BOUNDARY STATES)

<u>AND</u>

<u>IMPORTANT</u>: IN SOME CASE SUPERGRAVITY E.O.M. ARE RELIABLE (WHEN CURVATURE OF SOLUTIONS IS SMALLER THAN M_s).

<u>QUESTION</u>: CAN WE USE THIS DUAL ROLE OF D_p BRANES TO BETTER UNDERSTAND STRING THEORY AND/OR FIELD THEORY?