Pisa – February 9-17, 2017

Knots and Applications

Dalla matematica dei nodi alla topologia del DNA

Renzo L. Ricca

Department of Mathematics & Applications – U. Milano-Bicocca

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- **2. Lecture 2:** ribbon concept and self-linking invariant, writhe and twist, DNA topology, skein relations and knot polynomials, examples of computation.

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- **2. Lecture 2:** ribbon concept and self-linking invariant, writhe and twist, DNA topology, skein relations and knot polynomials, examples of computation.
- **3. Laboratory:** explorations with KnotAtlas, LinKnot, KnotPlot.

Lecture 1

- ***An intuitive approach:***

  - Knots in art and science: from arts and crafts to DNA biology

  - Knots in mathematics: theorems, numerics and platonic figures

- ***A little bit of history:***

  - Origin of knot theory: Kelvin's string theory and Tait's tabulation

- ***The key concept:***

  - Topological equivalence and invariants: playing with continuity

- ***Let's get serious:***

  - Elements of knot theory: classification issues and basic definitions

  - Gauss linking number: from definition to application

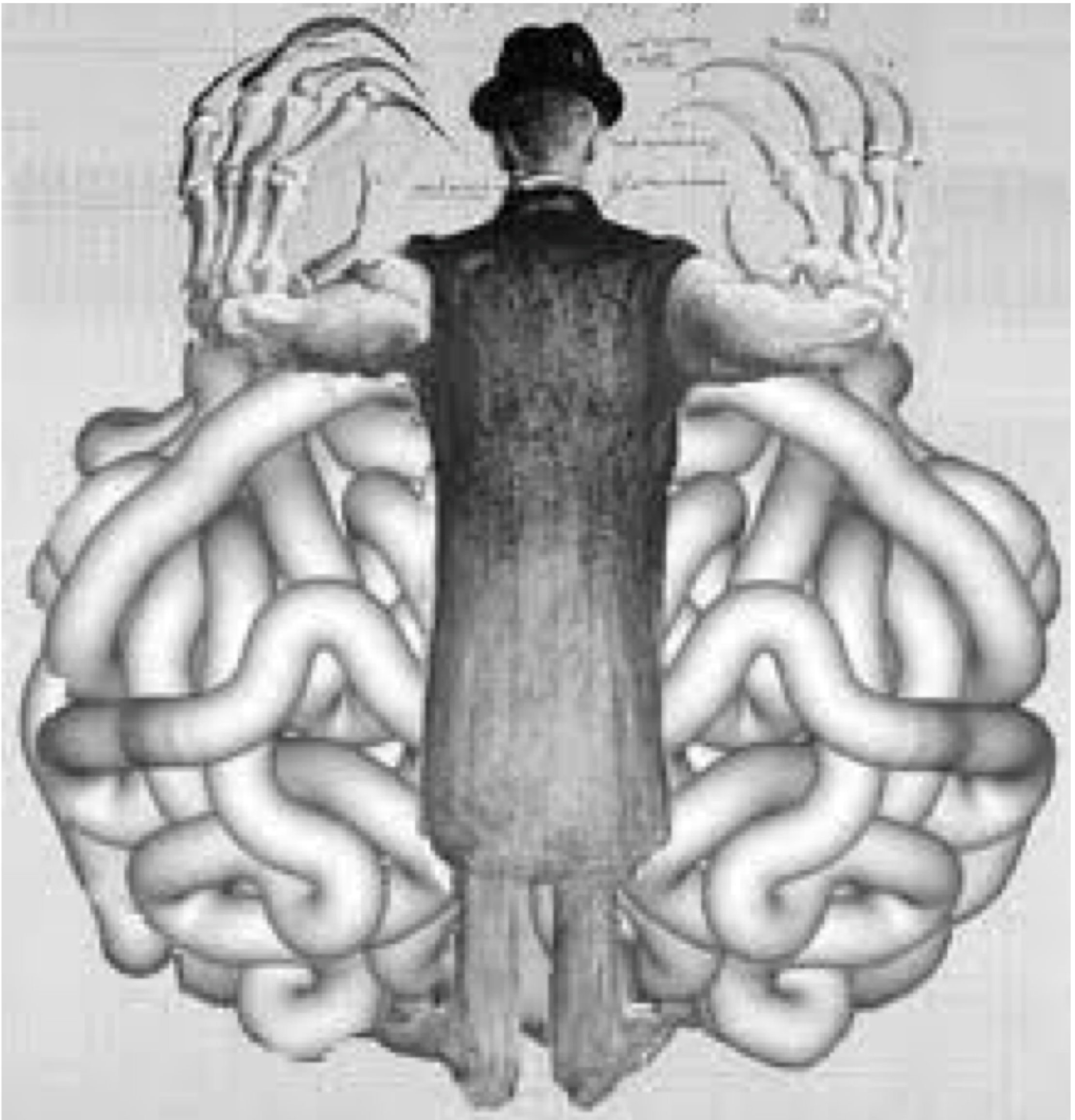
  - Minimal crossing number and minimal diagram

  - Tait's tabulation re-visited: a source of inspiration

  - Reidemeister's moves to encode topological equivalence

Funeral Celtic stone

Church of S. Croce (Florence): detail



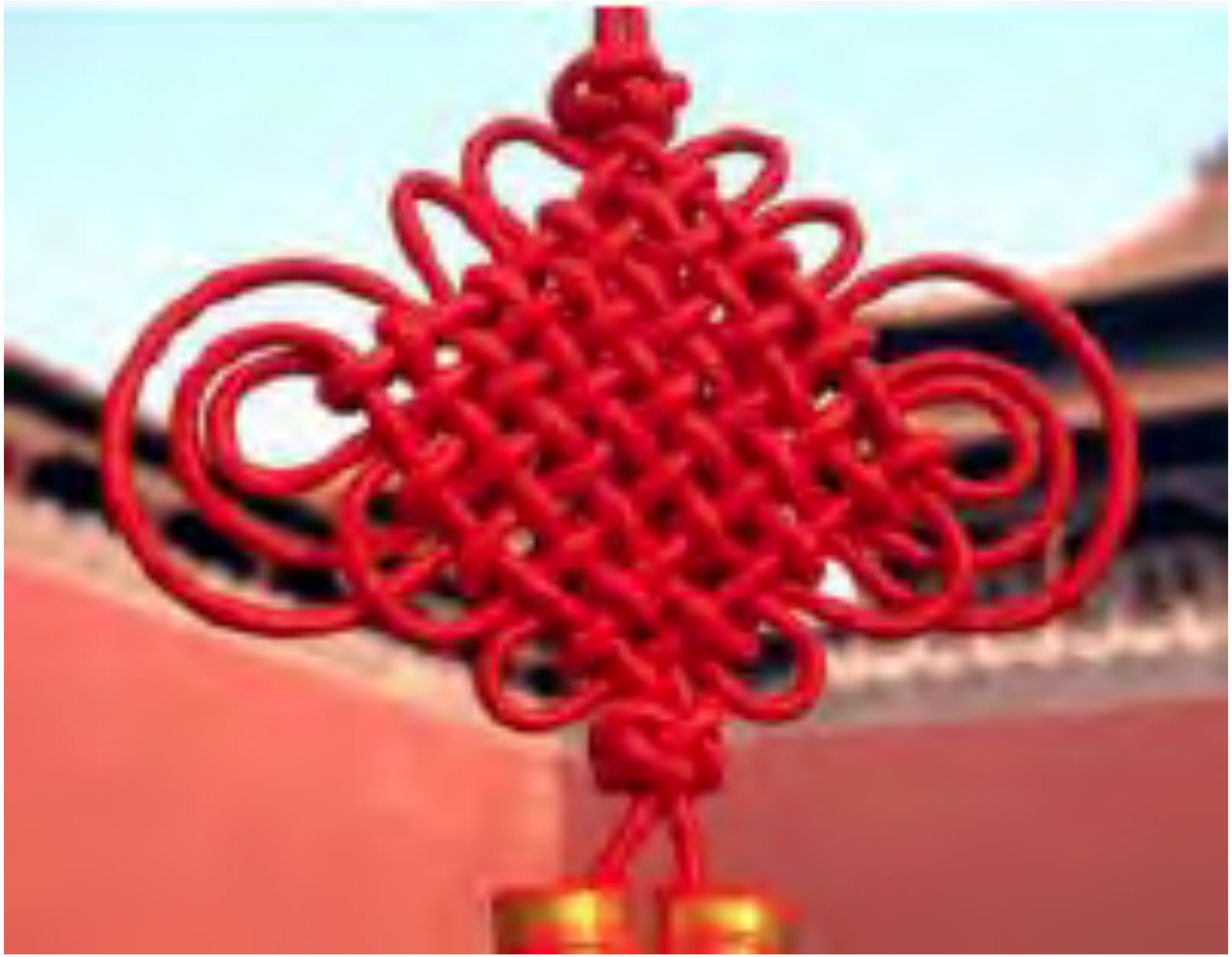
Pictorial representation of psyche

Knots in manifactures

Woven ropes of jute

Loop of rope

Knots in manifactures



Chinese ornament

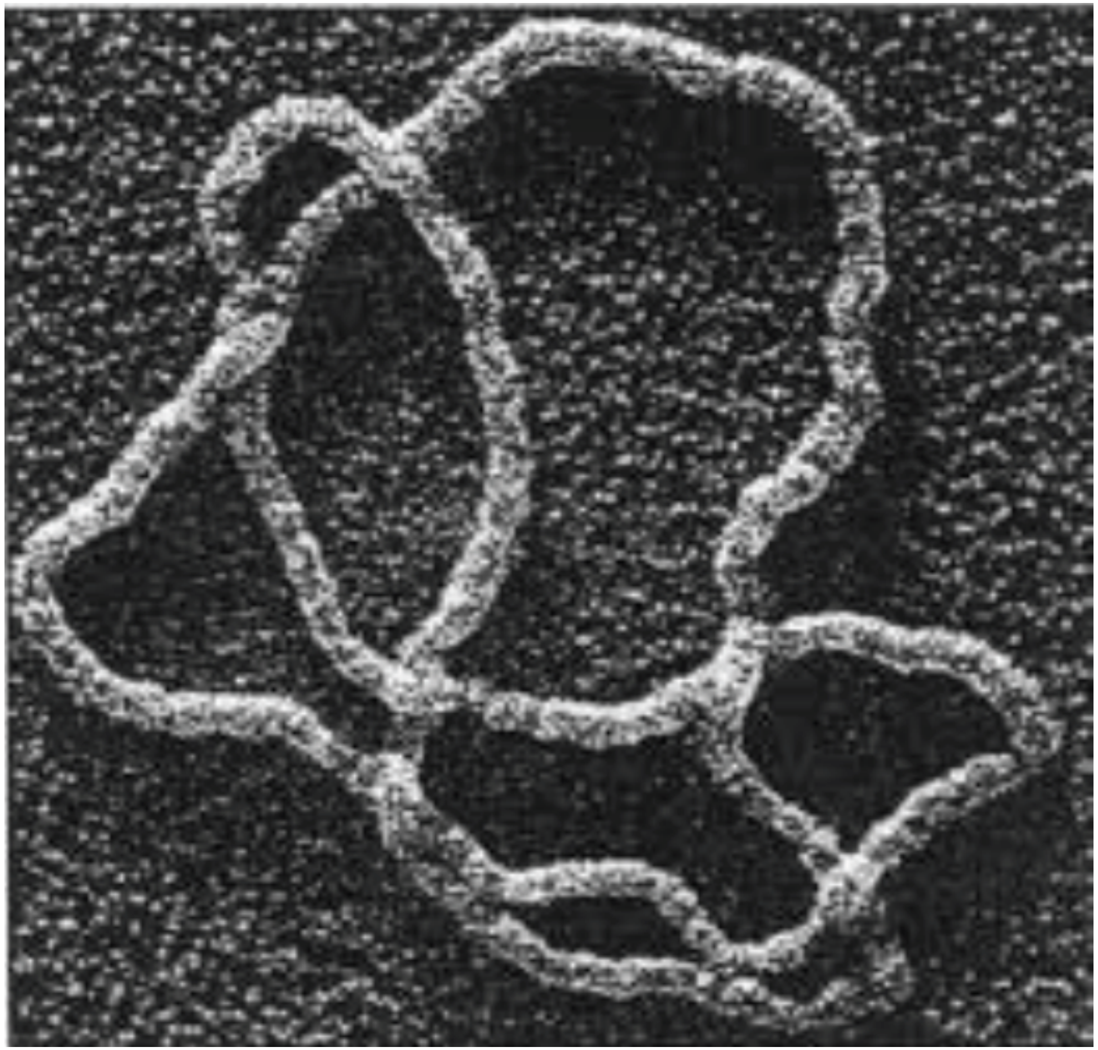
Knots in nature

Knotted grapevine

Knots in nature

Knotting of hagfish (Missinoide)

Knots in nature

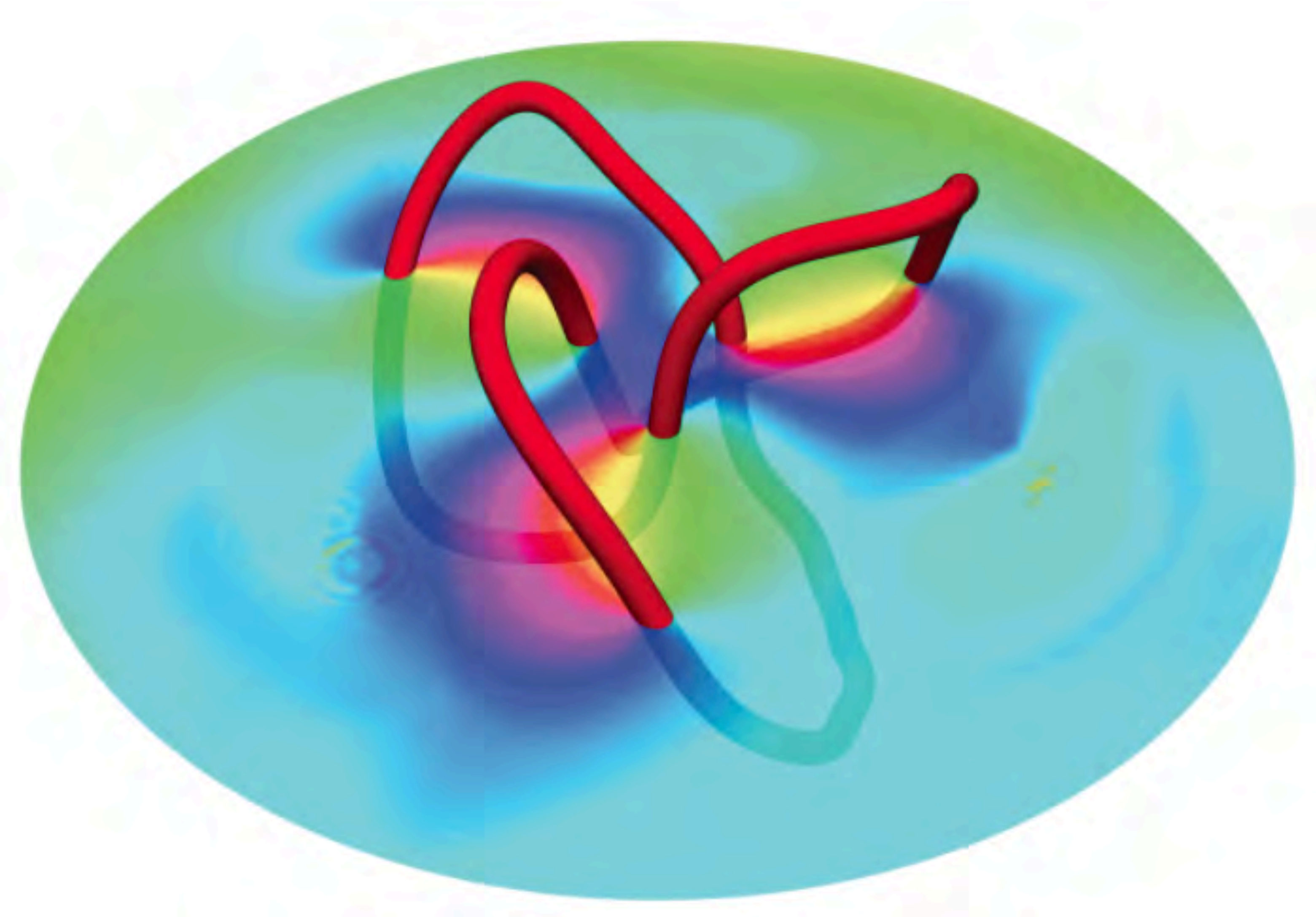


Knotted DNA (Dean et al., 1985)

Knots in technology

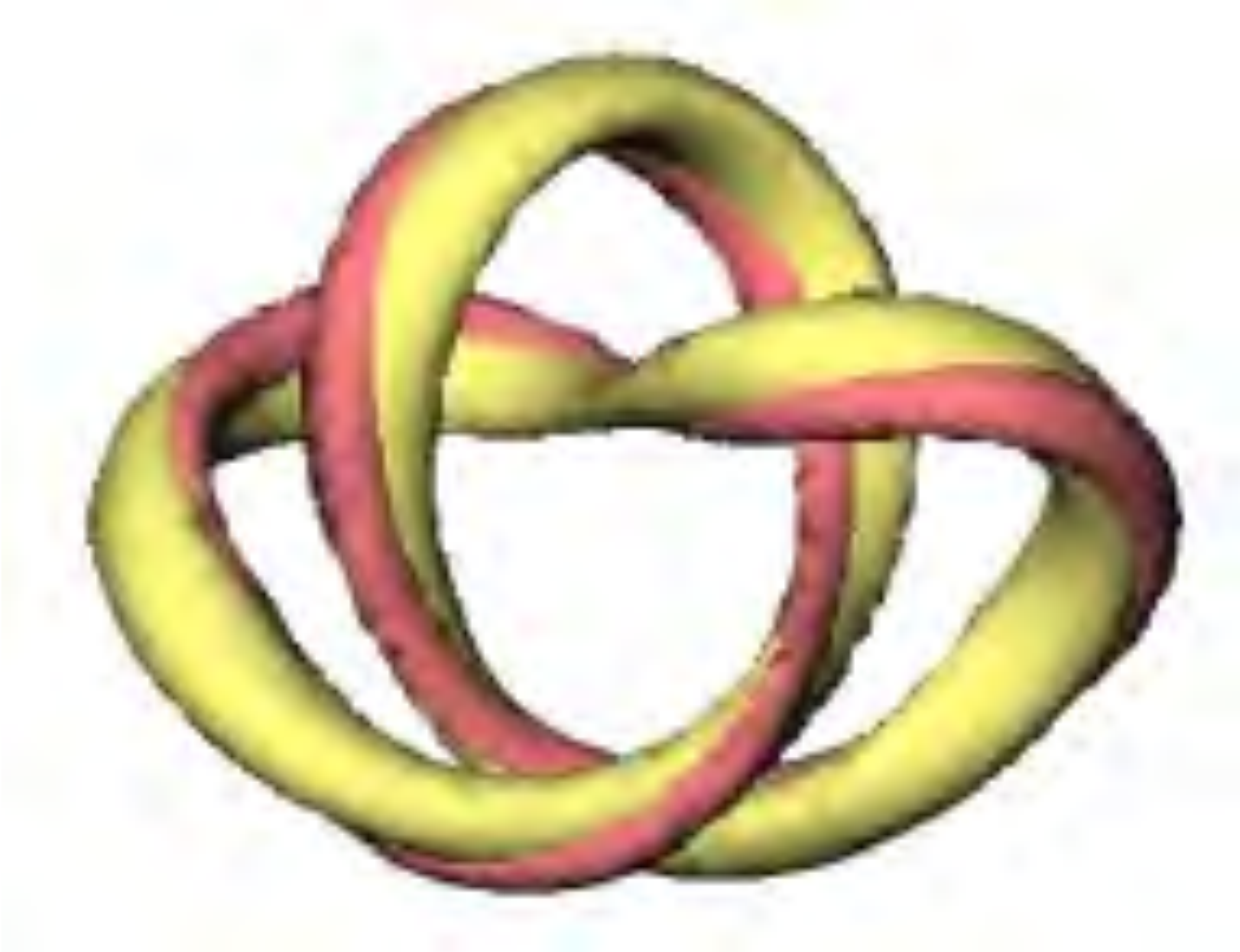
Linkage of crystal structures (Matsuura et al., 2006)

Knots in technology

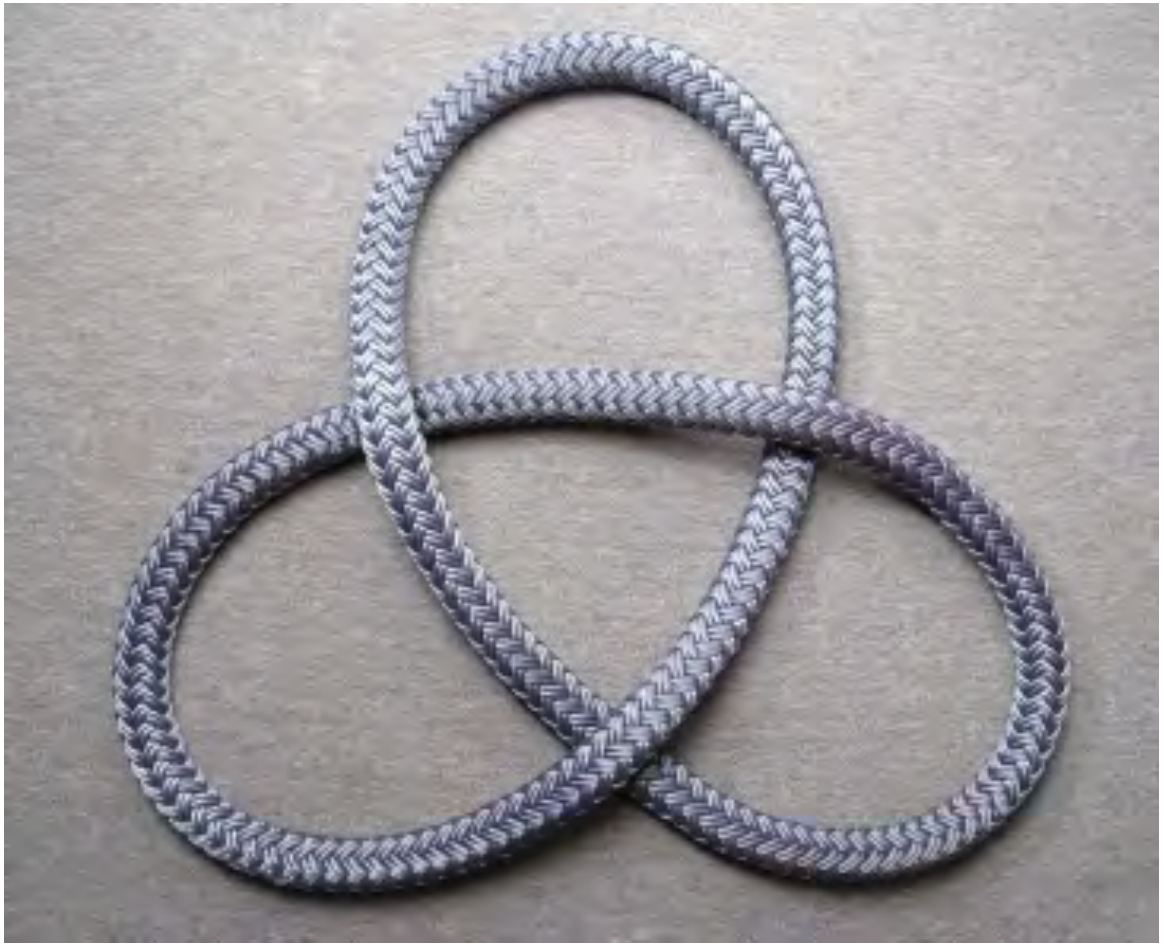


Phase defects in optics (Dennis et al, 2010)

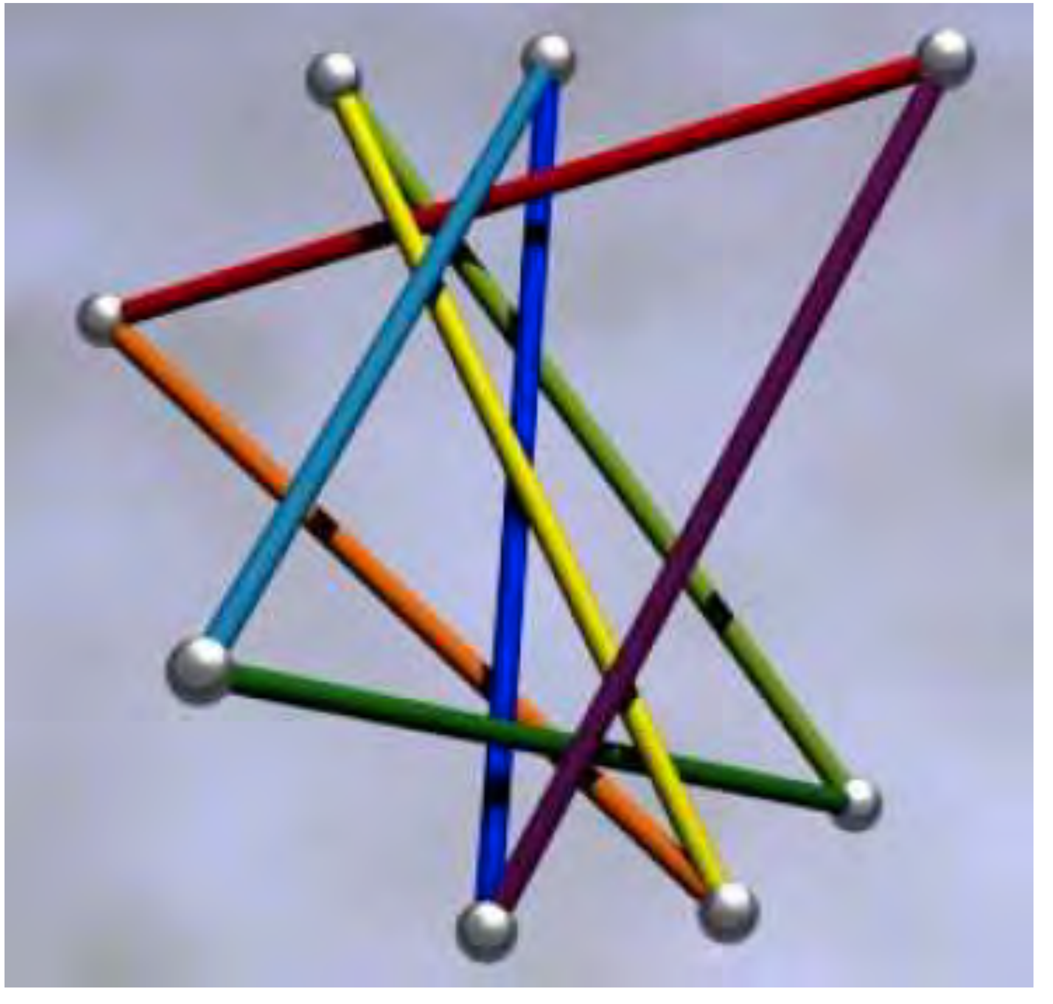
Knots in physical theory



***Solitons in field theory** (Sutcliffe, 2007)*

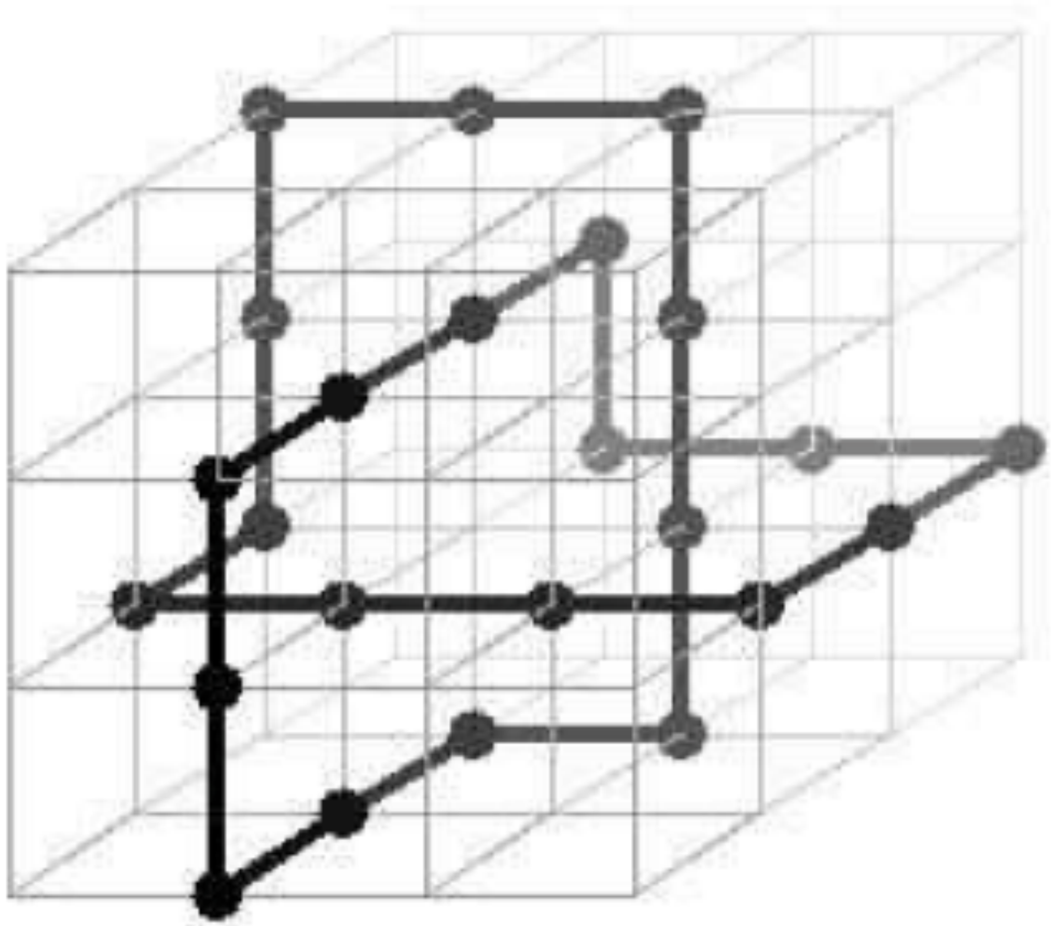


Trefoil knot



Polygonal knot of struts in computer simulation

Knots in mathematics

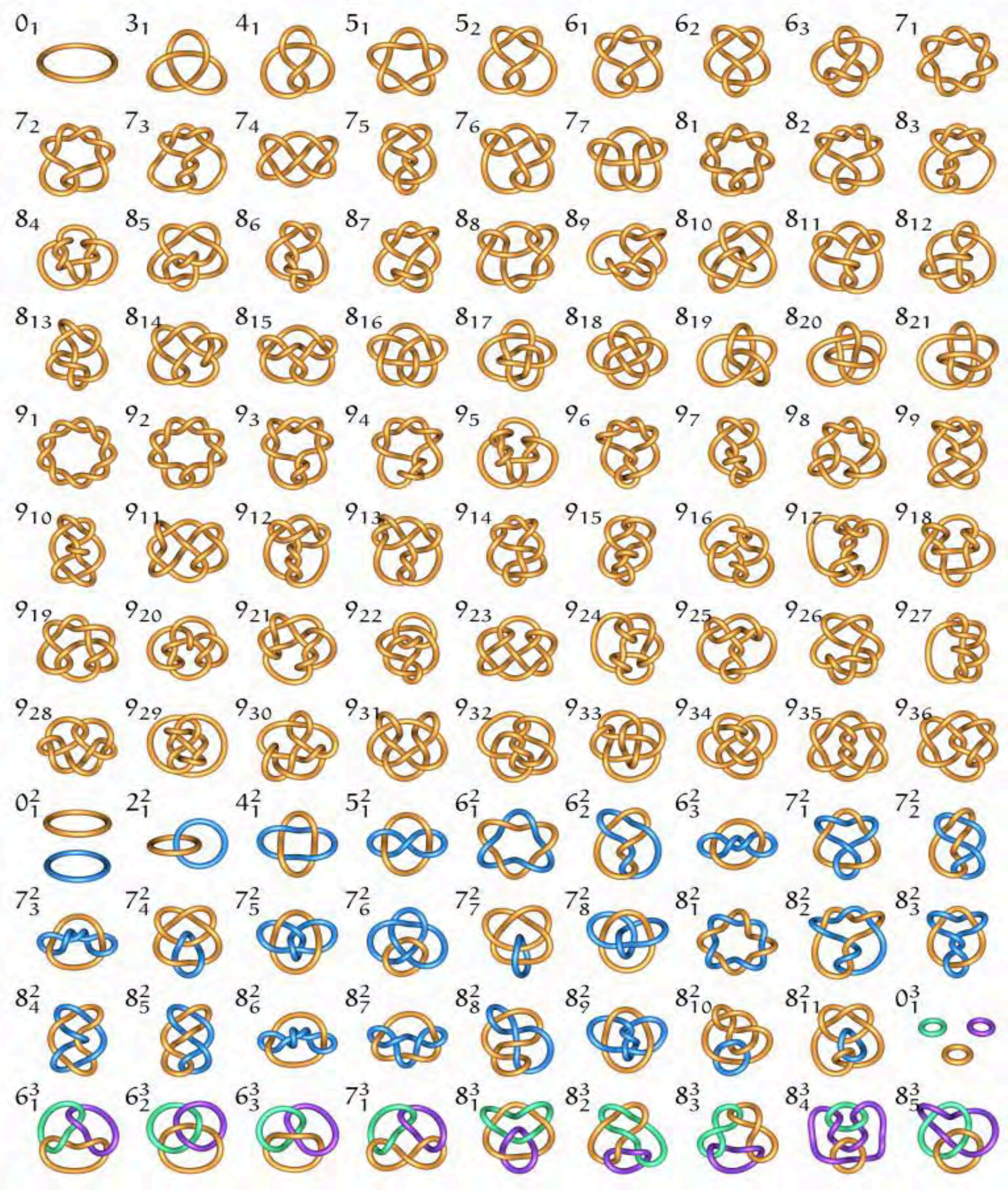


Knot in a cubic lattice

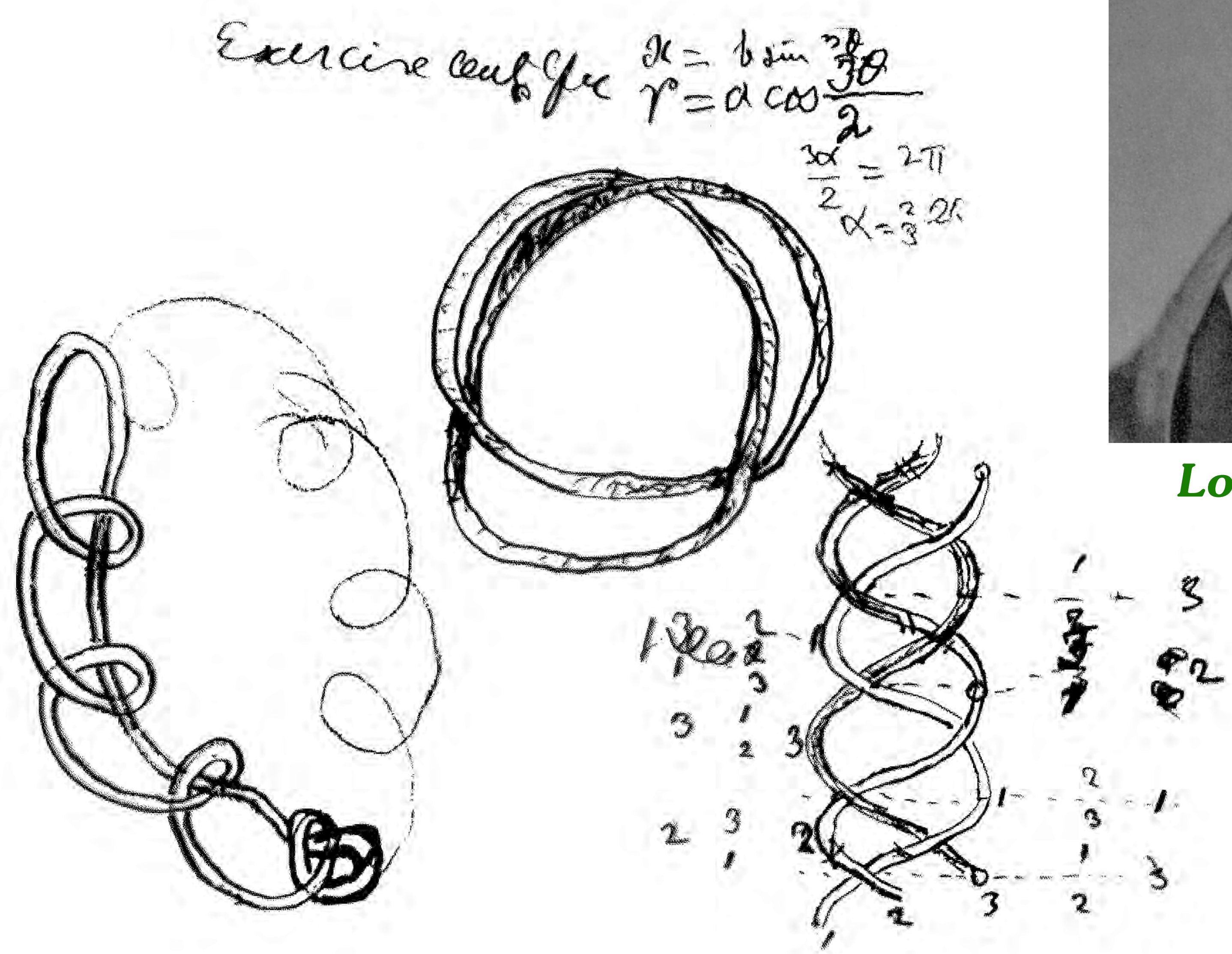
Knots in mathematics

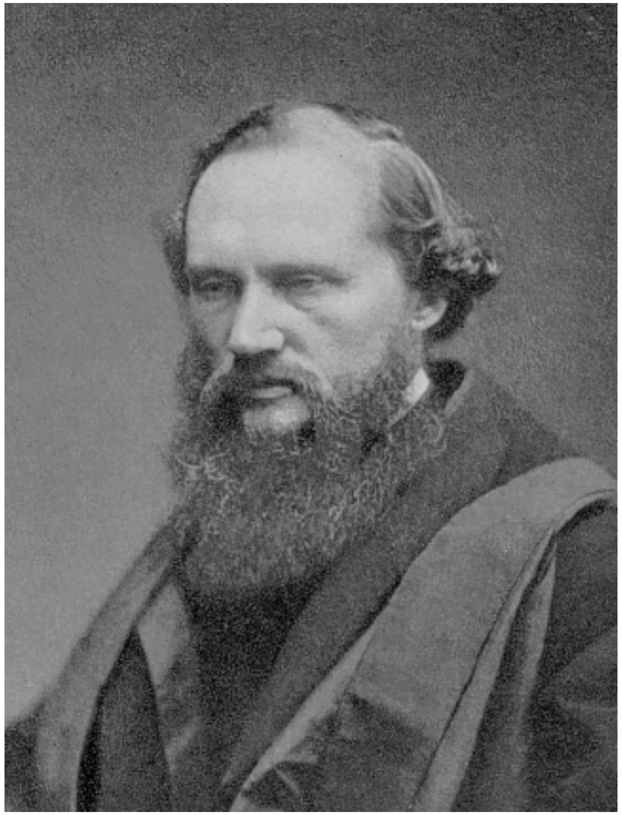
Plato's ideal trefoil knot

Knot table by KnotPlot



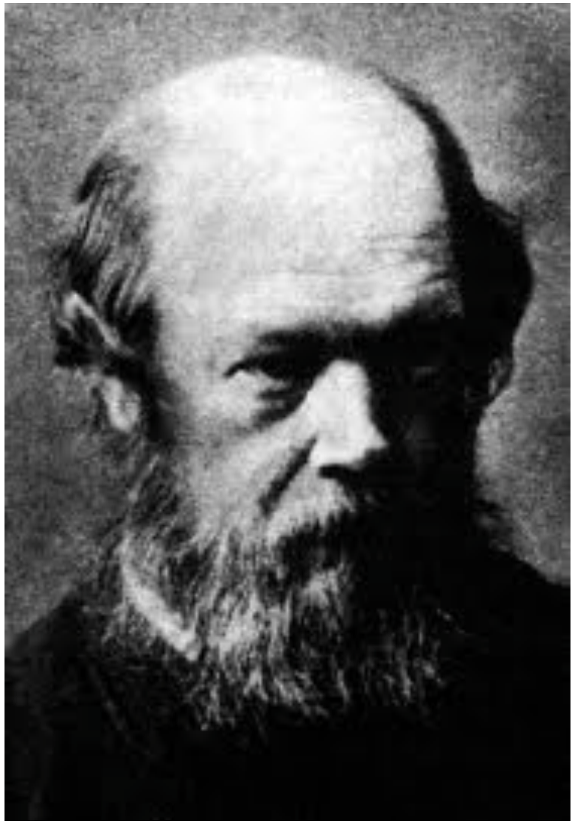
Knots in the vortex atom theory





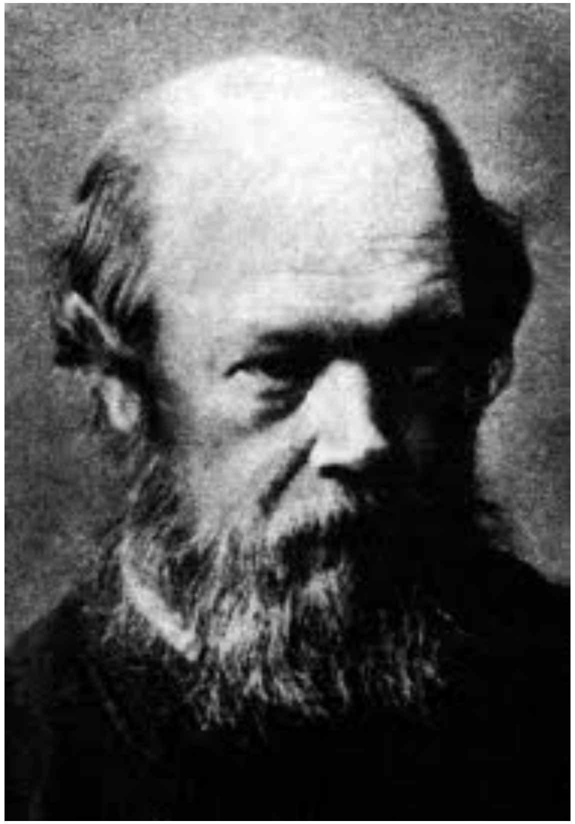
Lord Kelvin (1870)

Knot tabulation



P.G. Tait (1870)

Knot tabulation



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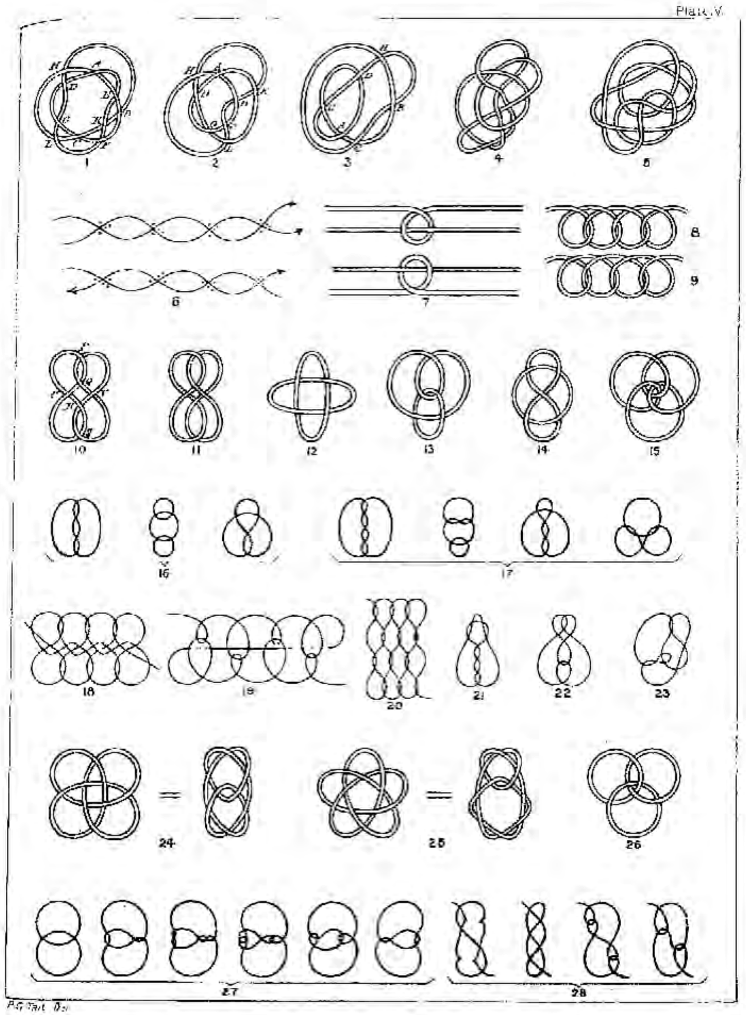
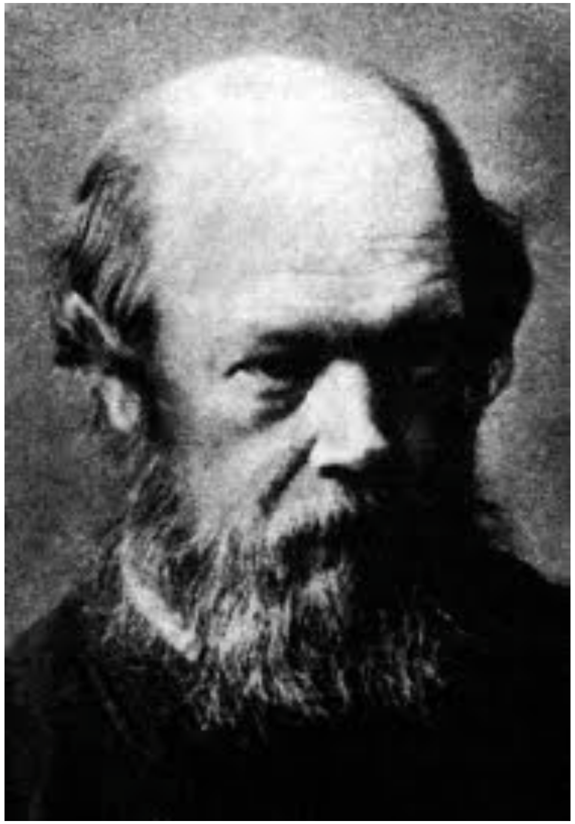


Knot tabulation



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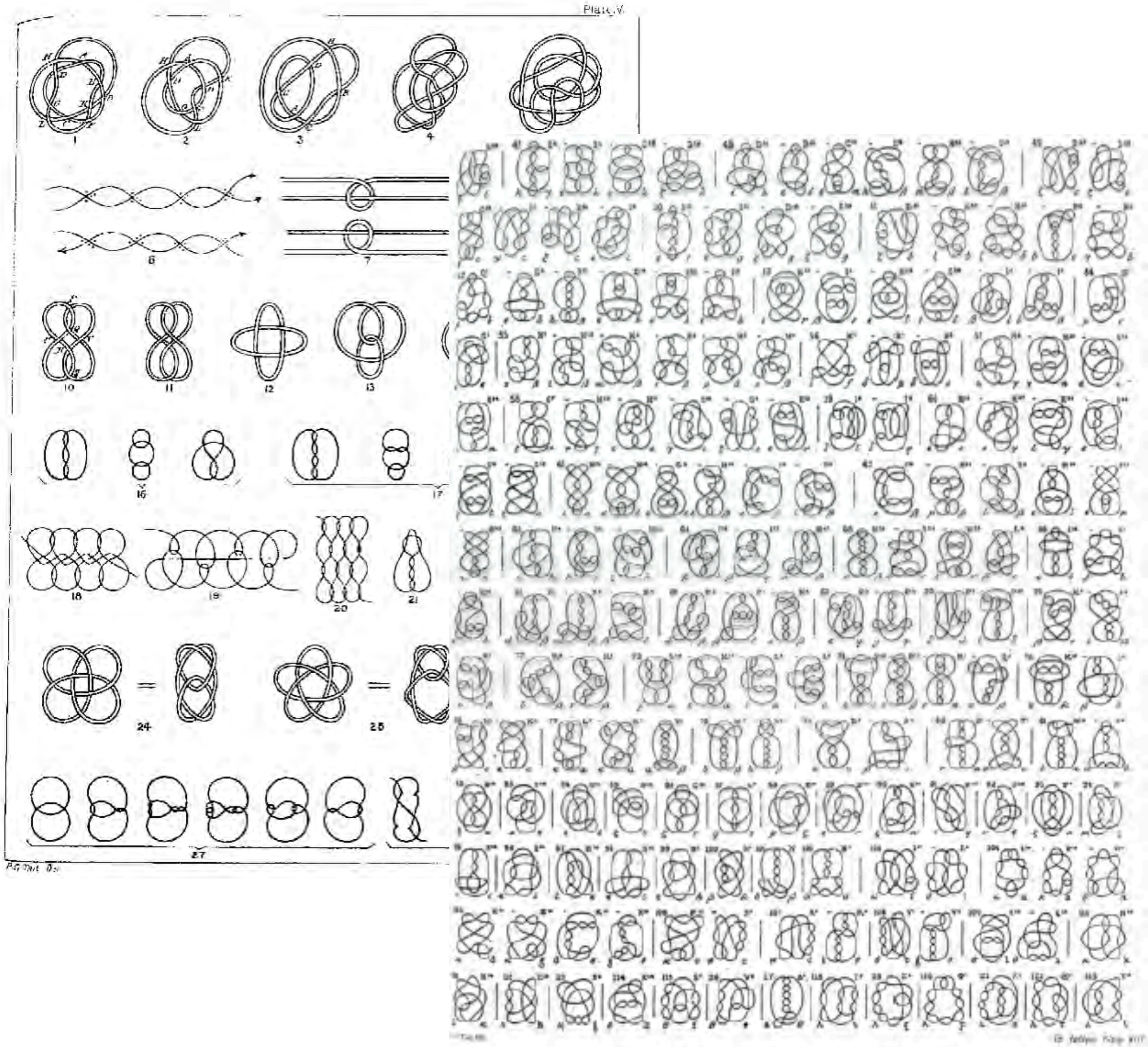

First equations

Es seien die Coordinaten eines unbestimmten Punkts der ersten Linie x, y, z; der zweiten x', y', z' und

$$\iint \frac{(x'-x)(\mathrm{d}y\,\mathrm{d}z'-\mathrm{d}z\,\mathrm{d}y')+(y'-y)(\mathrm{d}z\,\mathrm{d}x'-\mathrm{d}x\,\mathrm{d}z')+(z-z')(\mathrm{d}x\,\mathrm{d}y'-\mathrm{d}y\,\mathrm{d}x')}{[(x'-x)^2+(y'-y)^2+(z'-z)^2]^{\frac{3}{2}}} = V$$

dann ist dies Integral durch beide Linien ausgedehnt

$$= 4m\pi$$

und m die Anzahl der Umschlingungen.

Der Werth ist gegenseitig, d. i. er bleibt derselbe, wenn beide Linien gegen einander umgetauscht werden. 1833. Jan. 22.

K.F. Gauss (1833)

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K.F. Gauss (1833)

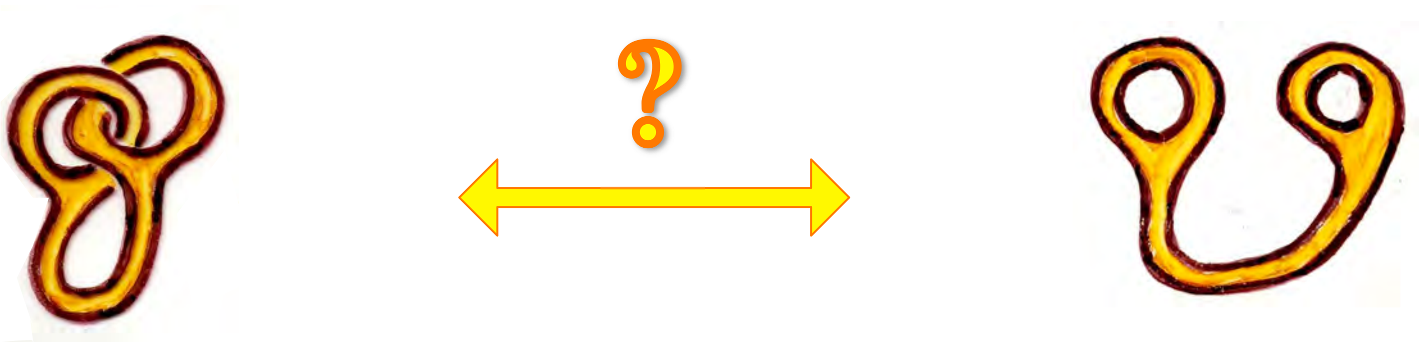
then
$$\iint \frac{ds\,d\sigma}{r^2} \begin{vmatrix} L & M & N \\ l & m & n \\ \lambda & \mu & \nu \end{vmatrix}$$

$$= \iint \frac{ds\,d\sigma}{r^2} \left[\left(1-\frac{dr}{ds}^2\right)\left(1-\frac{dr}{d\sigma}^2\right) - \left(r\frac{d^2r}{ds\,d\sigma}\right)^2 \right]^{\frac{1}{2}}$$

$$= 4\pi n$$

J.C. Maxwell (1867)

The concept of topological equivalence and invariants



The concept of topological equivalence and invariants

- ***Re-arrangement of internal structure***

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The concept of topological equivalence and invariants

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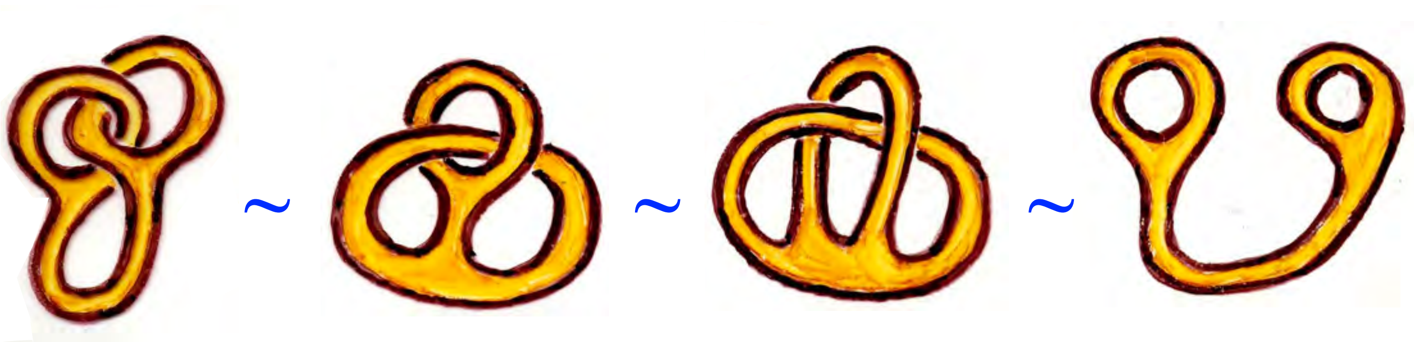
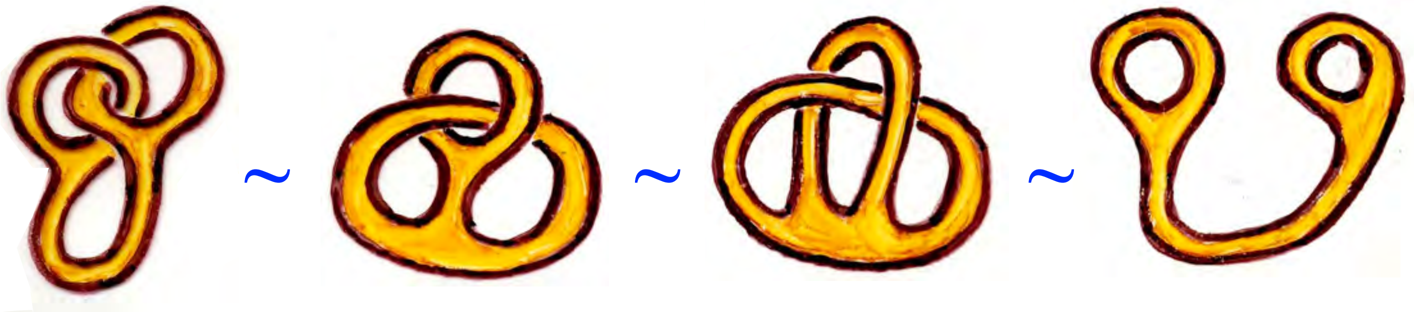
The concept of topological equivalence and invariants

- Re-arrangement of internal structure



The concept of topological equivalence and invariants

- *Re-arrangement of internal structure*



- *Linked pretzel*

The concept of topological equivalence and invariants

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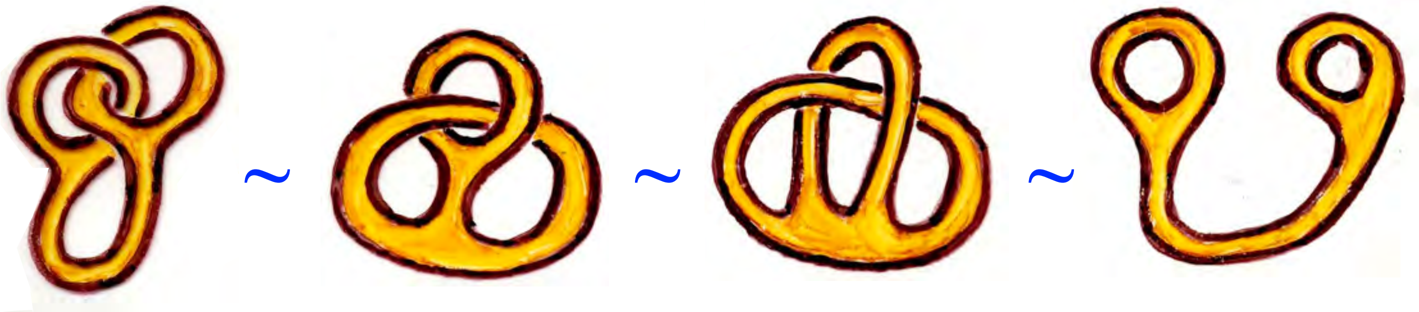

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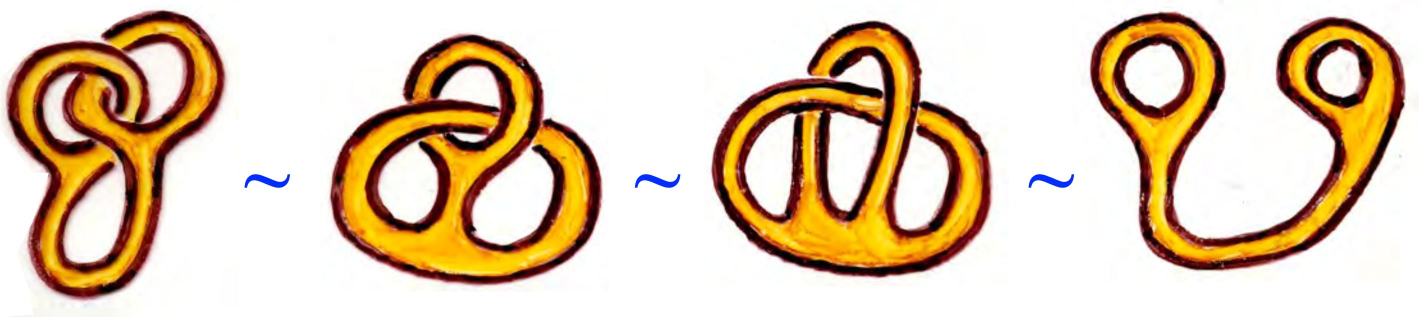

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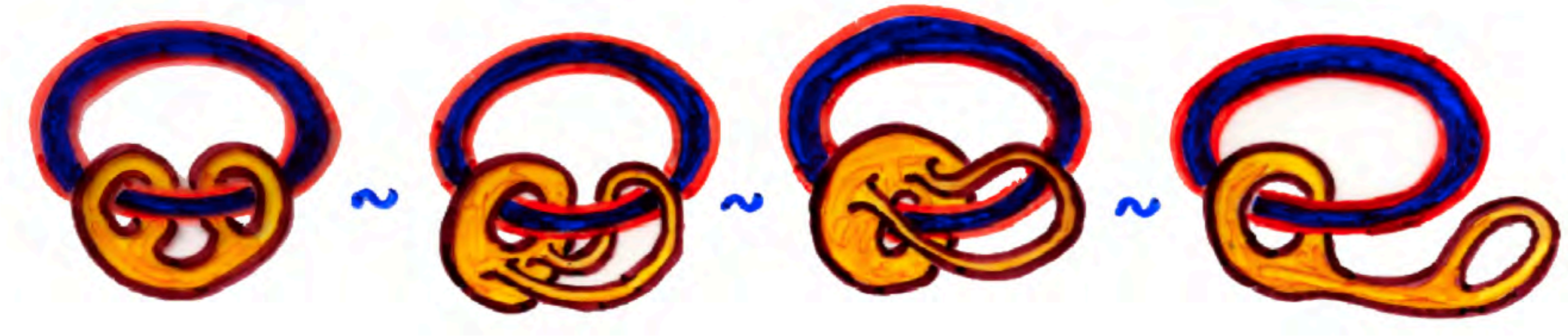

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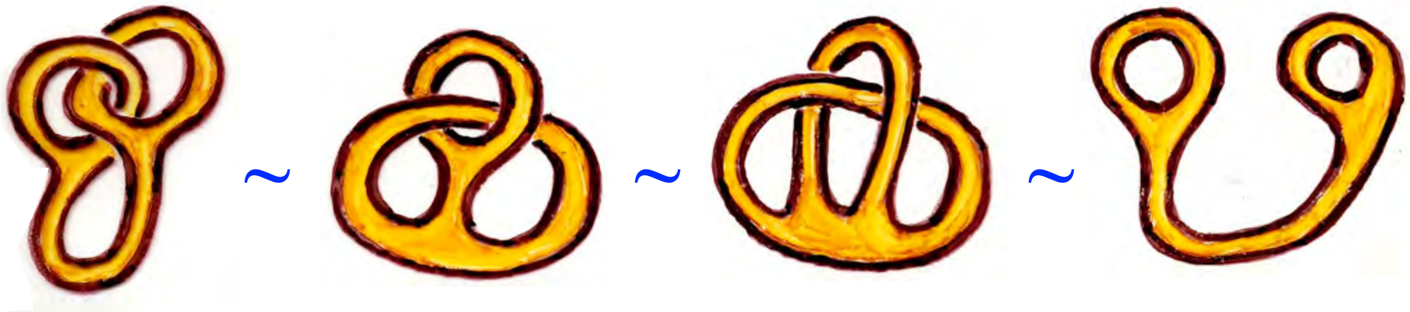

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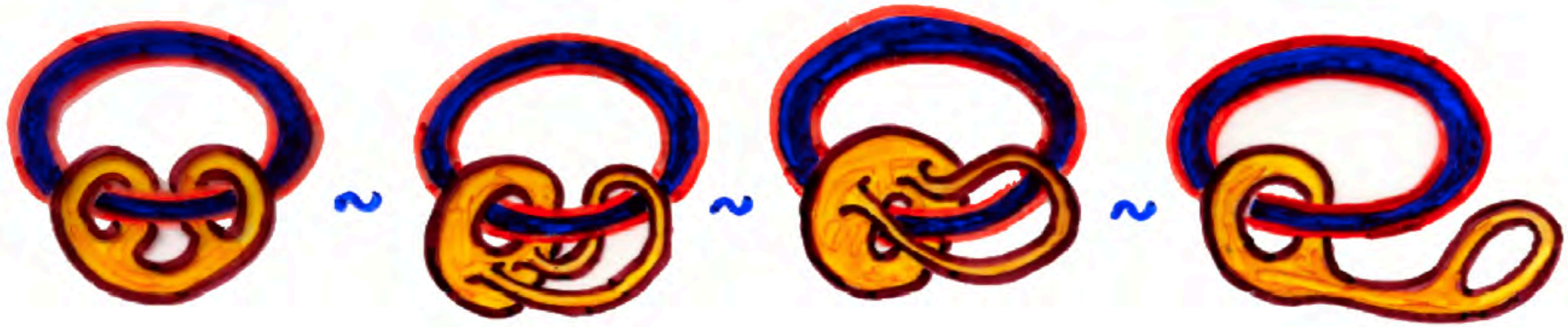

- ***Knotted pretzel***

The concept of topological equivalence and invariants

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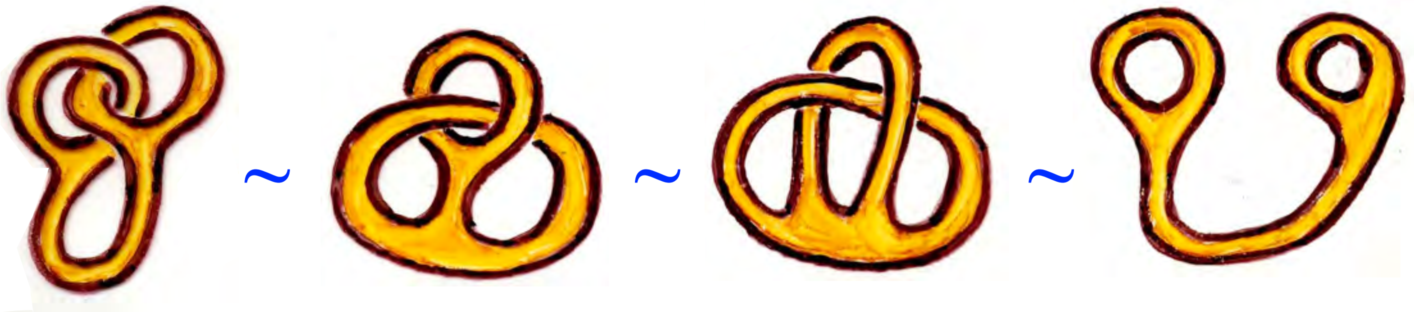

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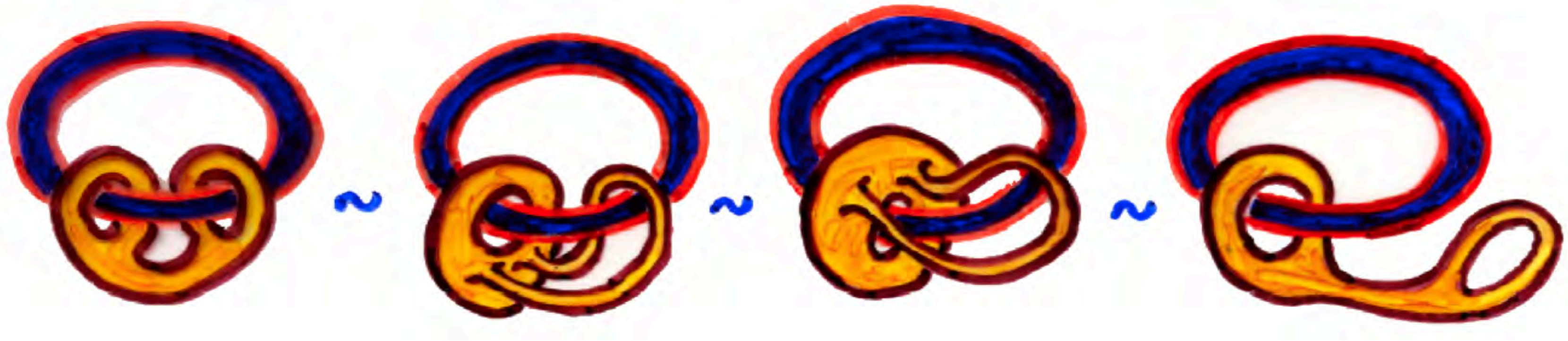

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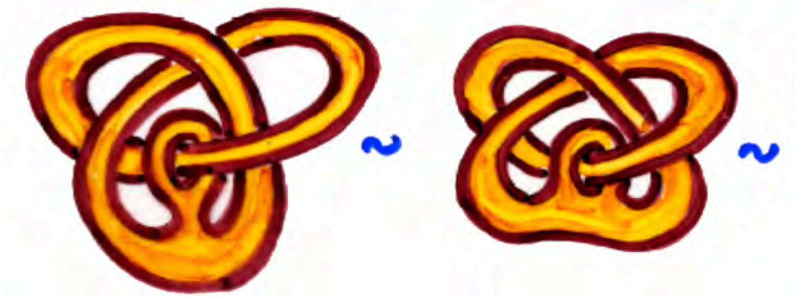

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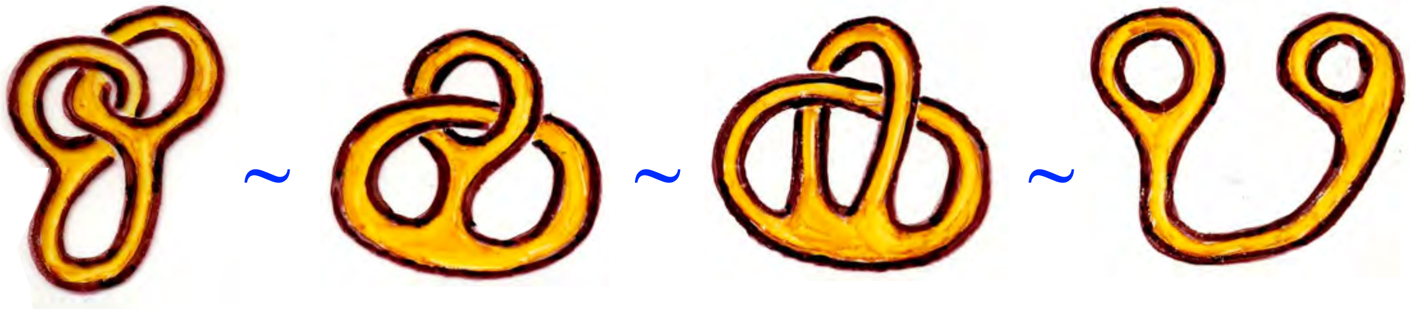

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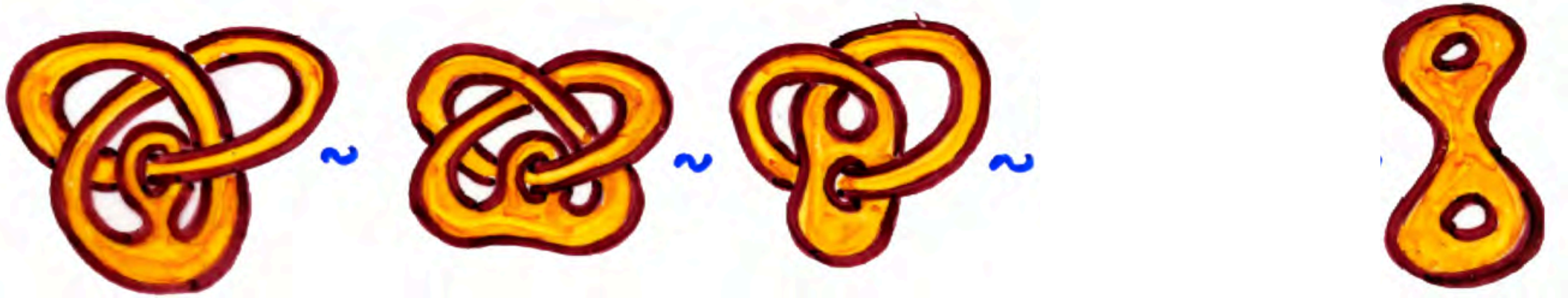

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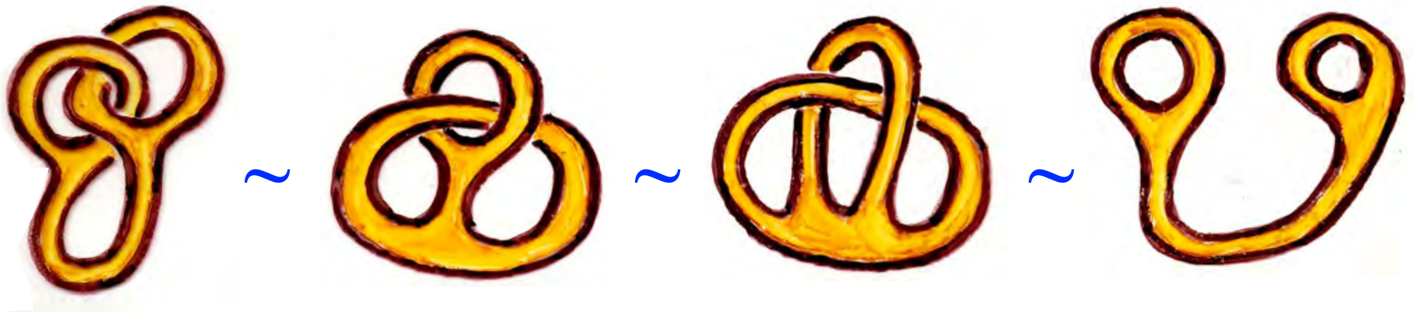

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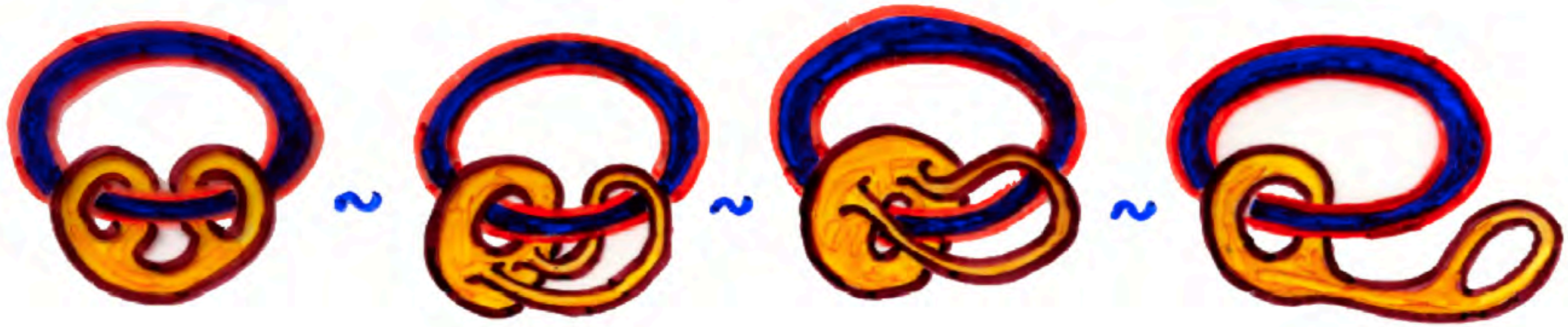

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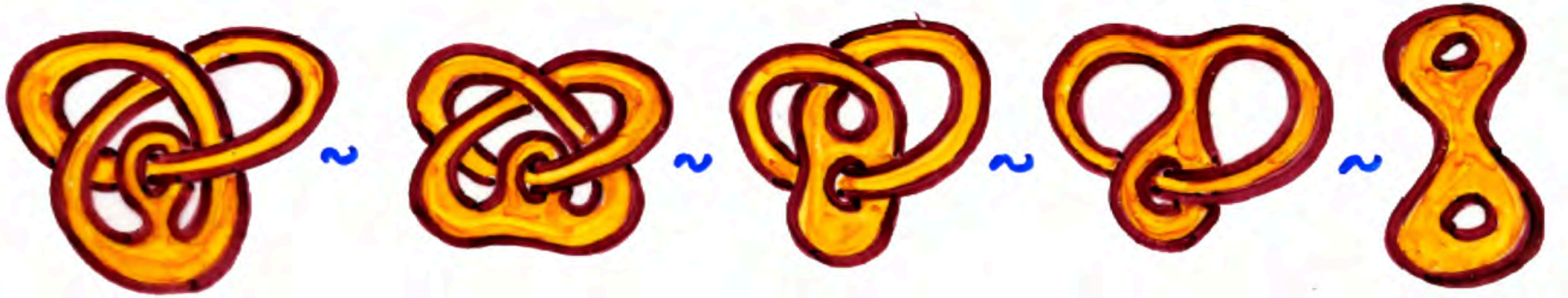
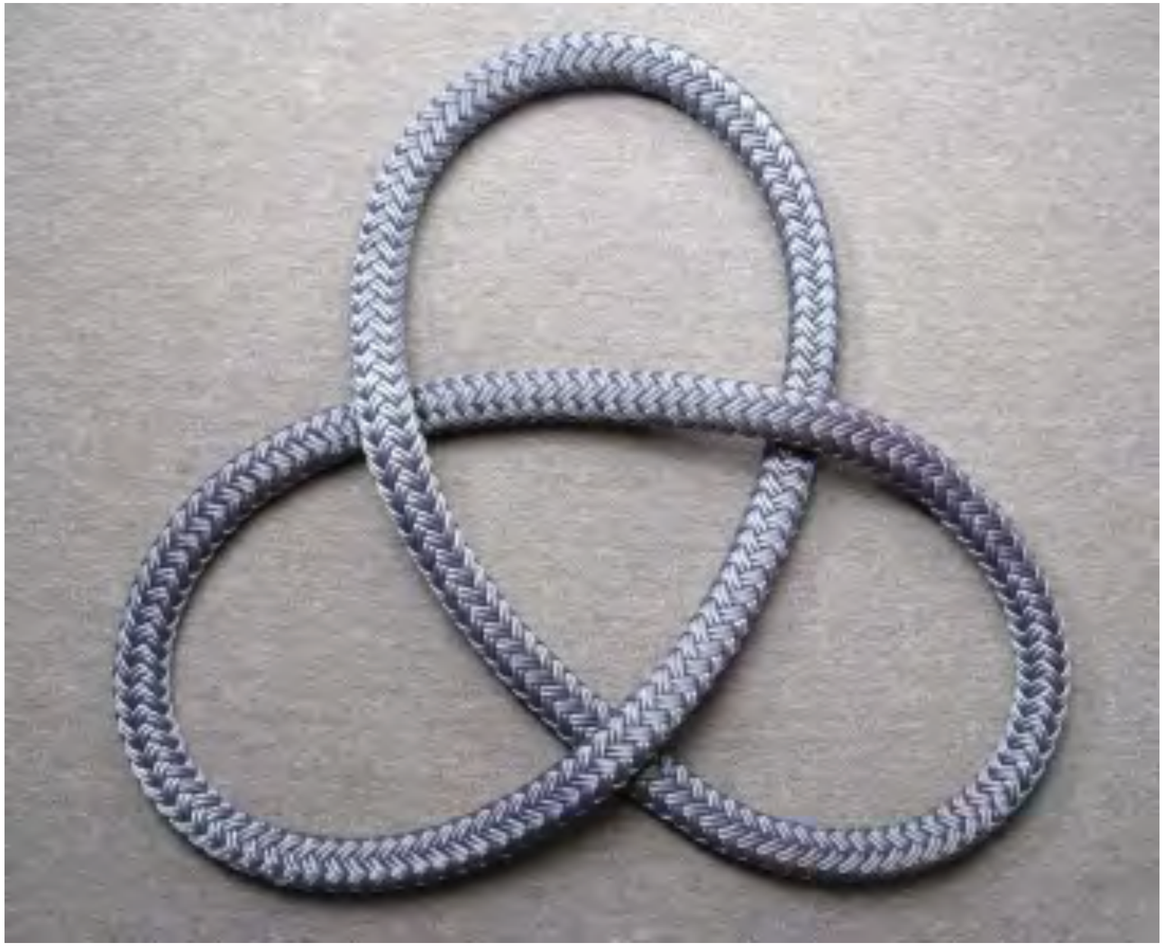


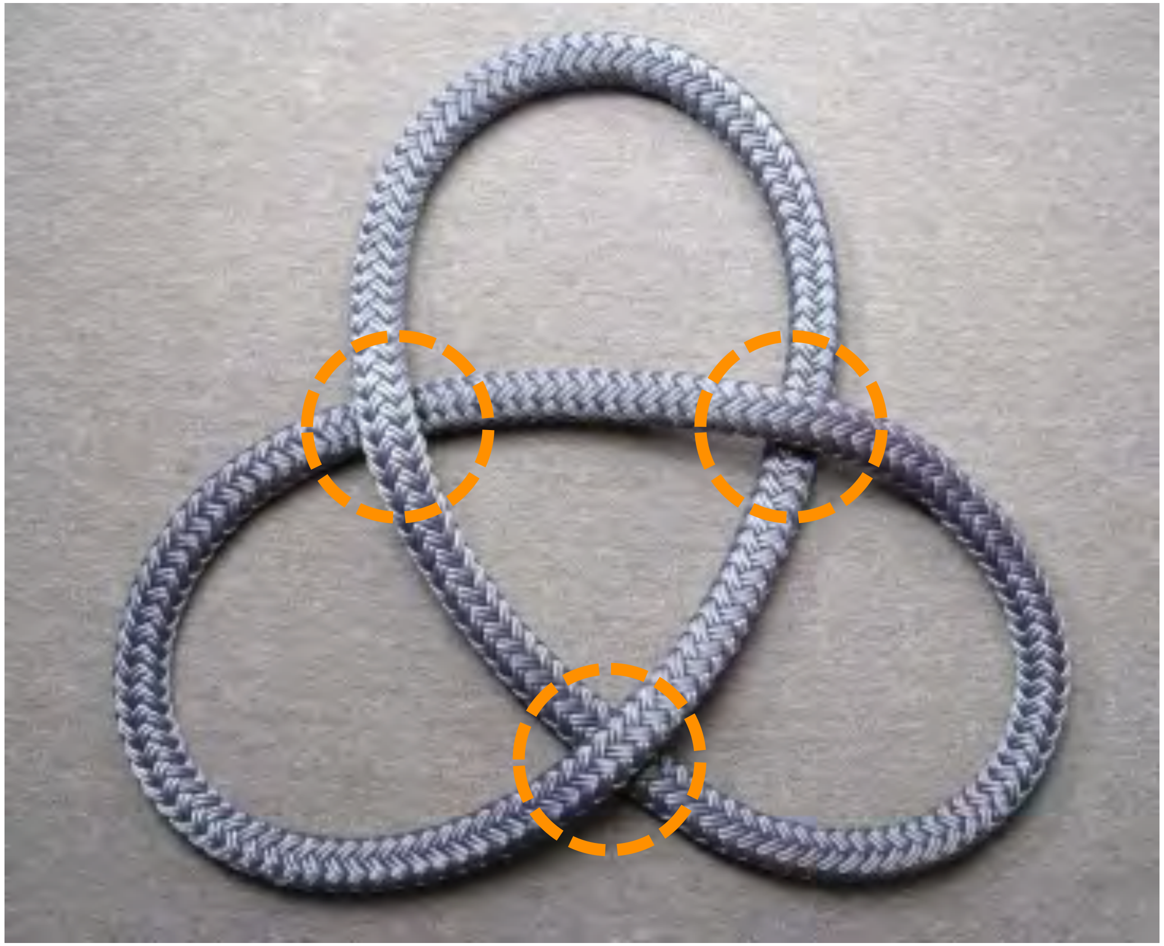
What to study on knots?



Trefoil knot

What to study on knots?

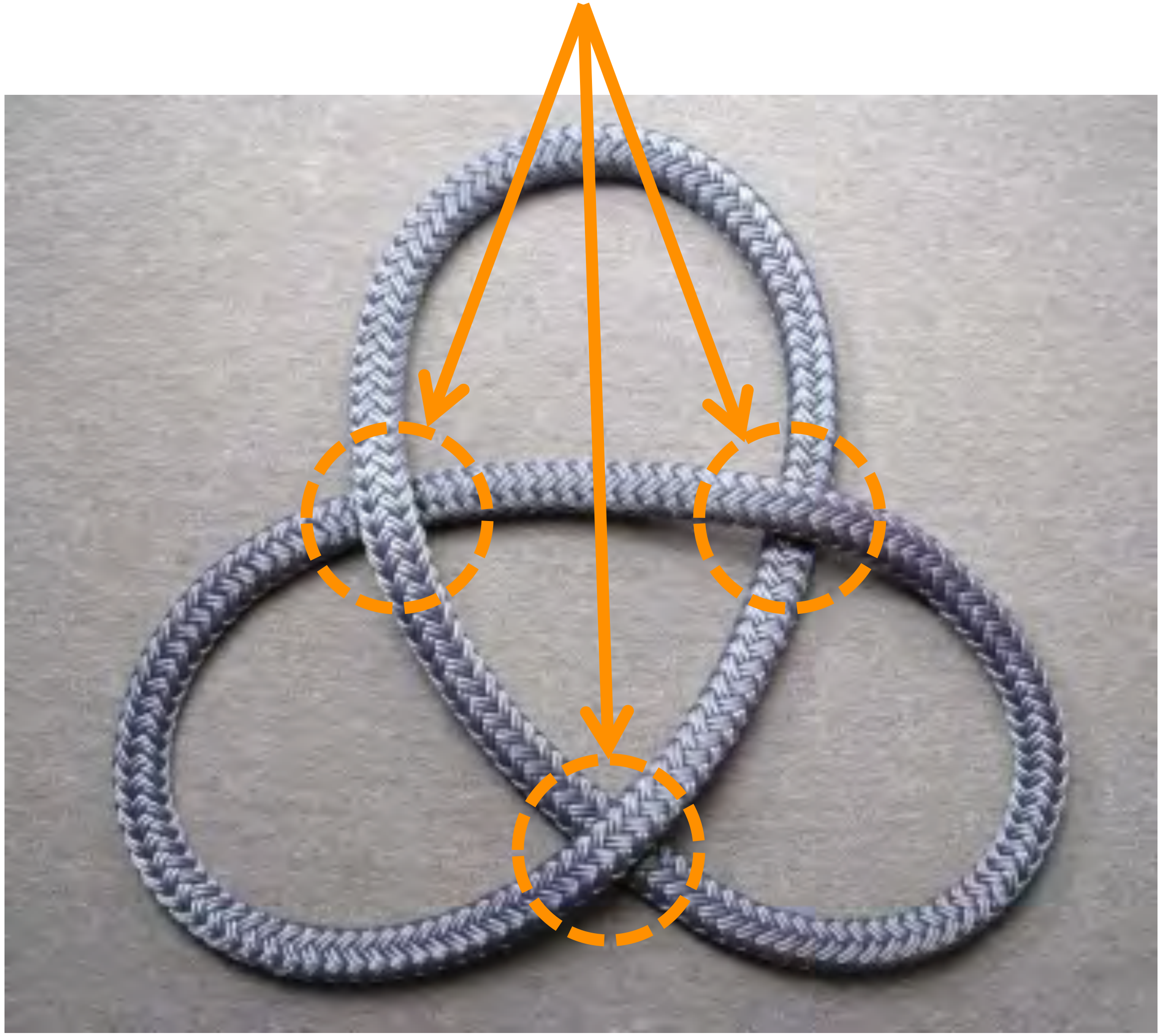
- ***Minimum number of crossings:*** $c_{\min} = \min(\#)$



Trefoil knot

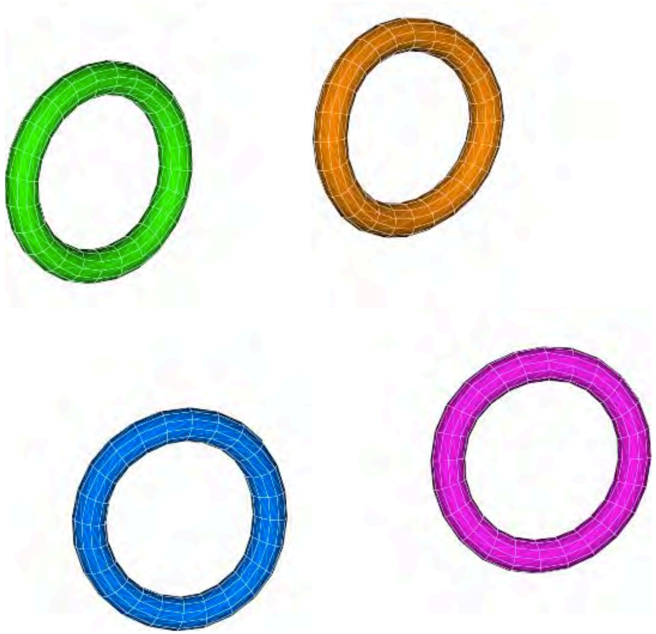
What to study on knots?

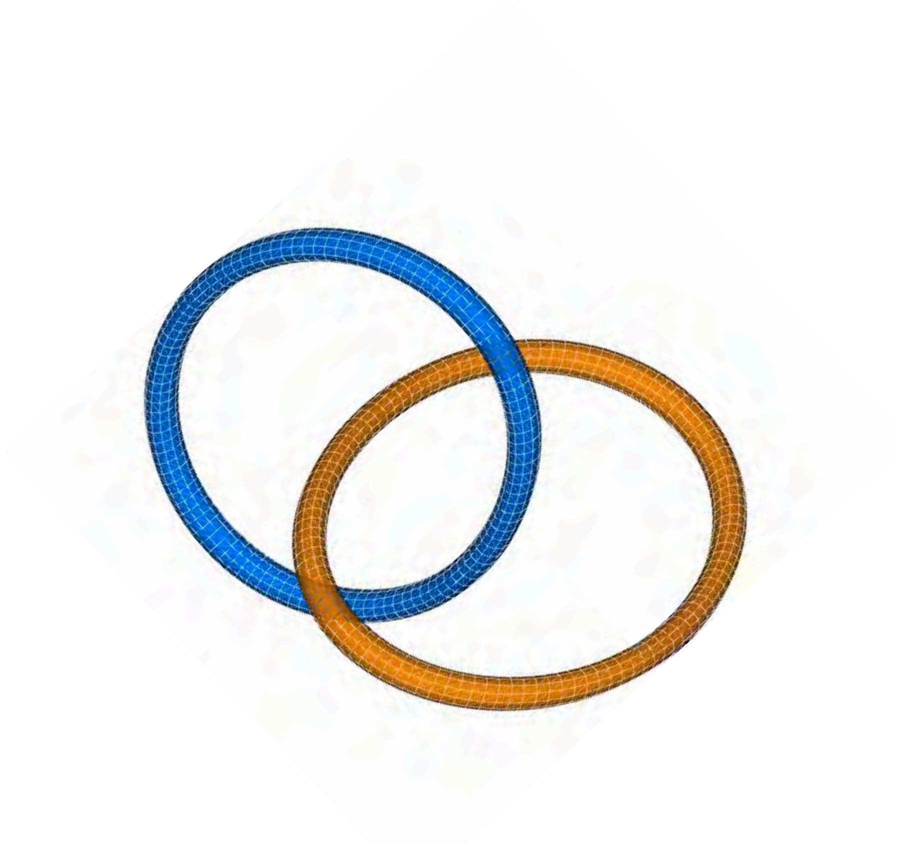
- **Minimum number of crossings:** $c_{\min} = 3$



Trefoil knot

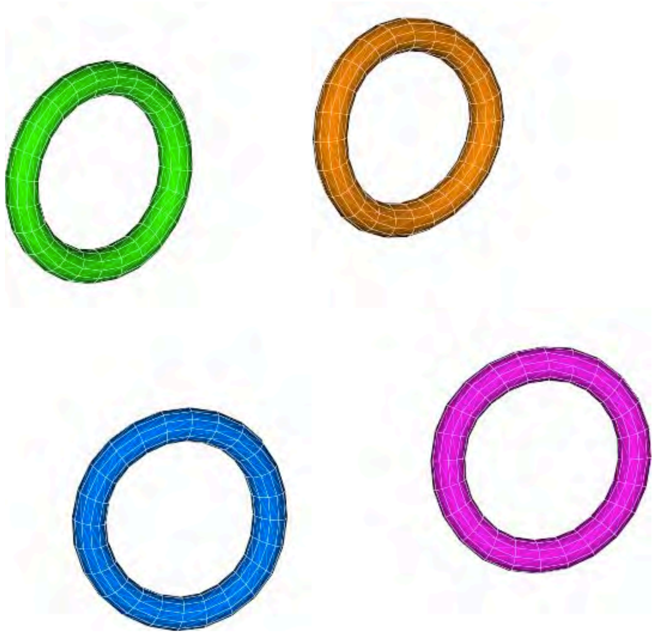
What to study on links?



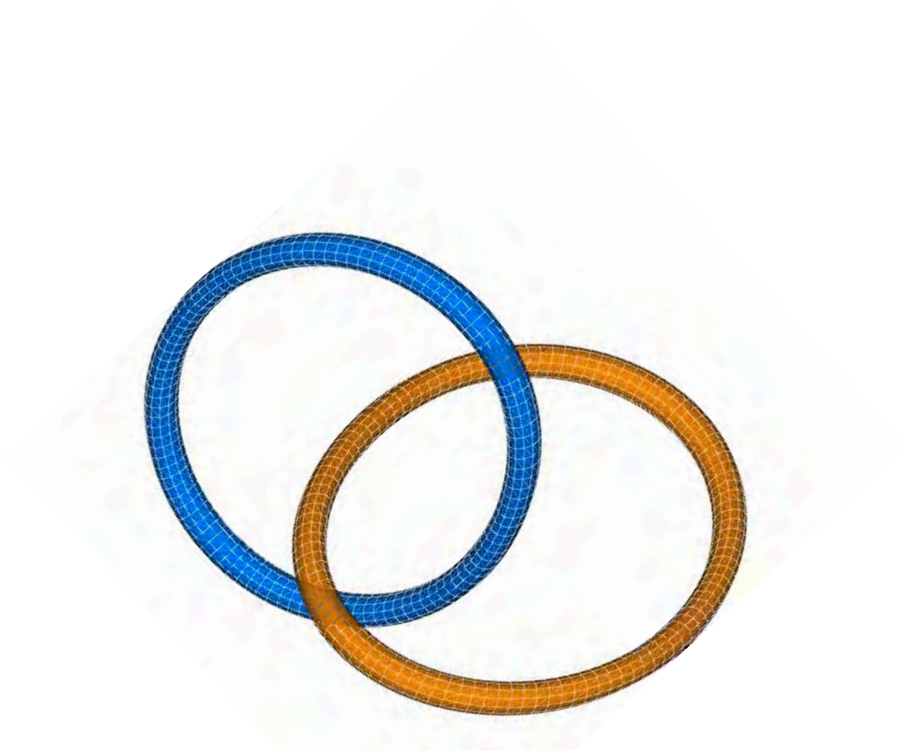


What to study on links?

- ***Number of components:*** N



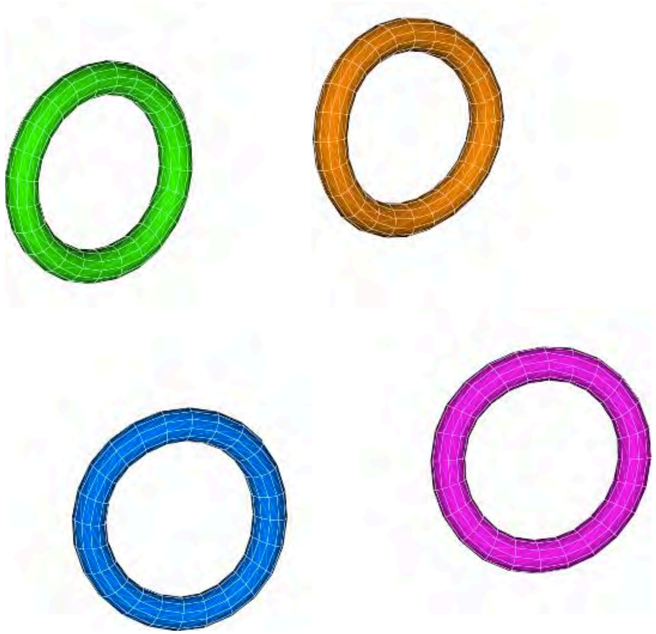
$N = 4$



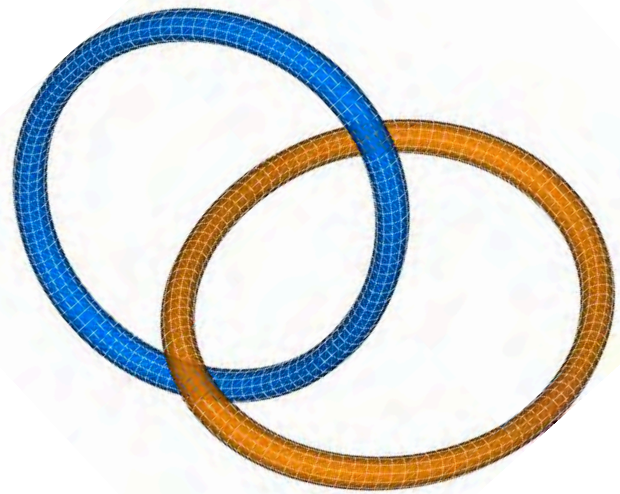
$N = 2$

What to study on links?

- ***Number of components:*** N
- ***Minimum number of crossings:*** $c_{\min} = \min(\#)$



$N = 4$, $c_{\min} = 0$



$N = 2$, $c_{\min} = 2$

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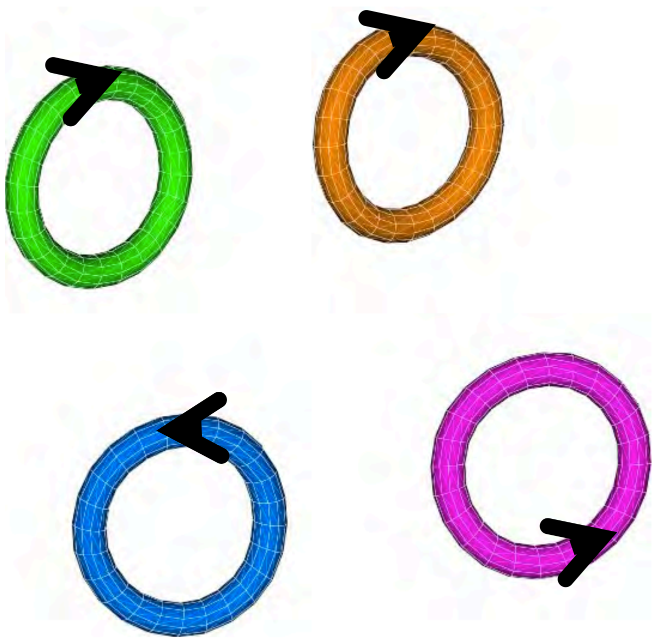
- ***Number of components:*** N
- ***Minimum number of crossings:*** $c_{\min} = \min(\#)$



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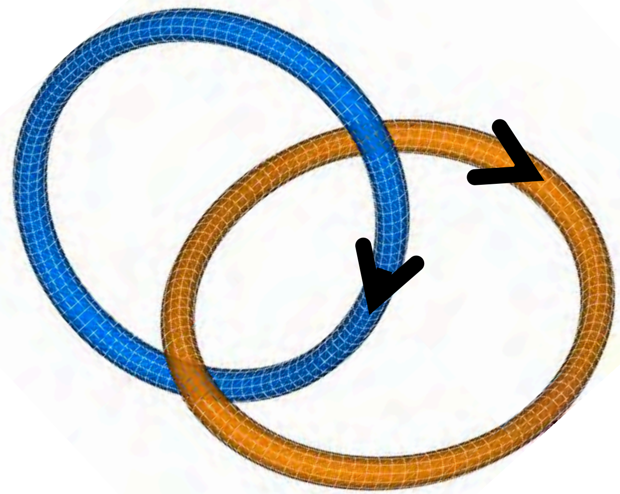
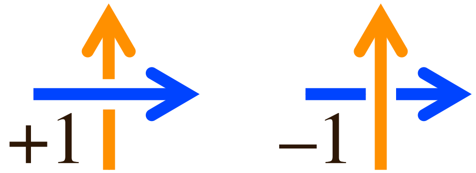


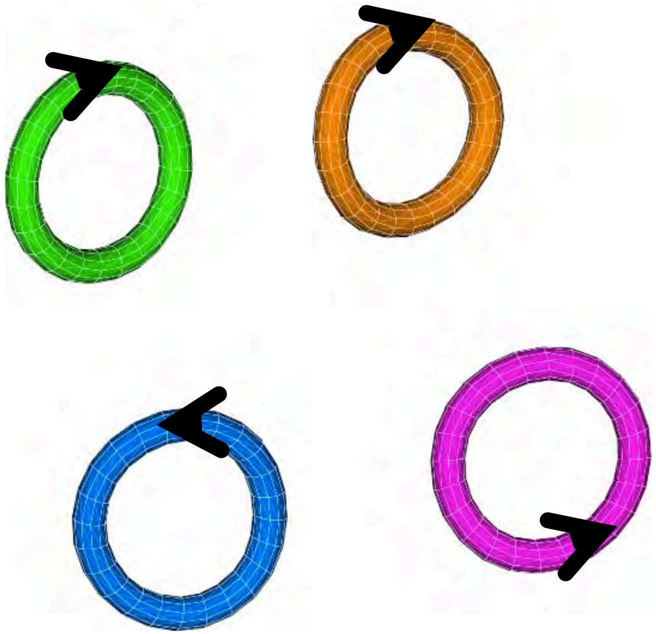
$N = 2$, $c_{\min} = 2$

What to study on links?

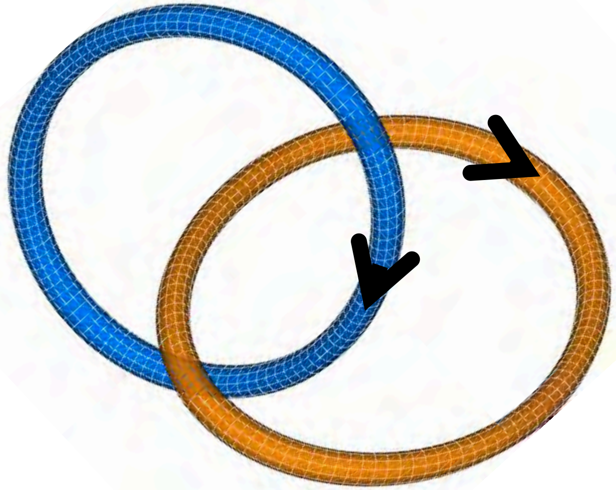
- ***Number of components:*** N
- ***Minimum number of crossings:*** $c_{\min} = \min(\#)$
- ***(Gauss) linking number between components:*** $Lk = \frac{1}{2}\sum_r \varepsilon_r$



$\varepsilon_r = \pm 1$



$N = 4\,,\ c_{\min} = 0$



$N = 2\,,\ c_{\min} = 2$

What to study on links?

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- ***Minimum number of crossings:*** $c_{\min} = \min(\#)$
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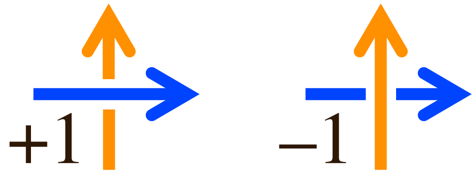

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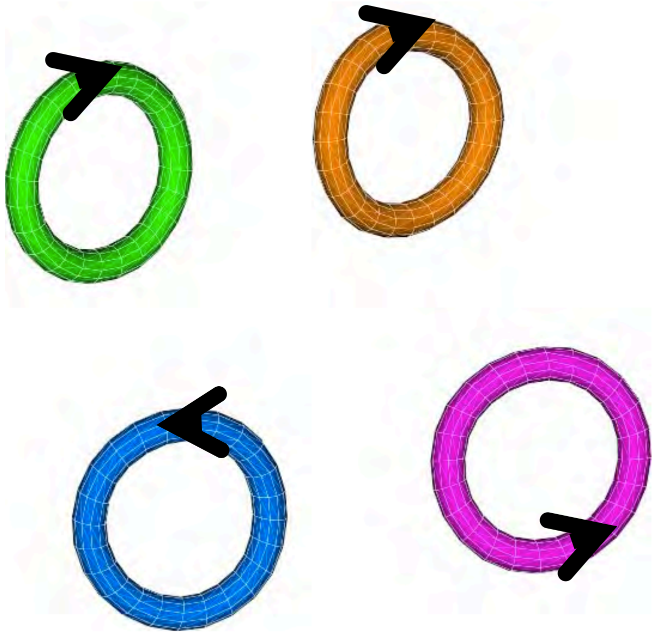

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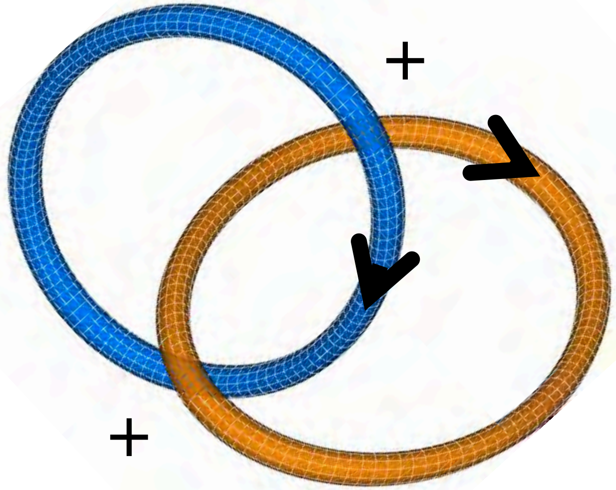

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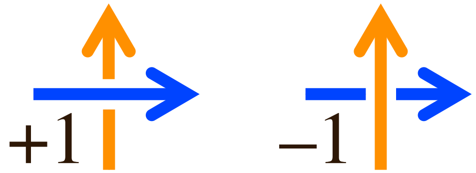
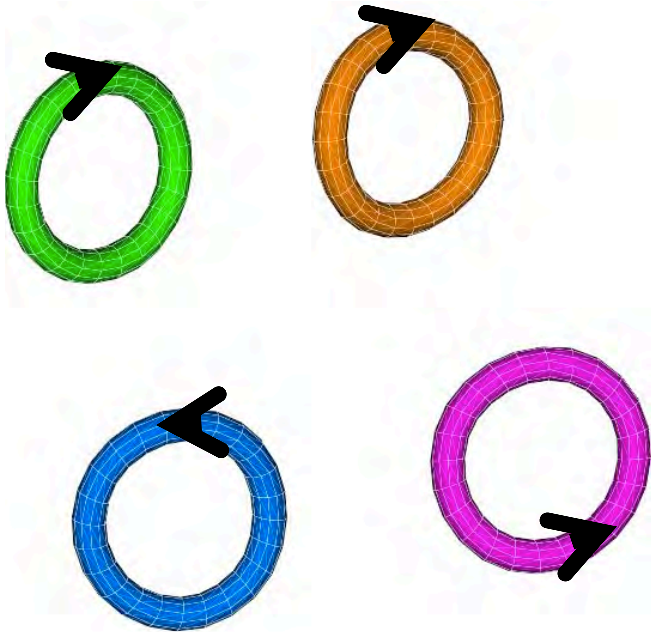
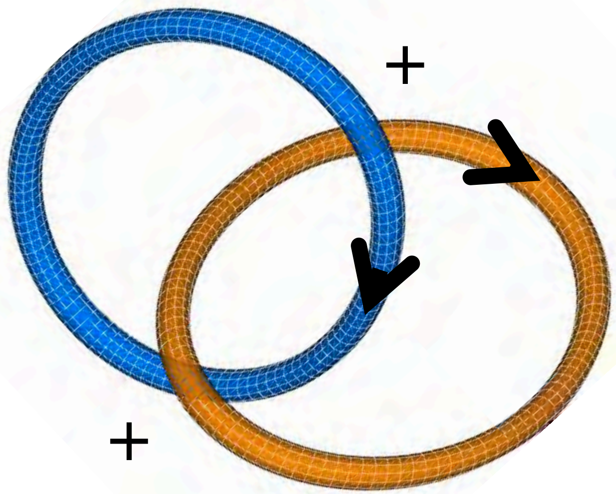


$\varepsilon_r = \pm 1$

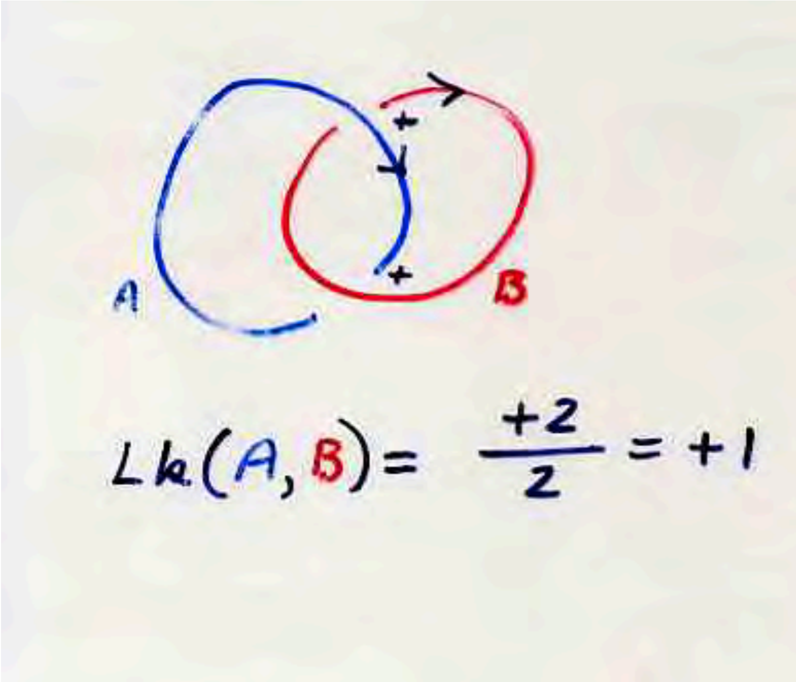




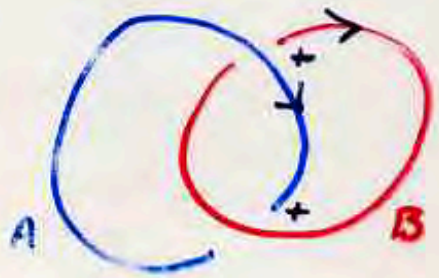
$N = 4\,,\;\; c_{\min} = 0\,,\;\; Lk = 0$

$N = 2\,,\;\; c_{\min} = 2\,,\;\; Lk = +1$

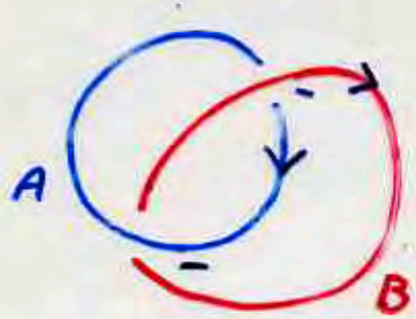
Computations by hand: some examples



Computations by hand: some examples

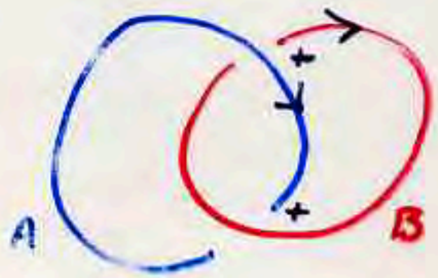


$$Lk(A,B) = \frac{+2}{2} = +1$$

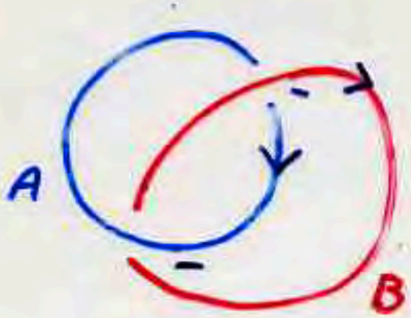


$$Lk(A,B) = \frac{-2}{2} = -1$$

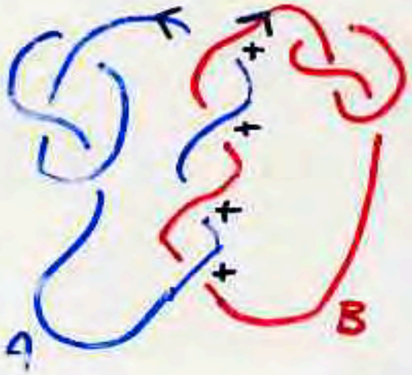
Computations by hand: some examples



$$Lk(A,B) = \frac{+2}{2} = +1$$

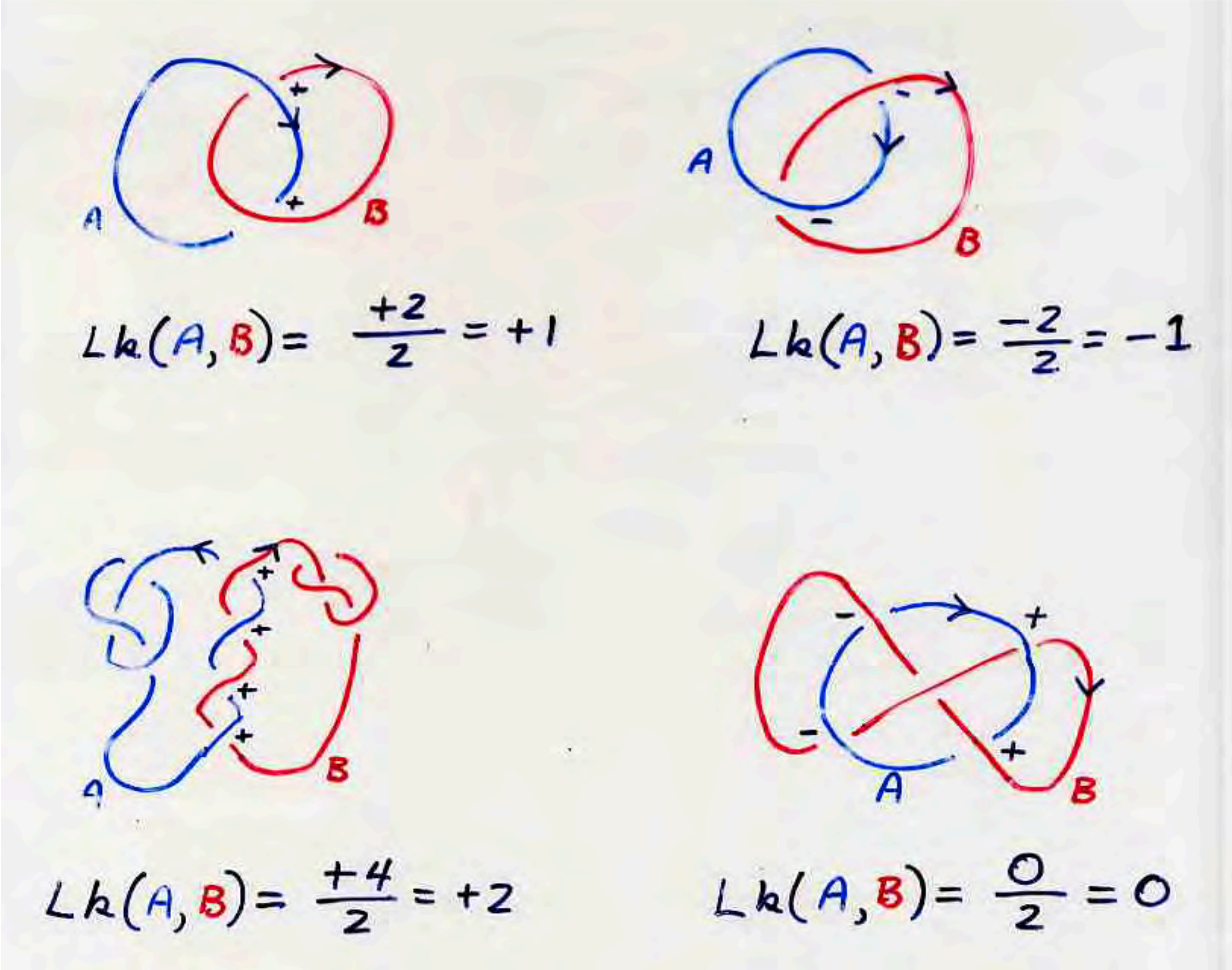


$$Lk(A,B) = \frac{-2}{2} = -1$$

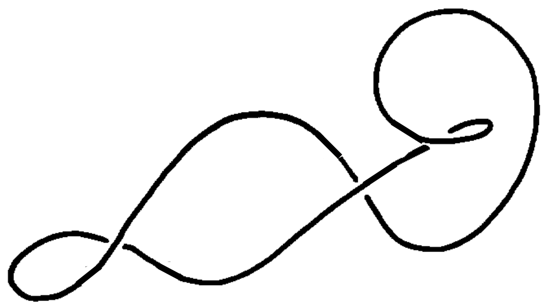


$$Lk(A,B) = \frac{+4}{2} = +2$$

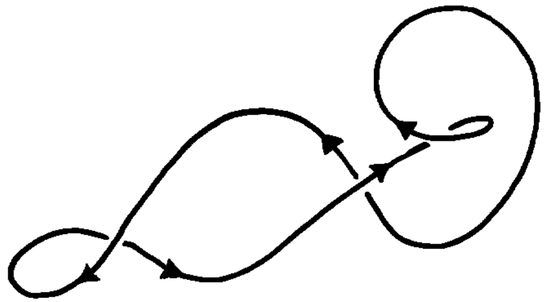
Computations by hand: some examples



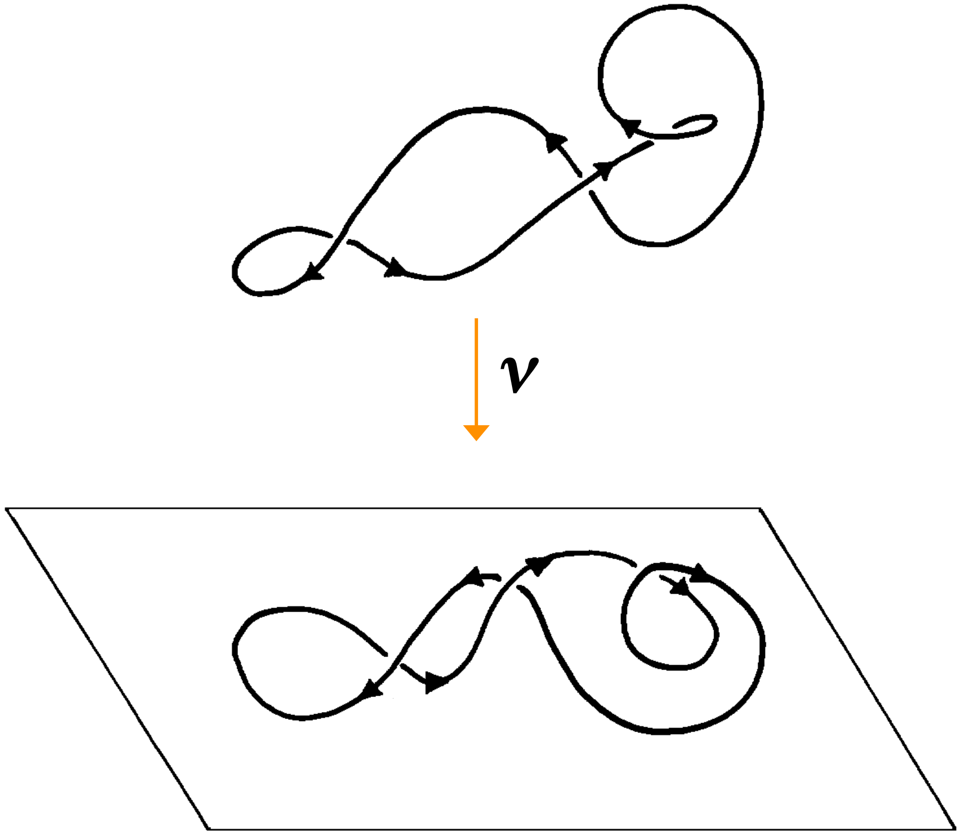
Indented diagram and signed crossings



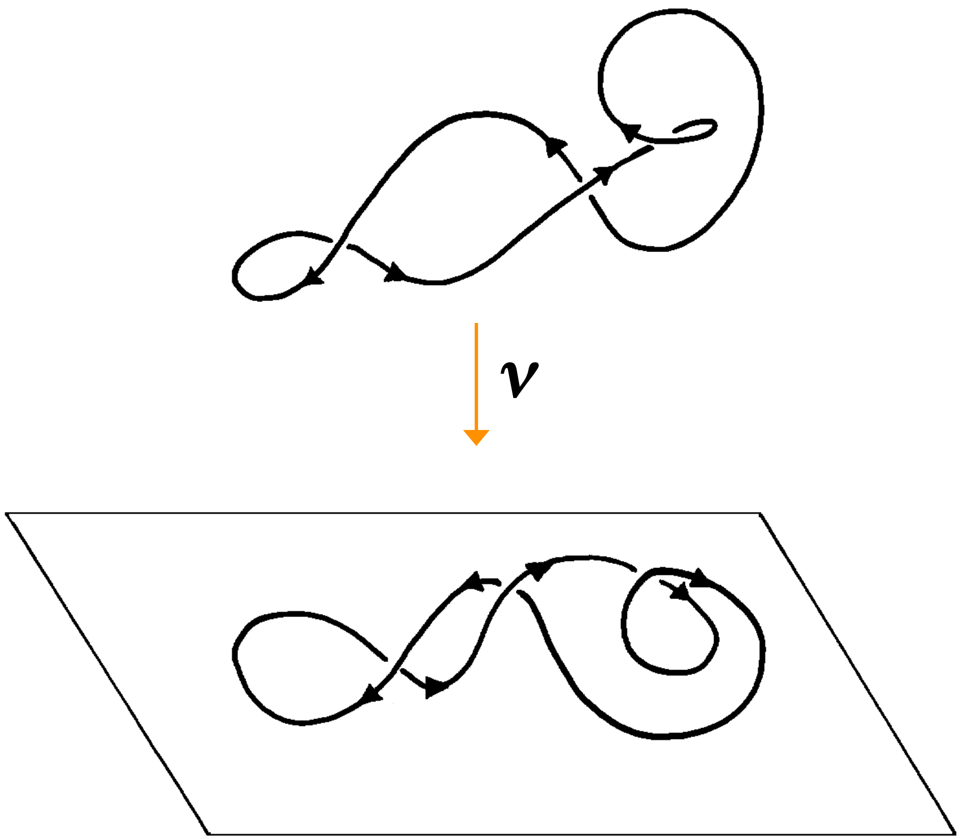
Indented diagram and signed crossings

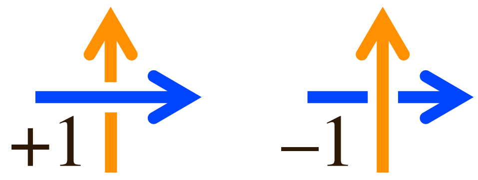


Indented diagram and signed crossings

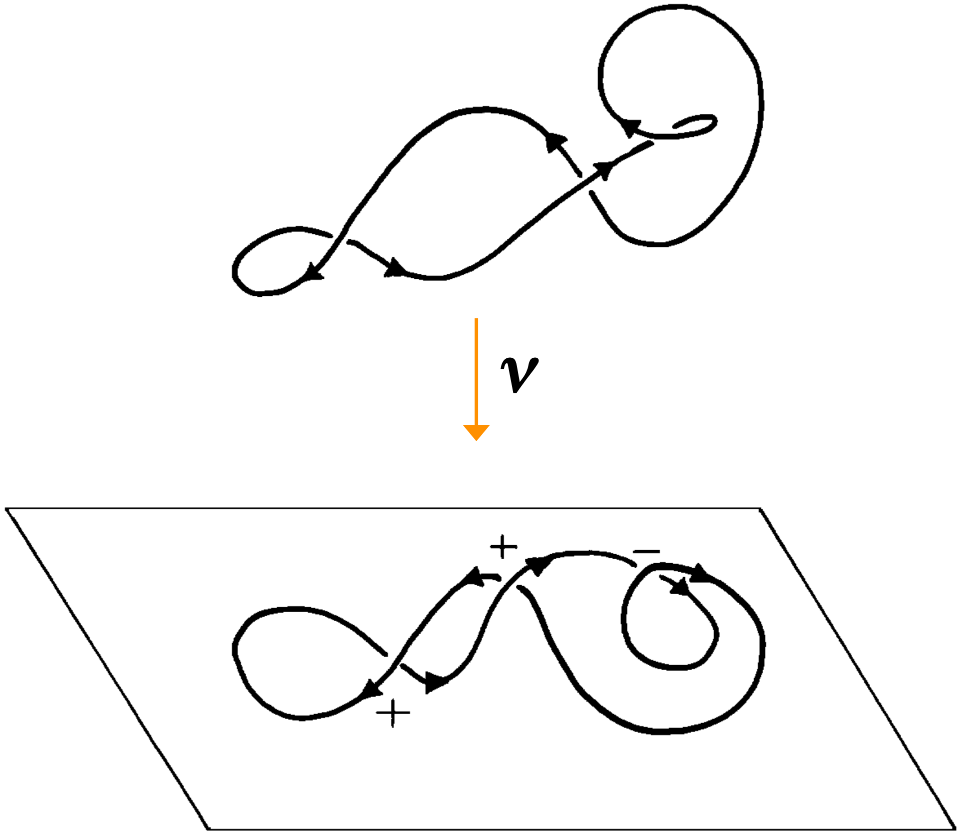


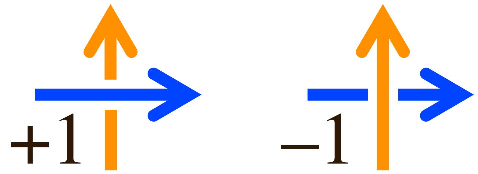
Indented diagram and signed crossings



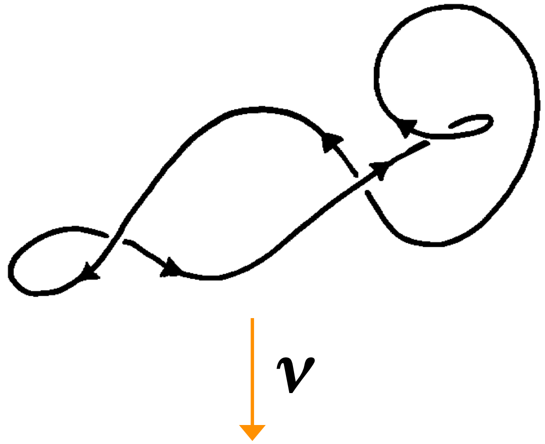


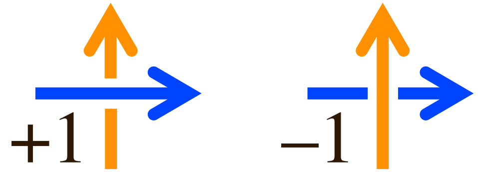
Indented diagram and signed crossings

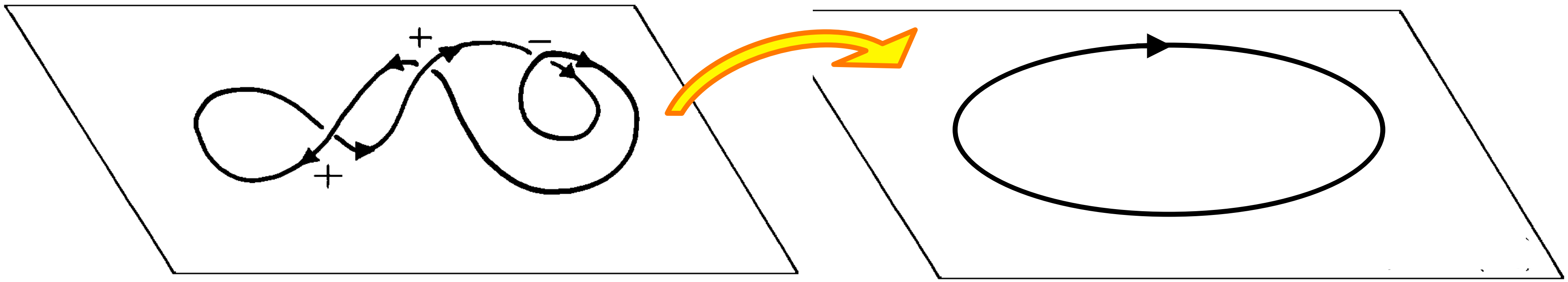




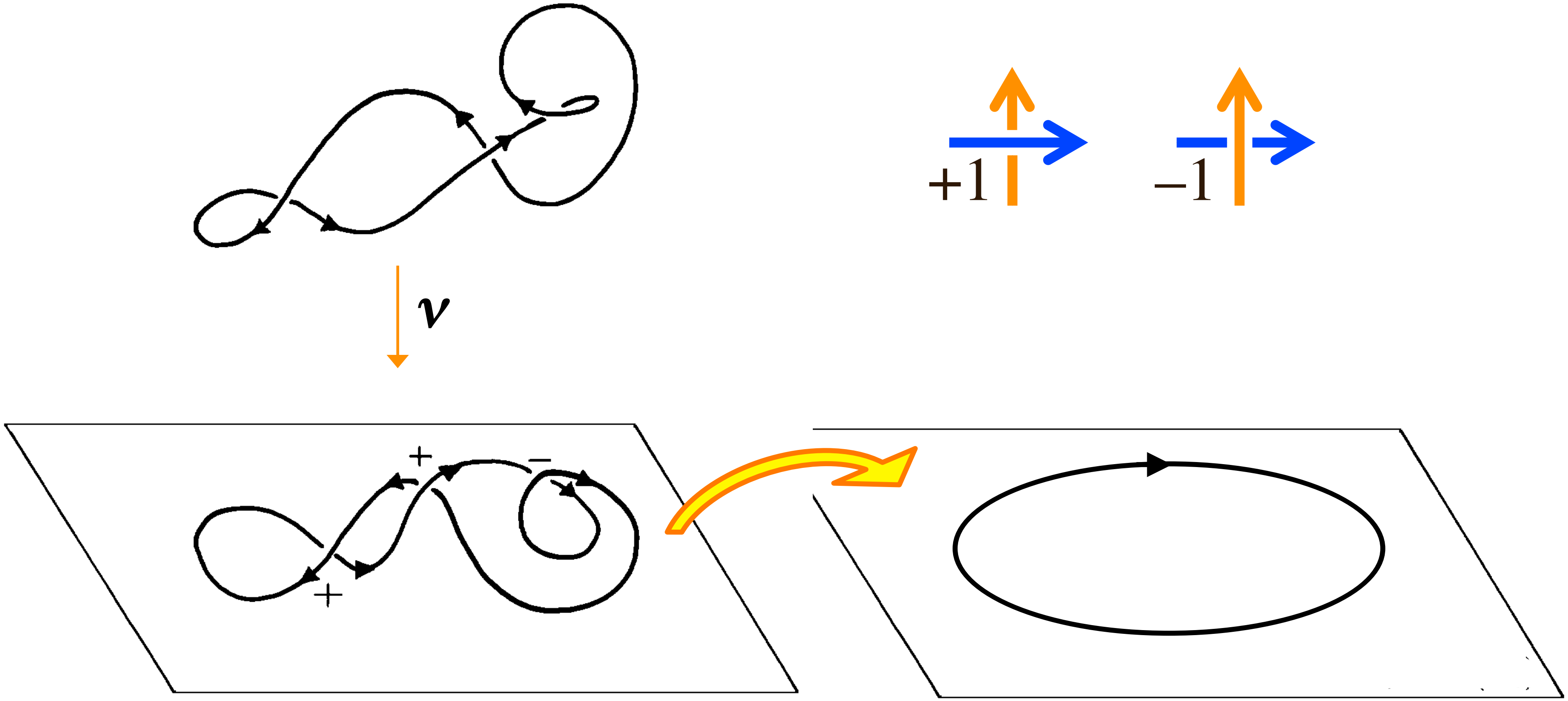
Indented diagram and signed crossings

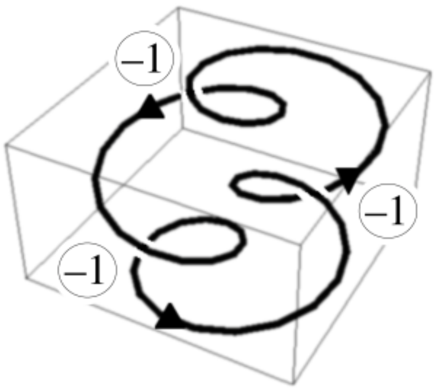




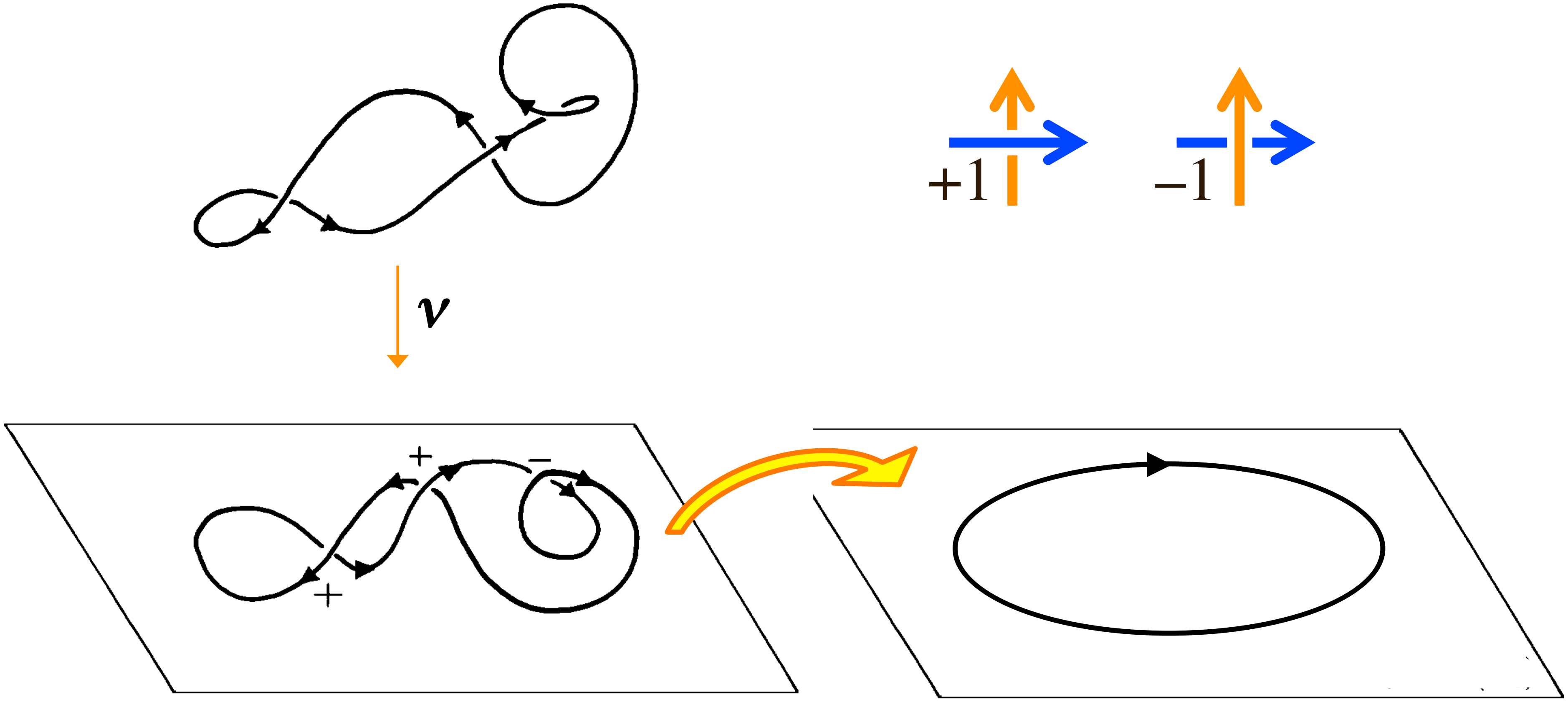


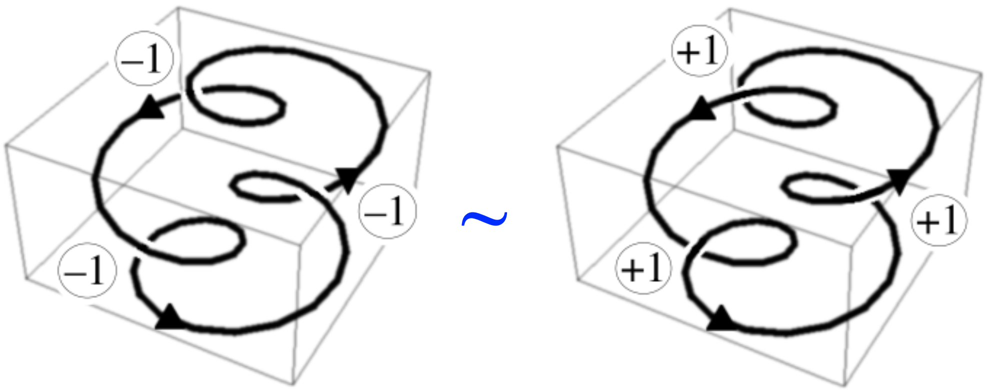
Indented diagram and signed crossings



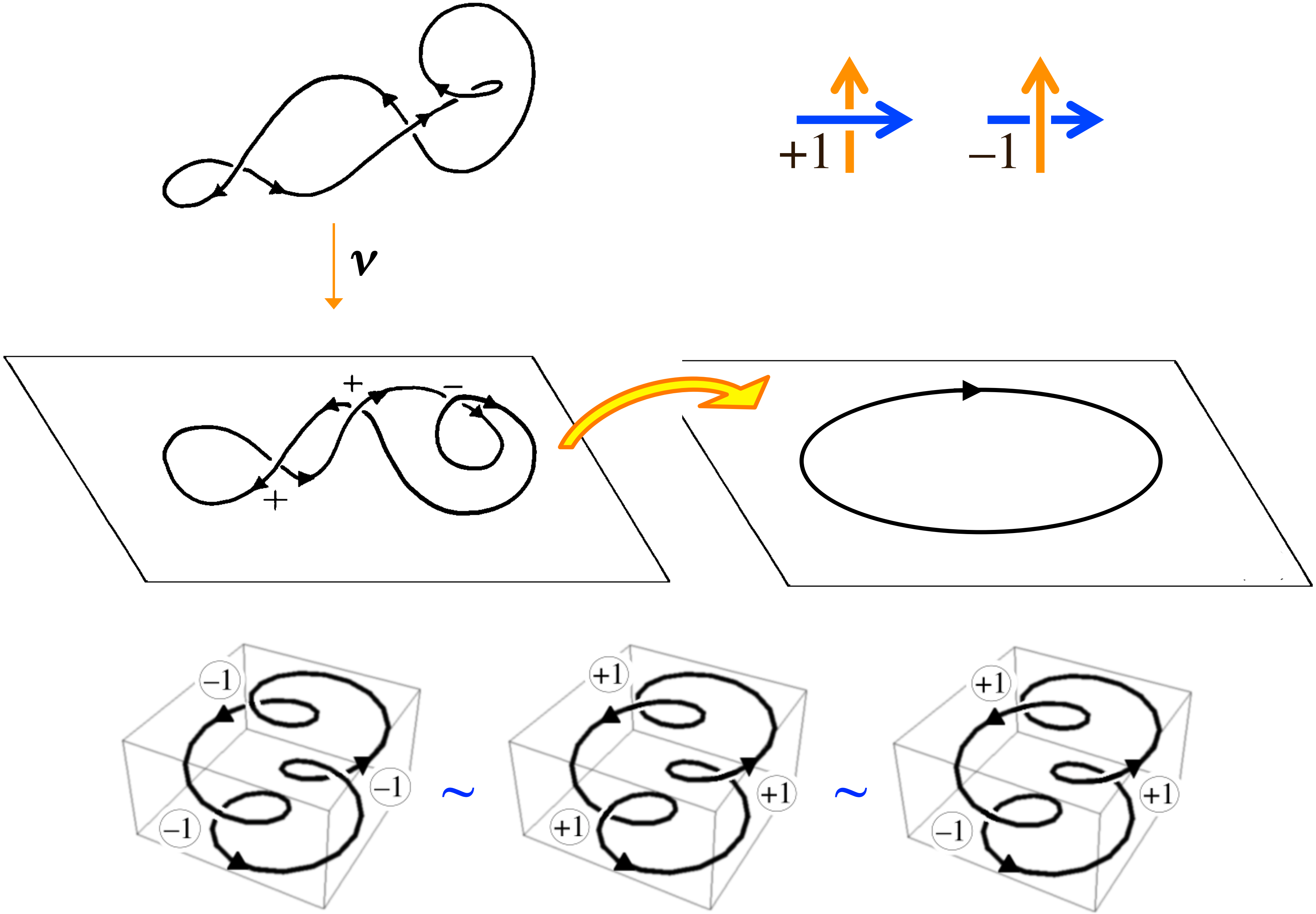


Indented diagram and signed crossings

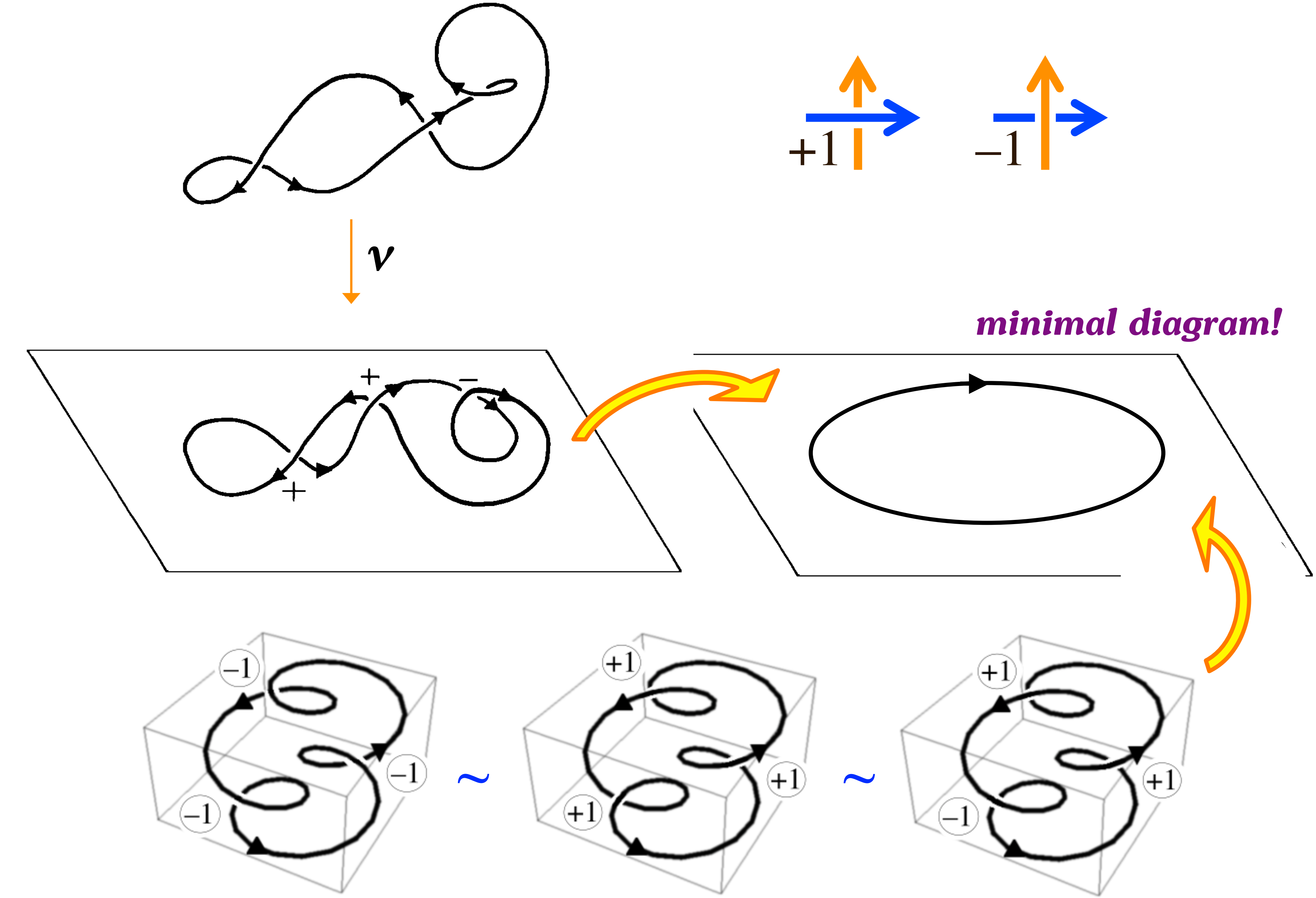




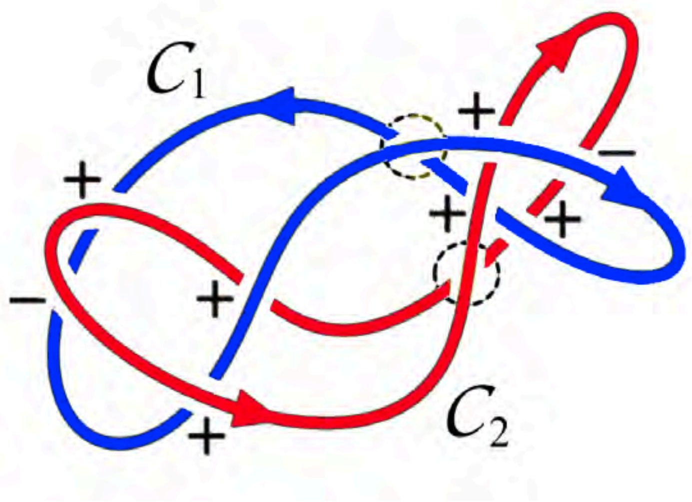
Indented diagram and signed crossings



Indented diagram and signed crossings

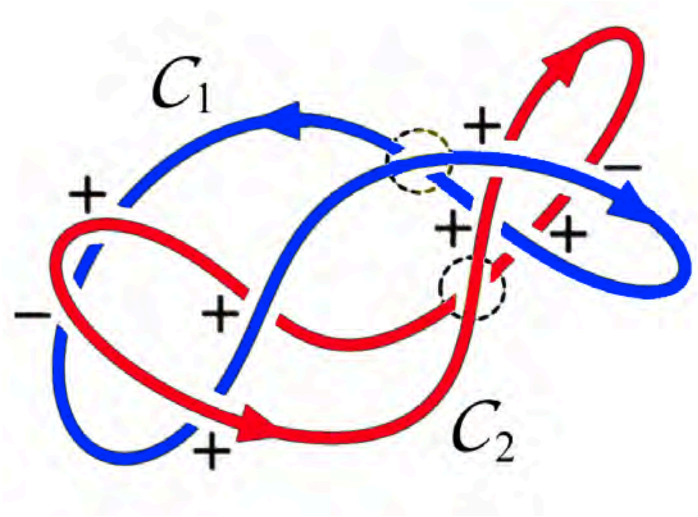


Minimal diagrams and topological invariants



generic projection

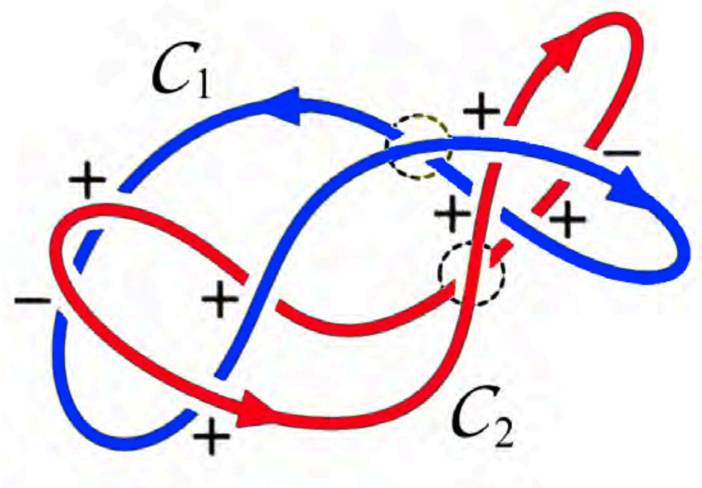
Minimal diagrams and topological invariants



generic projection

$$N = 2\,,\ Lk_{12} = +2\,,\ \bar{C} = 10$$

Minimal diagrams and topological invariants



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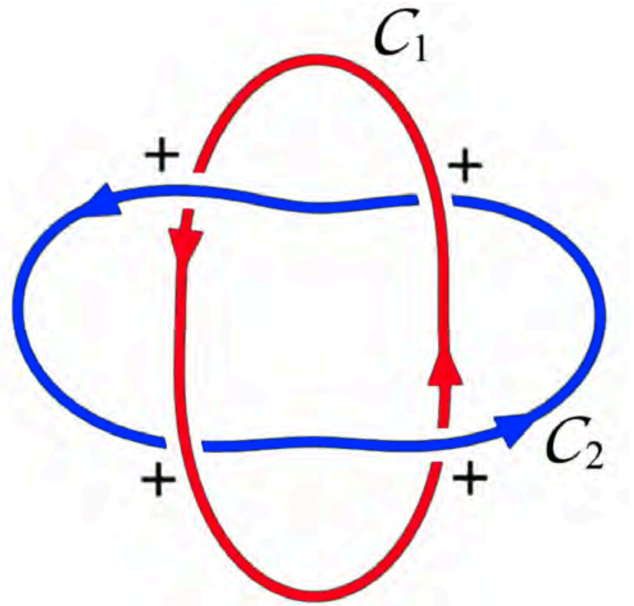


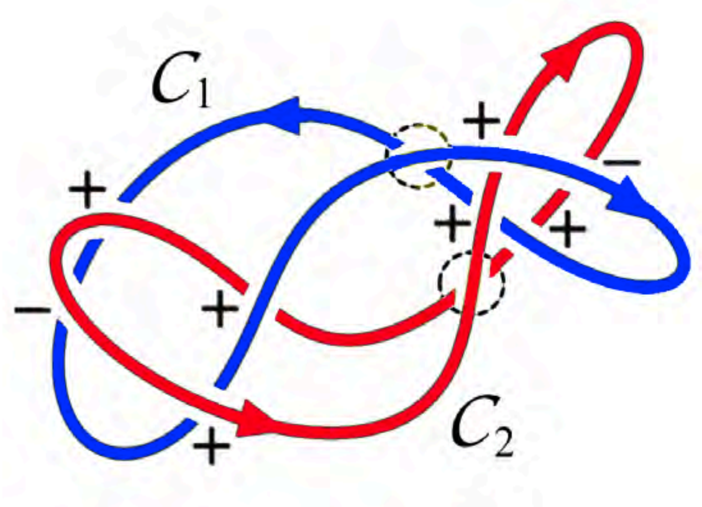
generic projection

$N = 2$, $Lk_{12} = +2$, $\bar{C} = 10$

minimal projection

$N = 2$

Minimal diagrams and topological invariants



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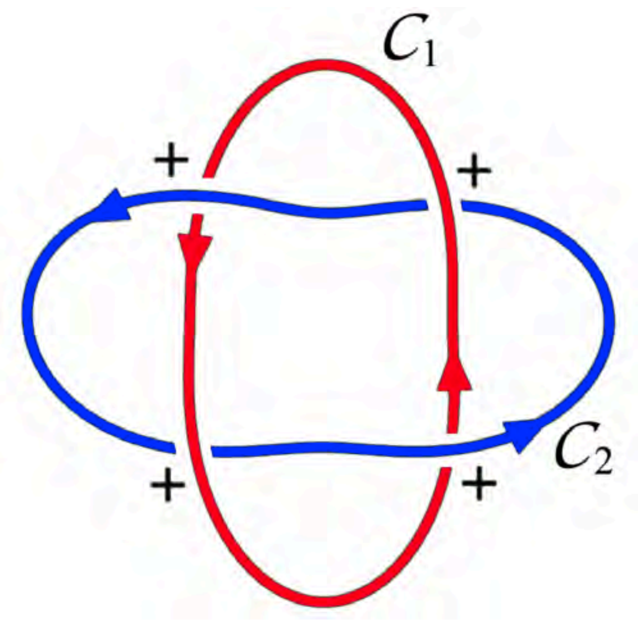


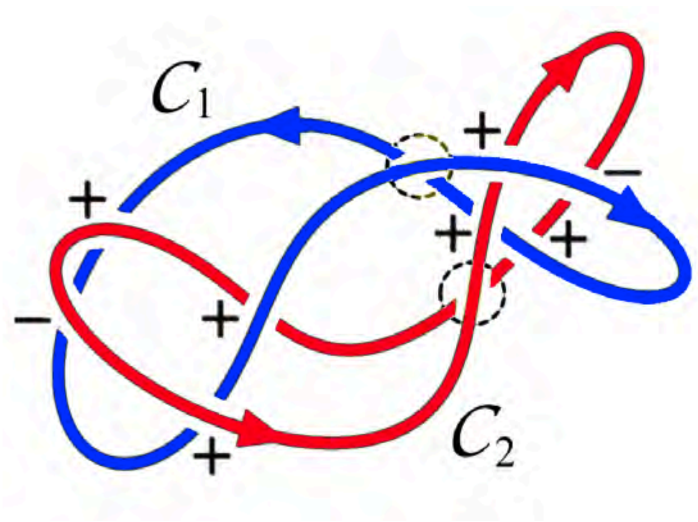
generic projection

$N = 2$, $Lk_{12} = +2$, $\bar{C} = 10$

minimal projection

$N = 2$, $Lk_{12} = +2$

Minimal diagrams and topological invariants



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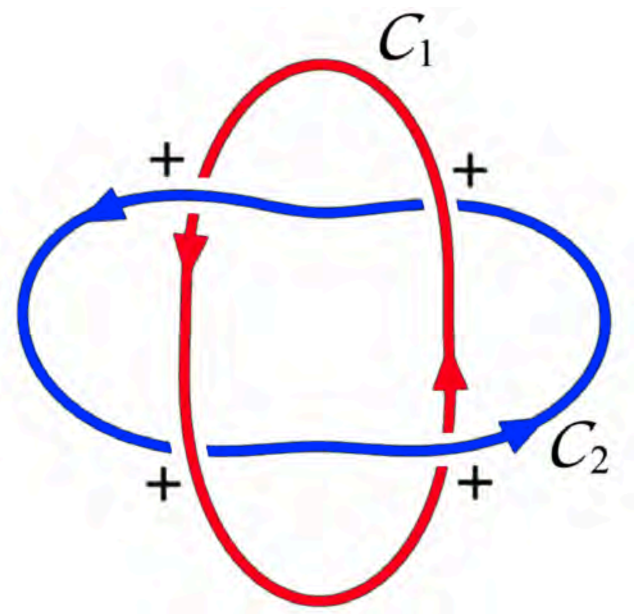


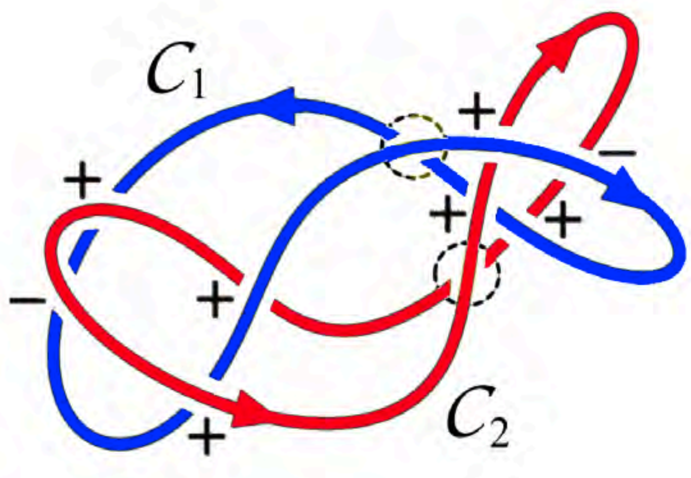
generic projection

$N = 2\,,\;\; Lk_{12} = +2\,,\;\; \bar{C} = 10$

minimal projection

$N = 2\,,\;\; Lk_{12} = +2\,,\;\; \bar{C} = c_{\min} = 4$

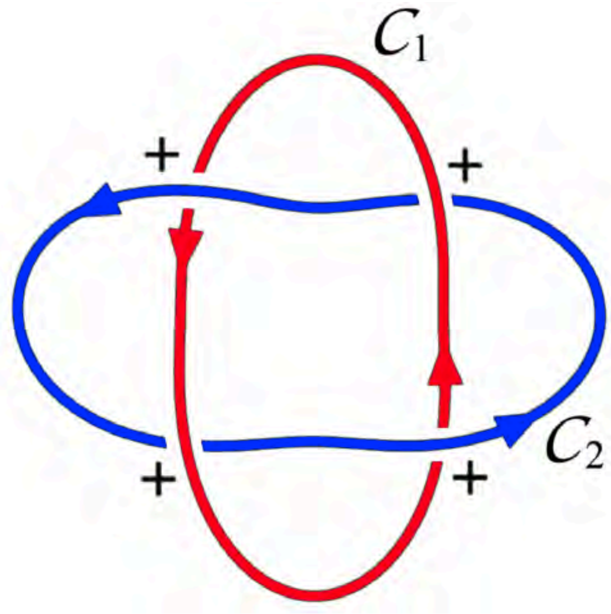
Minimal diagrams and topological invariants



generic projection

$N = 2$, $Lk_{12} = +2$, $\bar{C} = 10$

$\sim$

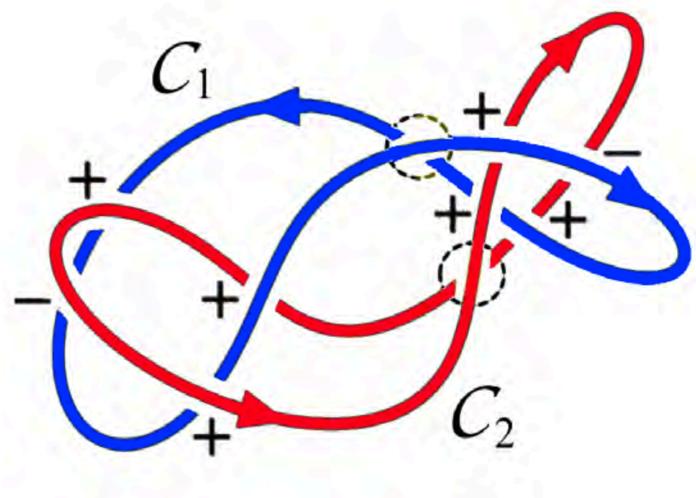


minimal projection

$N = 2$, $Lk_{12} = +2$, $\bar{C} = c_{\min} = 4$

- From generic diagrams: minimal diagram presentation;

Minimal diagrams and topological invariants



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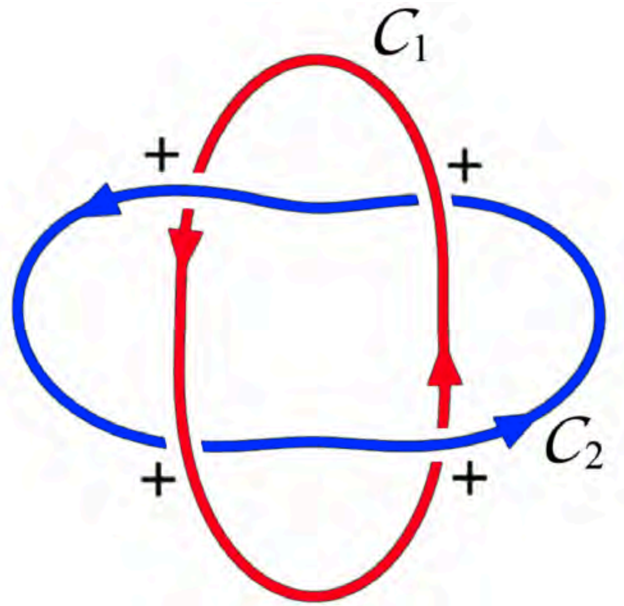


generic projection

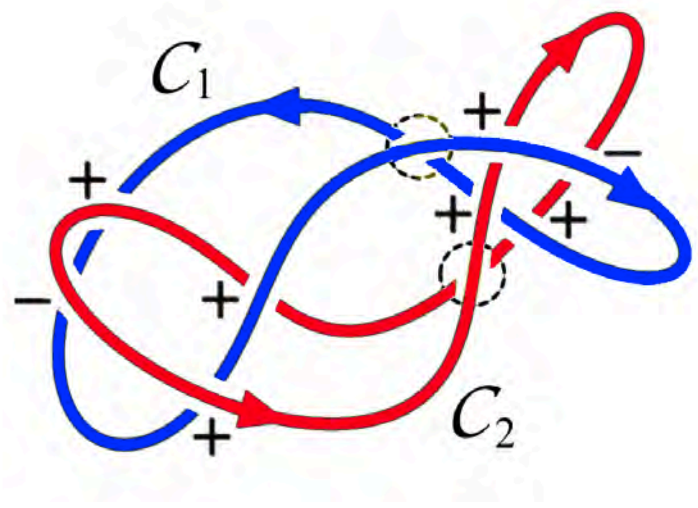
$N = 2$, $Lk_{12} = +2$, $\bar{C} = 10$

minimal projection

$N = 2$, $Lk_{12} = +2$, $\bar{C} = c_{\min} = 4$

- **From generic diagrams: minimal diagram presentation;**
- **From number of components N : topological linking number Lk_{12} ;**

Minimal diagrams and topological invariants



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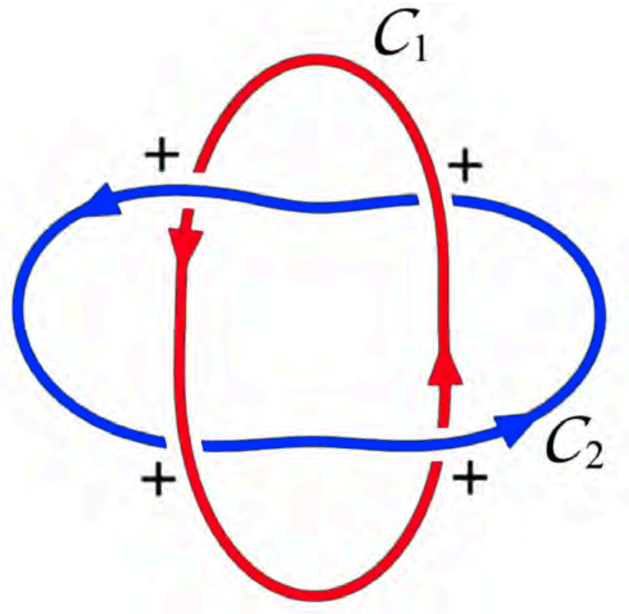


generic projection

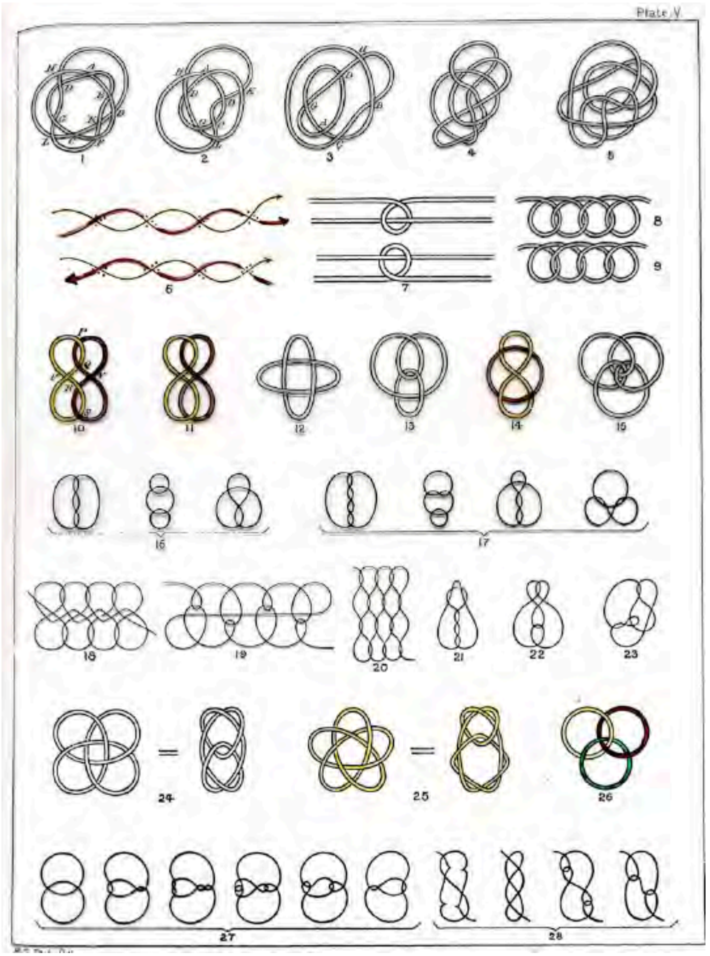
$N = 2$, $Lk_{12} = +2$, $\bar{C} = 10$

minimal projection

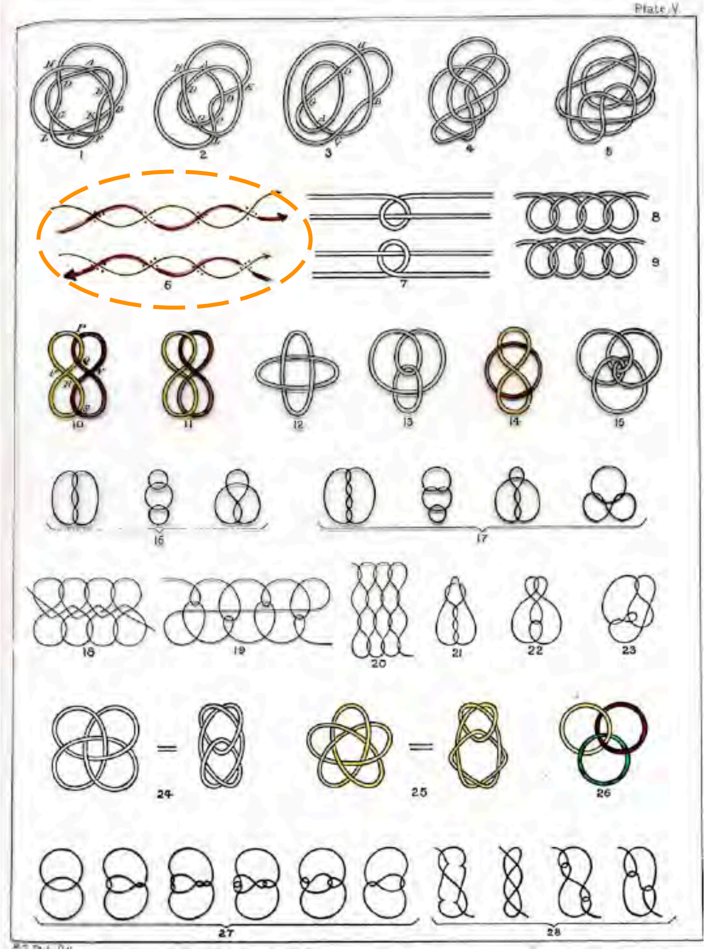
$N = 2$, $Lk_{12} = +2$, $\bar{C} = c_{\min} = 4$

- ***From generic diagrams: minimal diagram presentation;***
- ***From number of components*** N ***: topological linking number*** Lk_{12} ***;***
- ***From number of crossings*** $\bar{C}$ ***: topological crossing number*** $c_{\min}$ ***.***

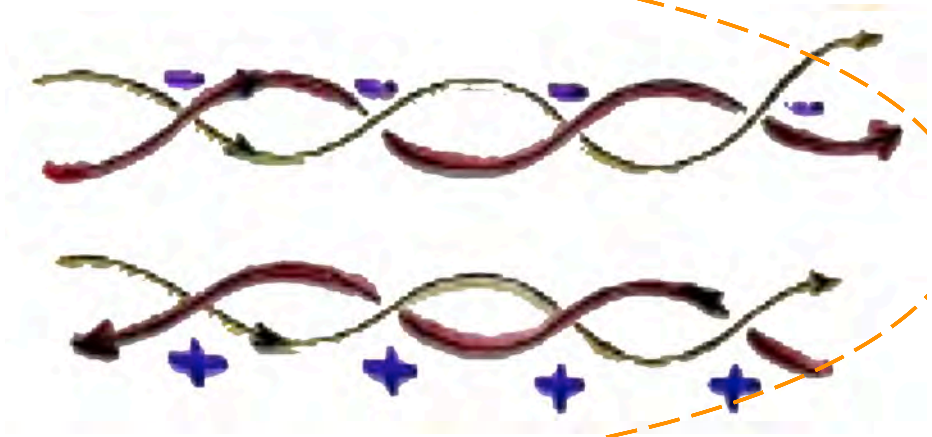
First results from Tait's tabulation

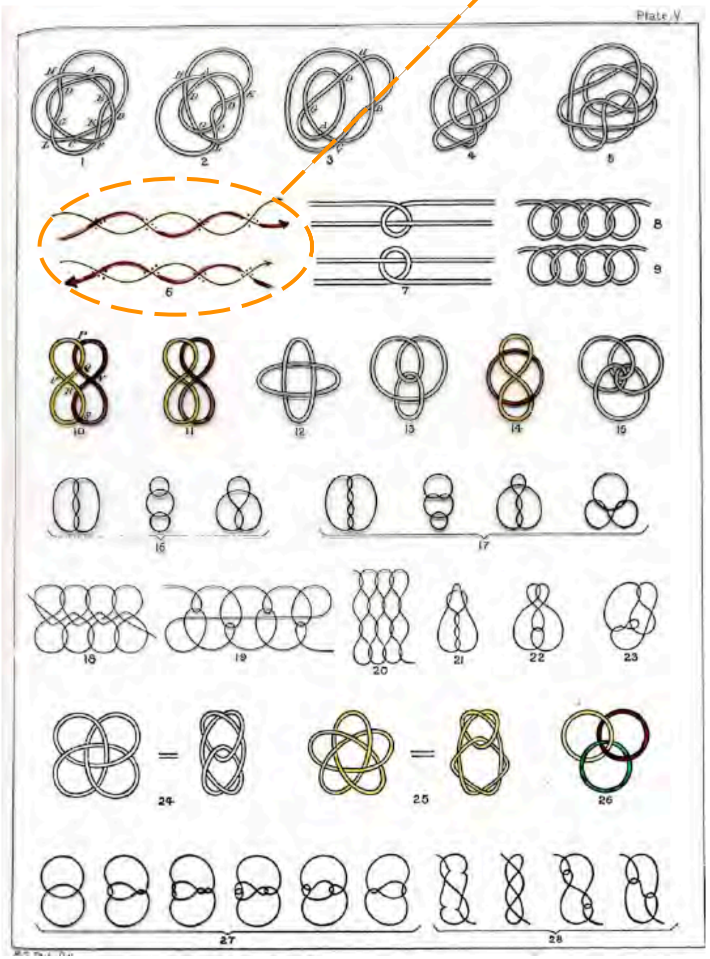


First results from Tait's tabulation



First results from Tait's tabulation

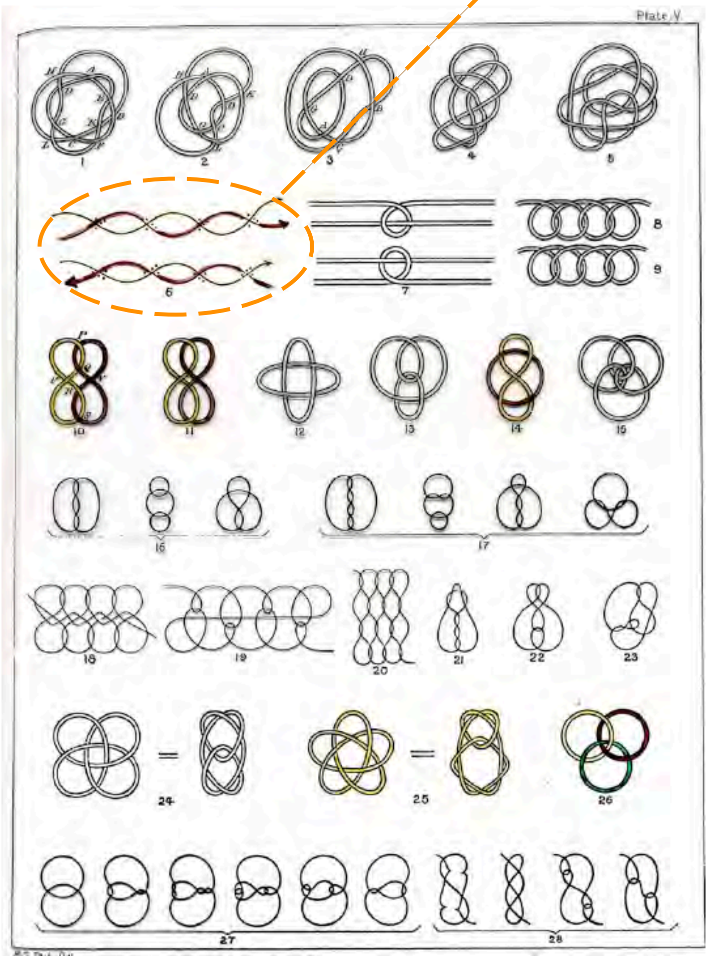




First results from Tait's tabulation

$$c_{\min} = 4$$

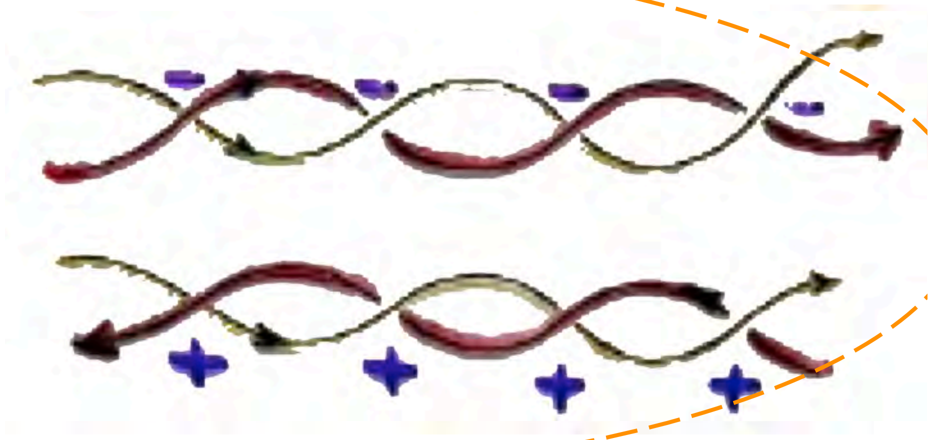
$$c_{\min} = 4$$

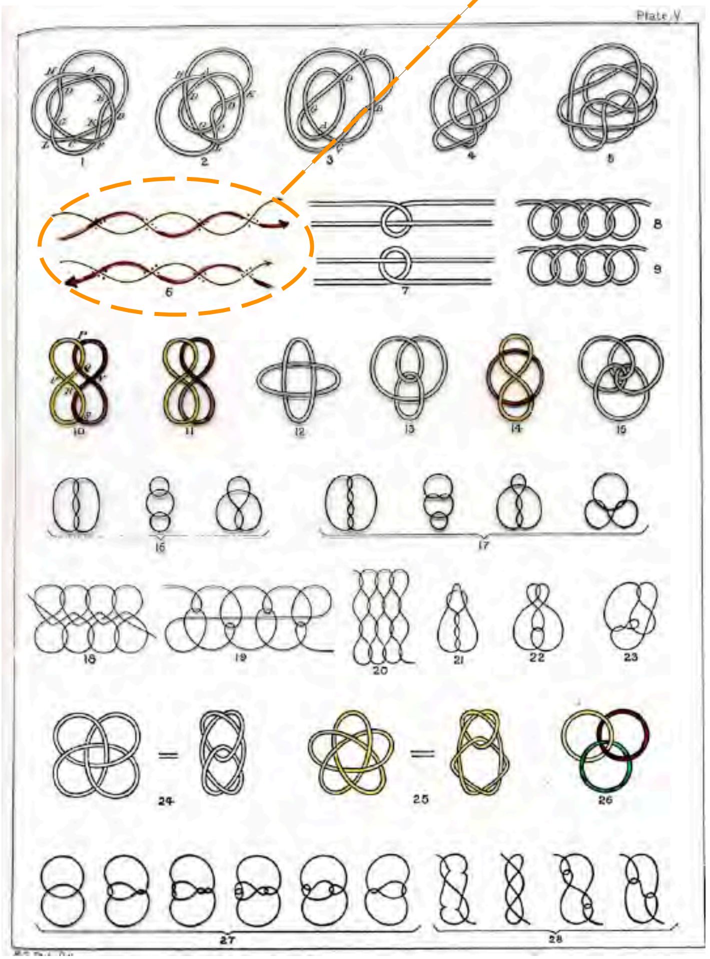


First results from Tait's tabulation

$$c_{min} = 4\ ,\ \ Lk = -2$$

$$c_{min} = 4\ ,\ \ Lk = +2$$

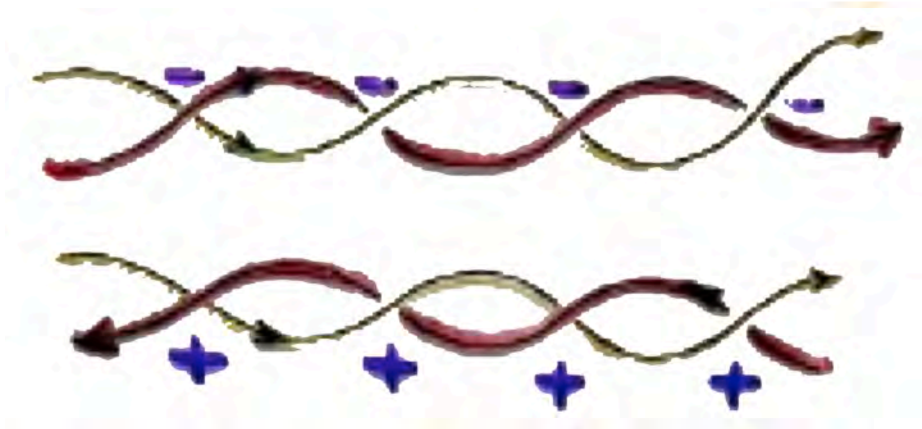


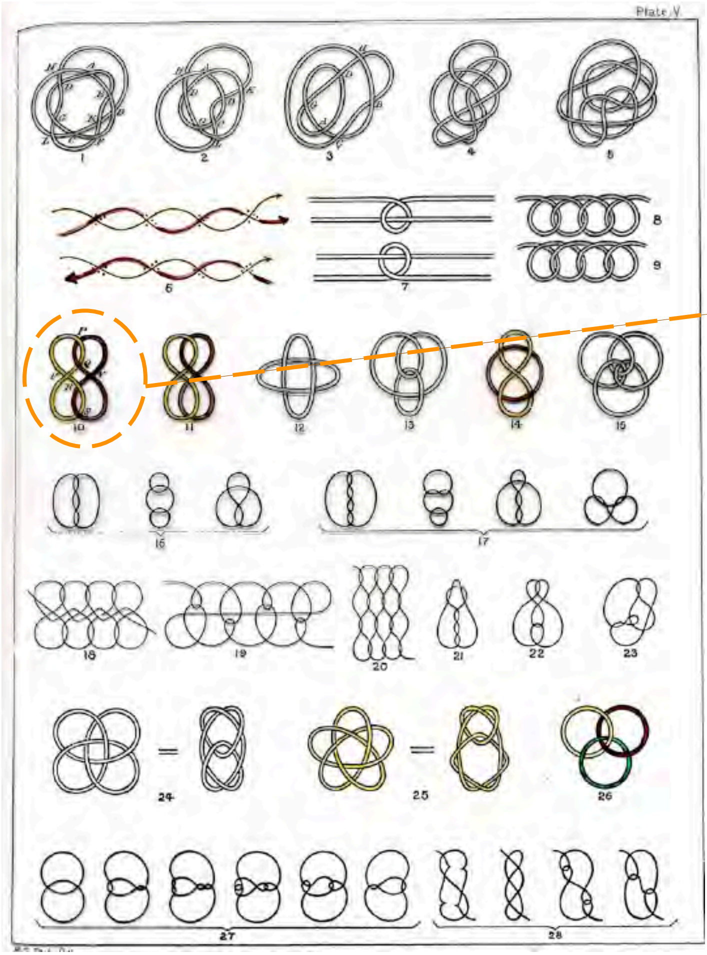


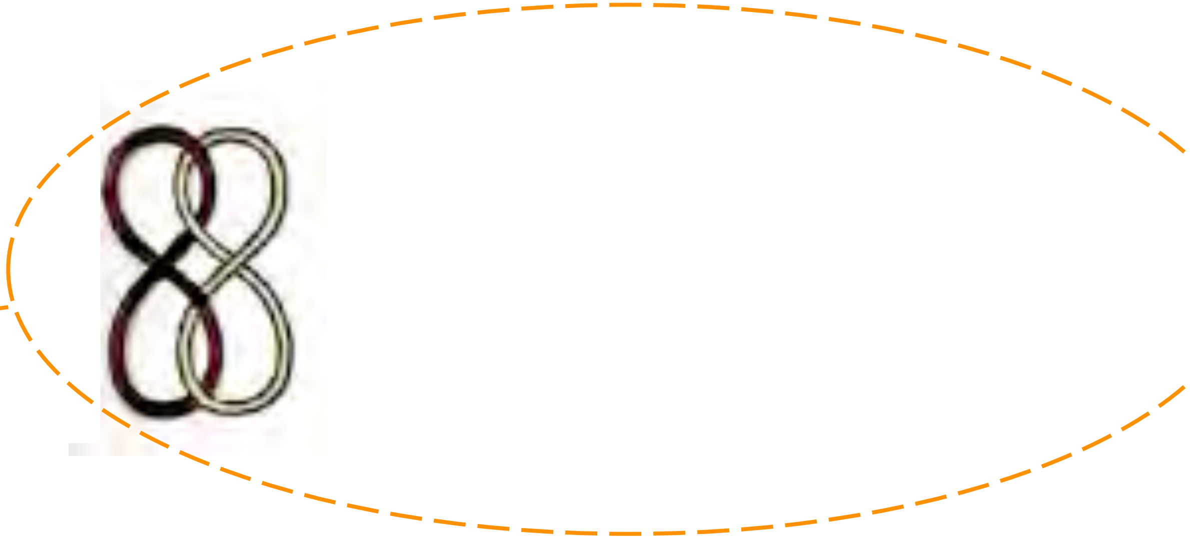
First results from Tait's tabulation

$$c_{\min} = 4\ ,\ Lk = -2$$

$$c_{\min} = 4\ ,\ Lk = +2$$



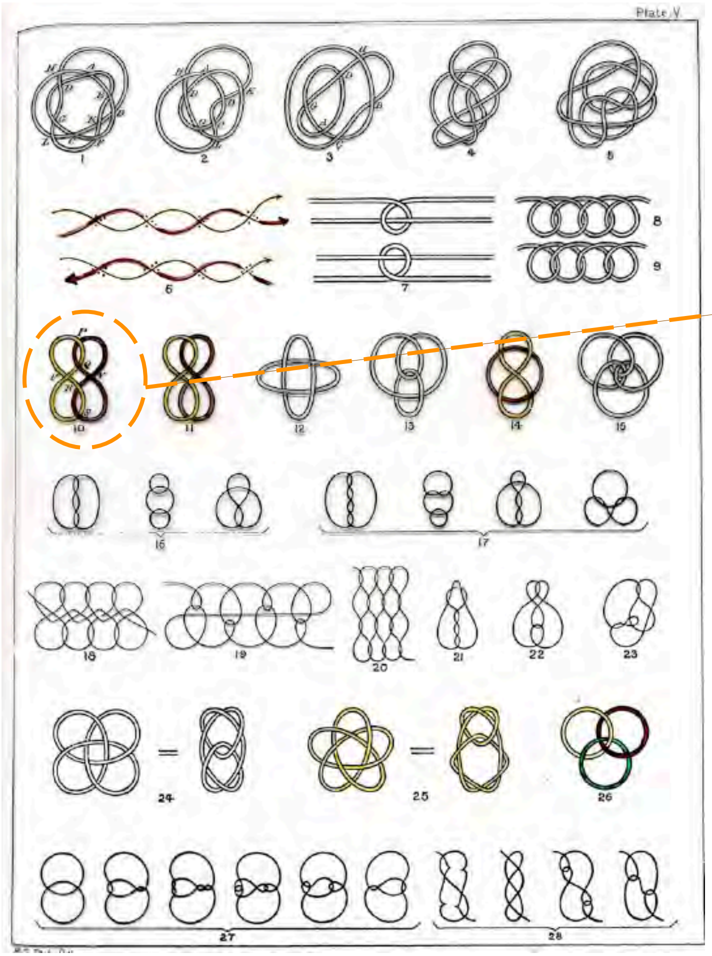


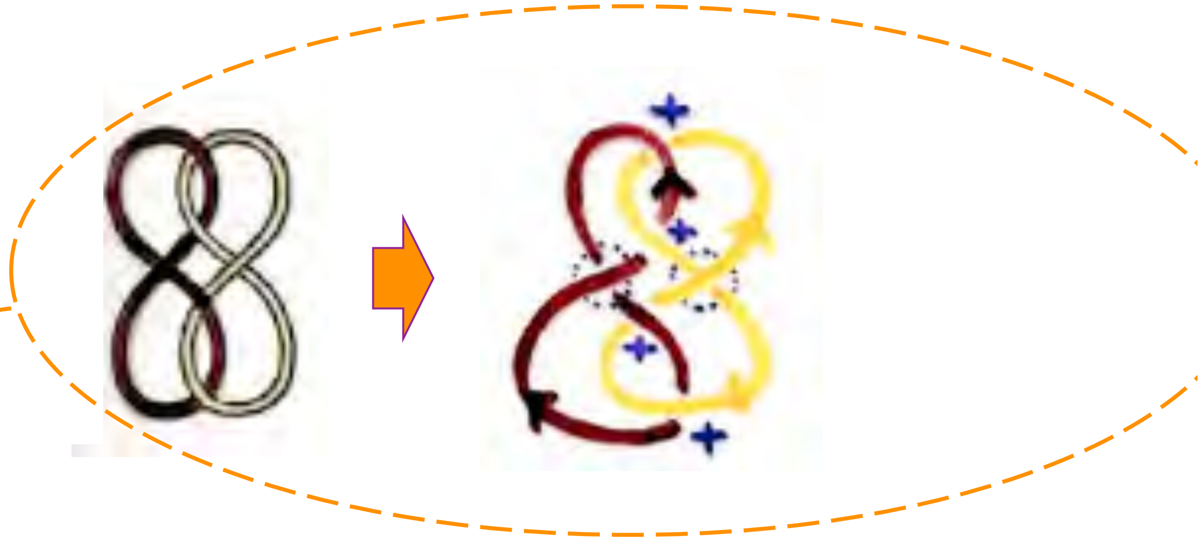


First results from Tait's tabulation

$$c_{\min} = 4 \,,\; Lk = -2$$

$$c_{\min} = 4 \,,\; Lk = +2$$

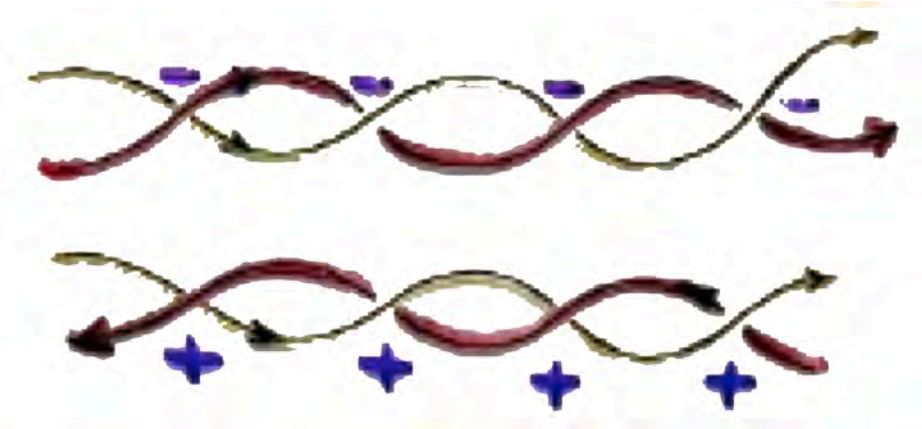


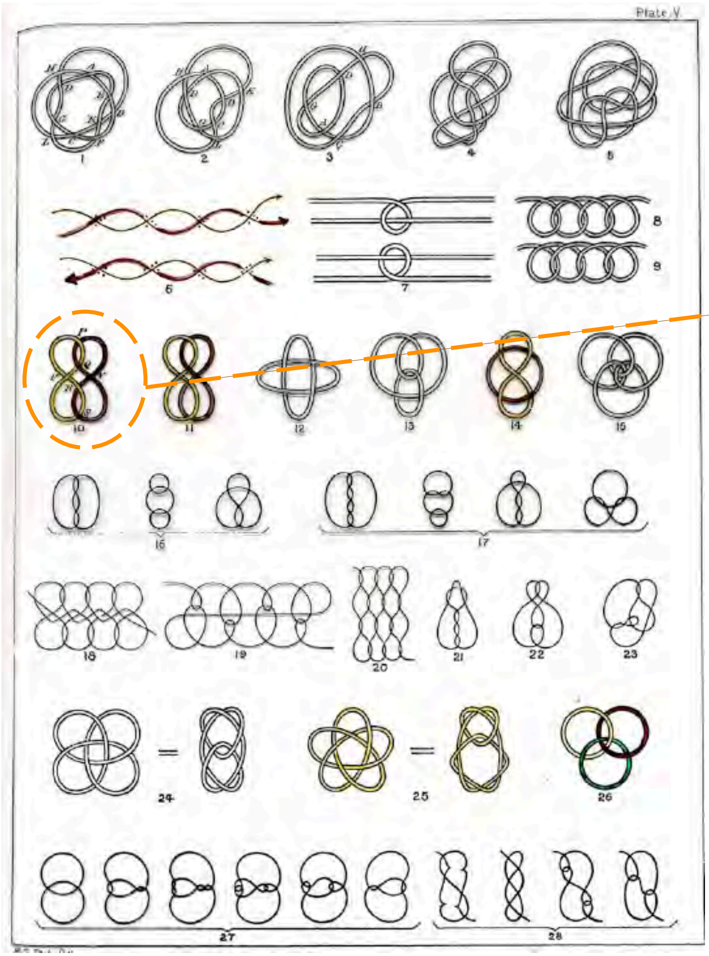


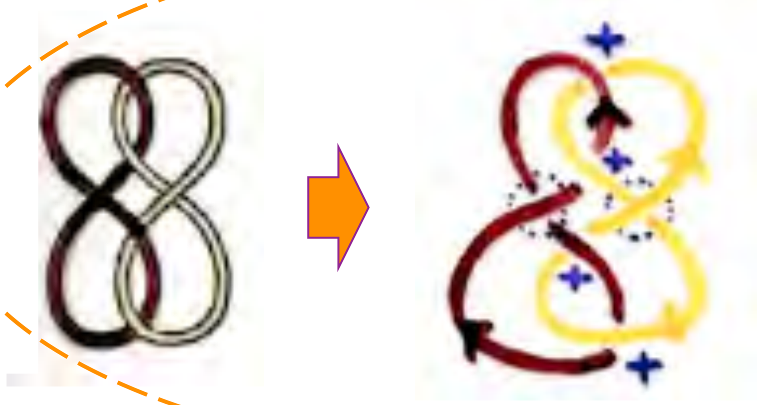
First results from Tait's tabulation

$$c_{\min} = 4 \;,\; Lk = -2$$

$$c_{\min} = 4 \;,\; Lk = +2$$







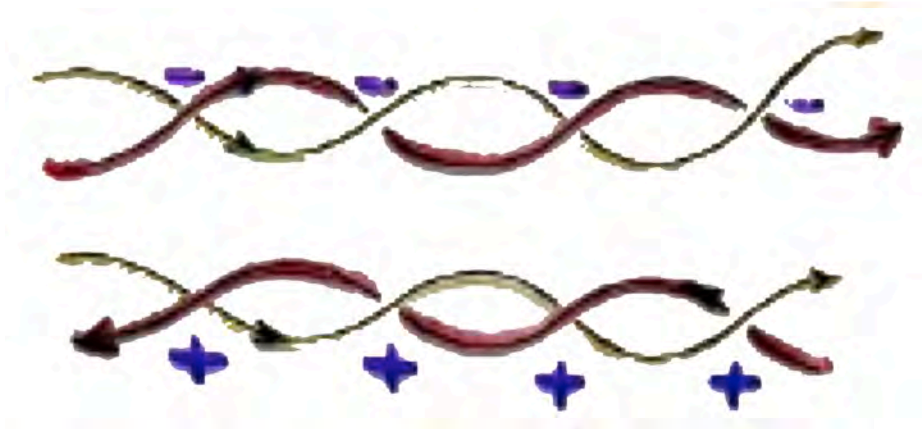
$$c_{\min} = 4$$

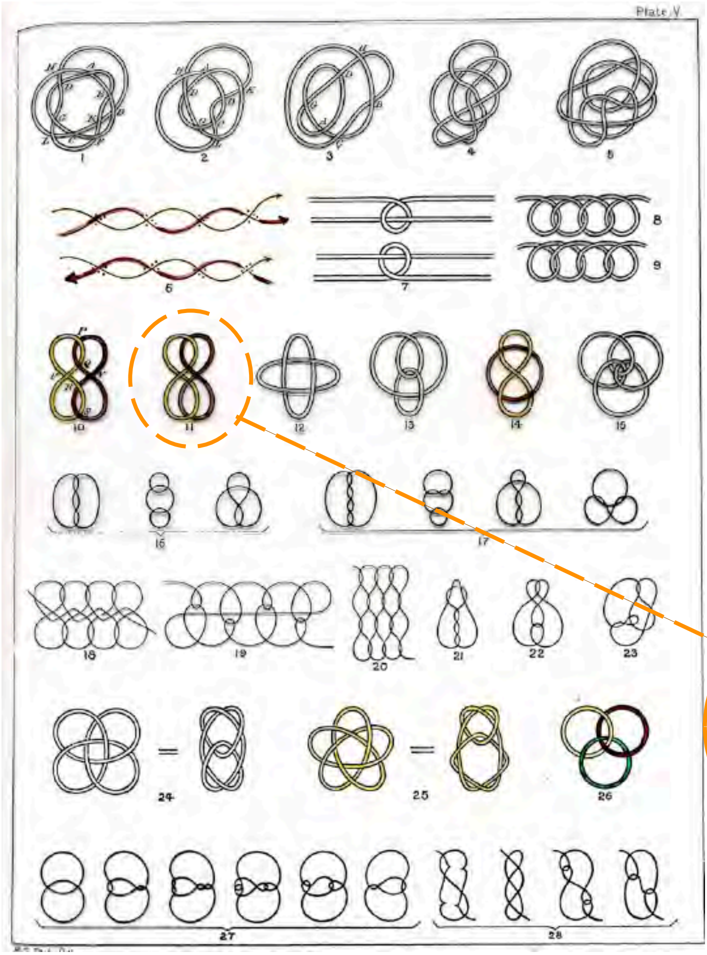
$$Lk = +2$$

First results from Tait's tabulation

$c_{\min} = 4\ ,\ Lk = -2$

$c_{\min} = 4\ ,\ Lk = +2$





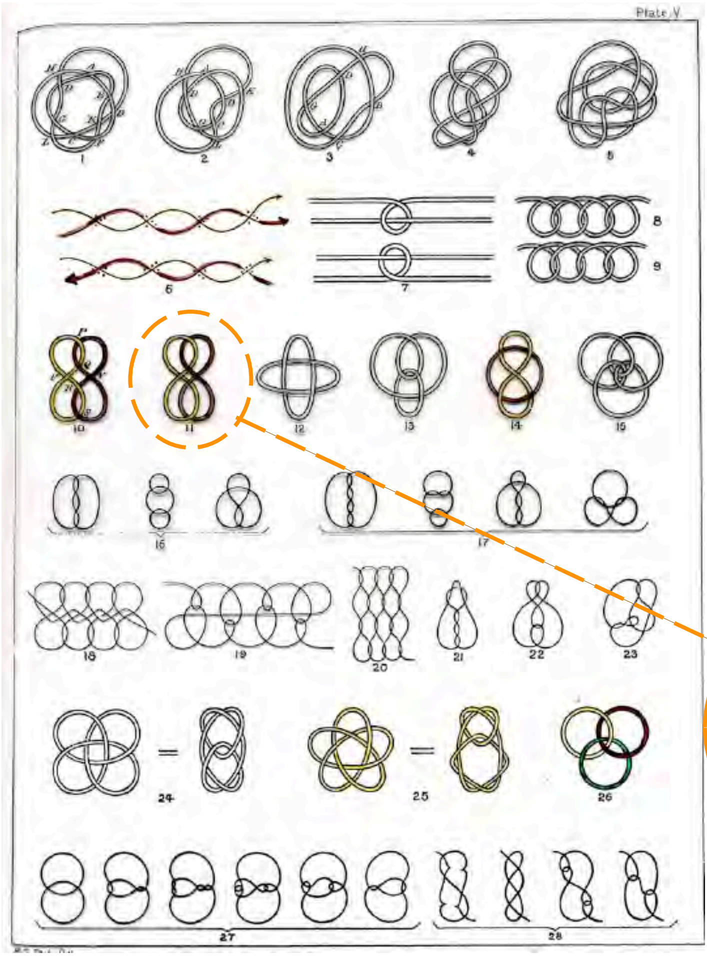
$c_{\min} = 4$

$Lk = +2$

First results from Tait's tabulation

$$c_{\min} = 4\ ,\ Lk = -2$$

$$c_{\min} = 4\ ,\ Lk = +2$$



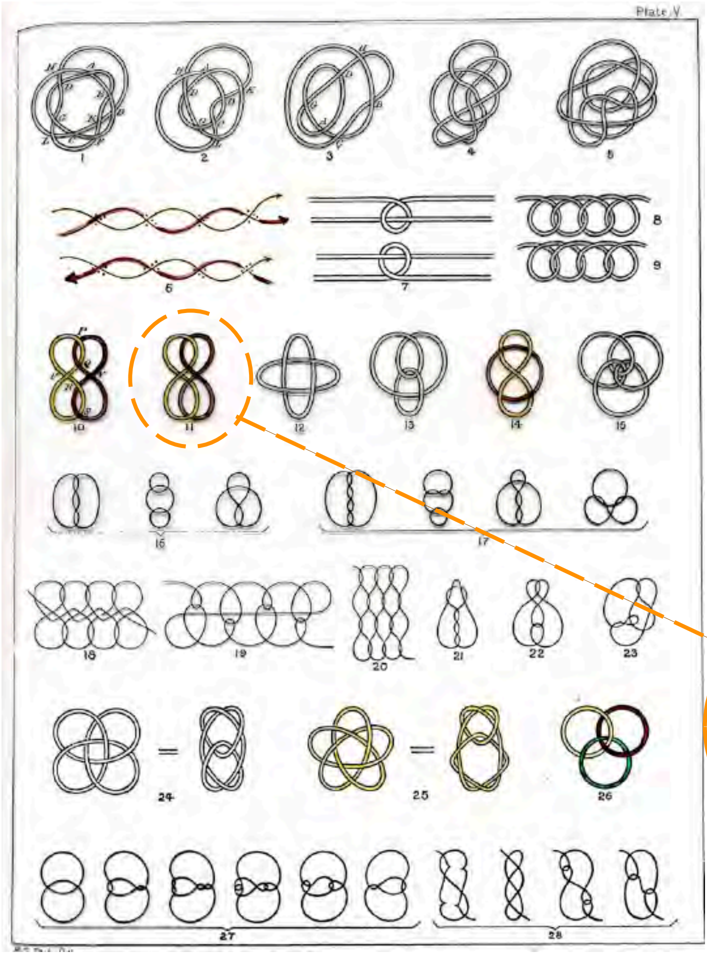
$$c_{\min} = 4$$

$$Lk = +2$$

First results from Tait's tabulation

$c_{\min} = 4$, $Lk = -2$

$c_{\min} = 4$, $Lk = +2$



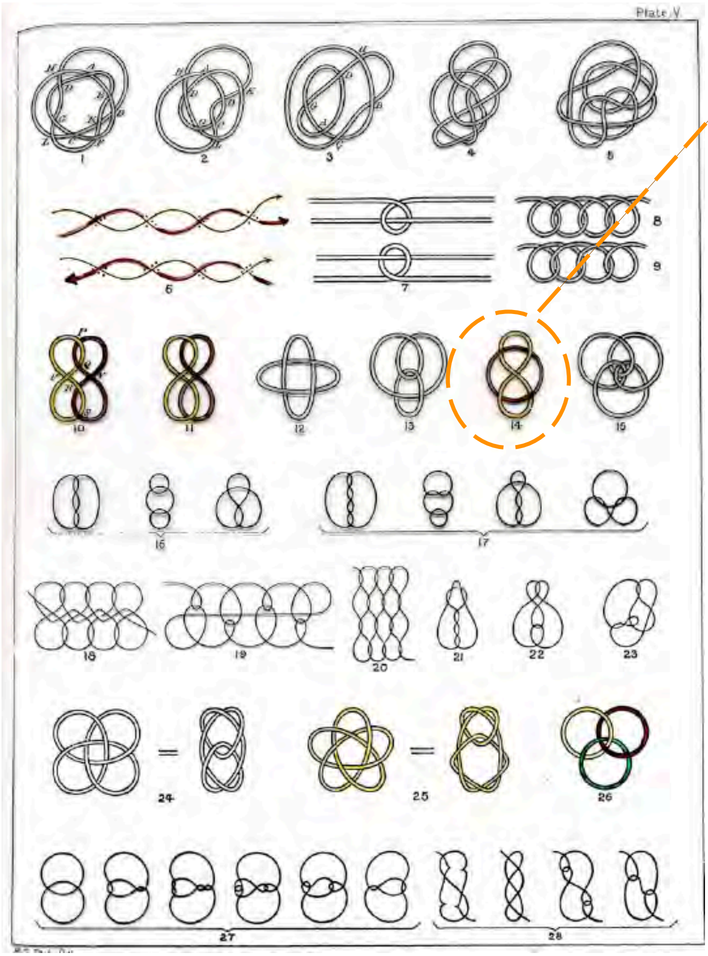
$c_{\min} = 4$

$Lk = +2$

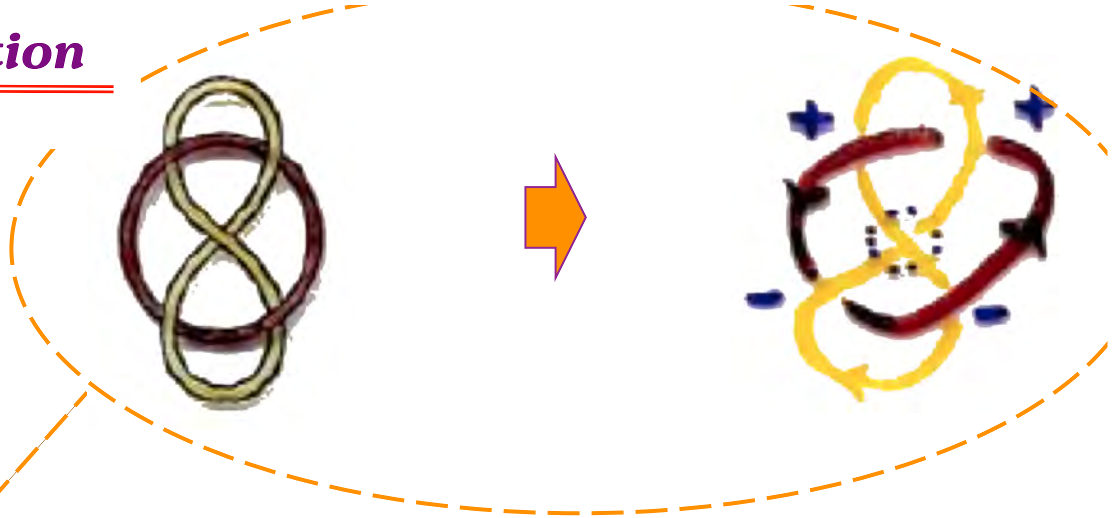
$c_{\min} = 0$

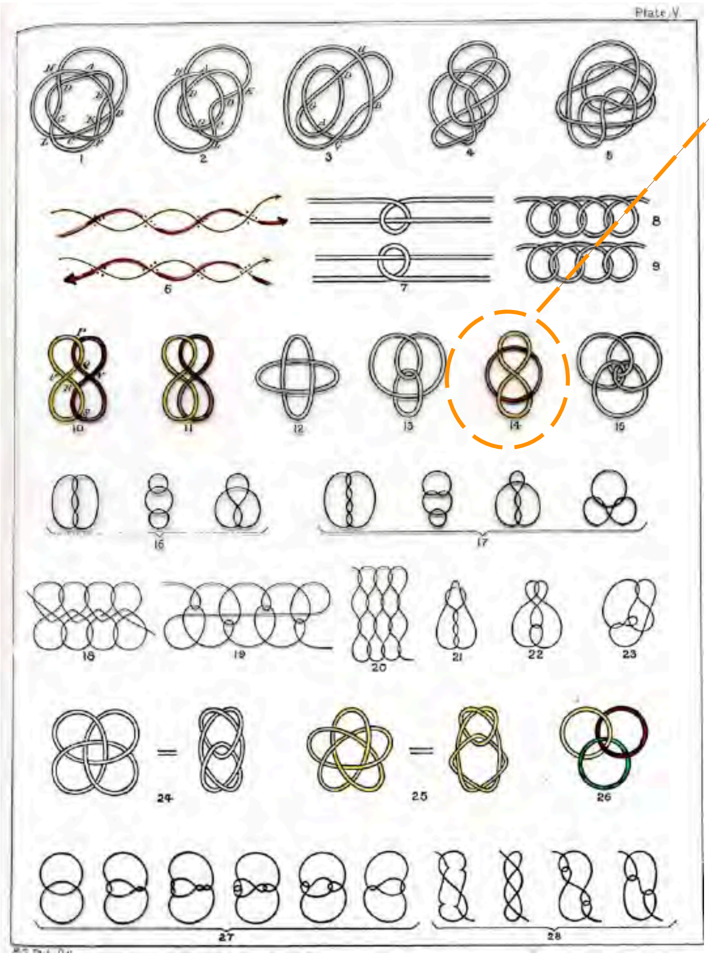
$Lk = 0$

First results from Tait's tabulation



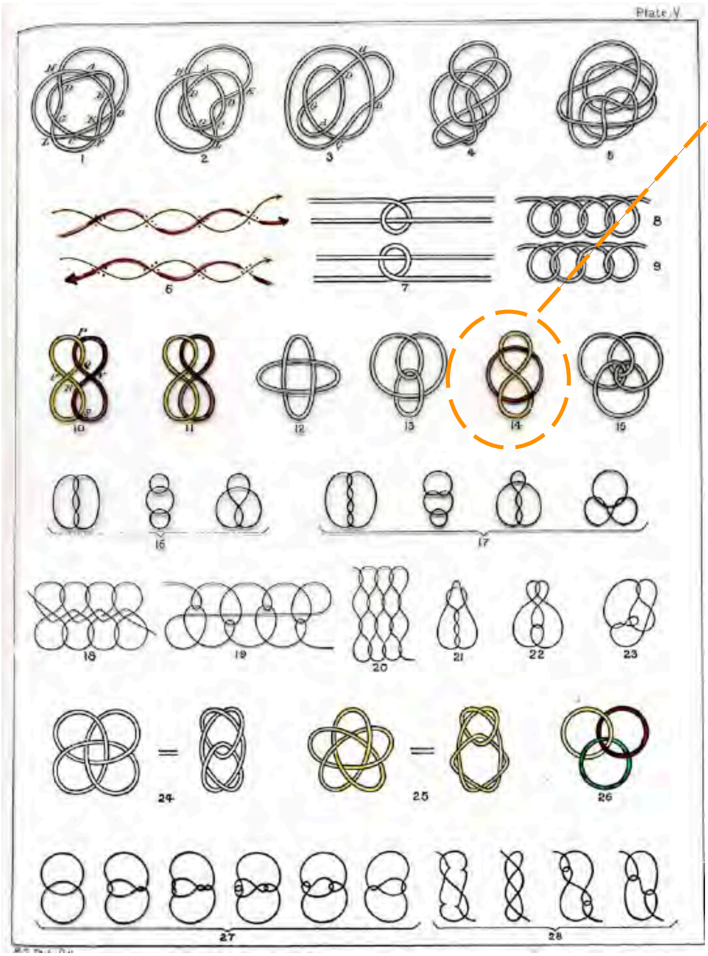
First results from Tait's tabulation





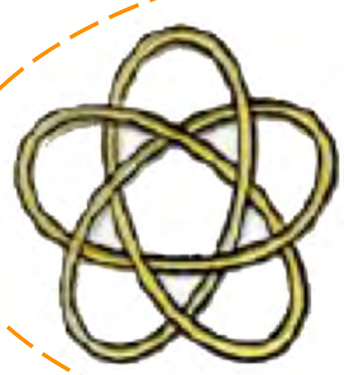
First results from Tait's tabulation

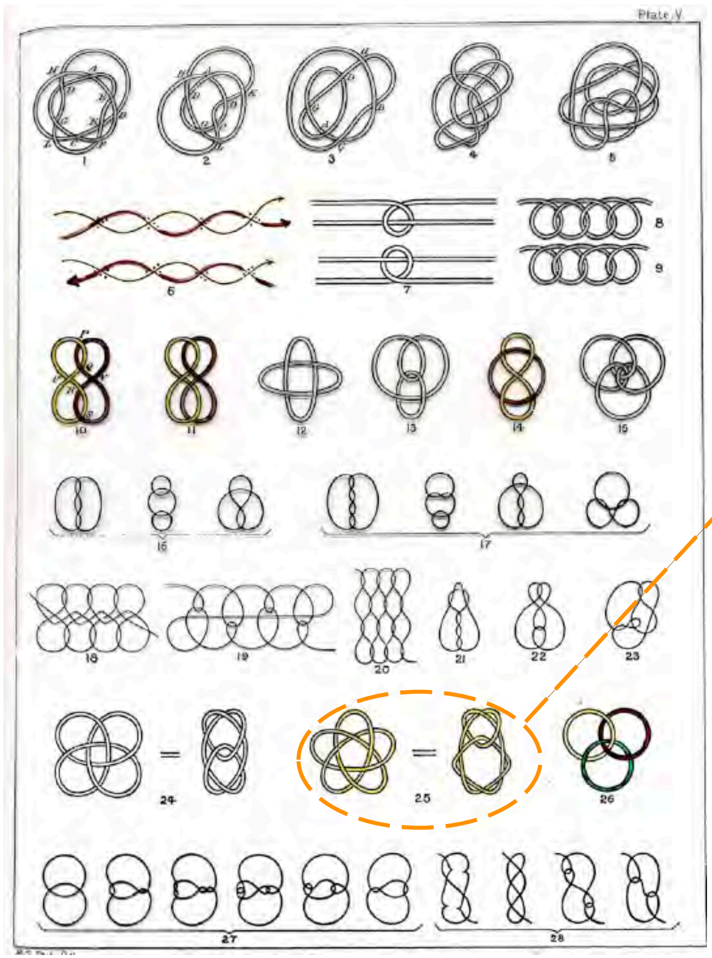
$c_{\min} = 5$, $Lk = 0$



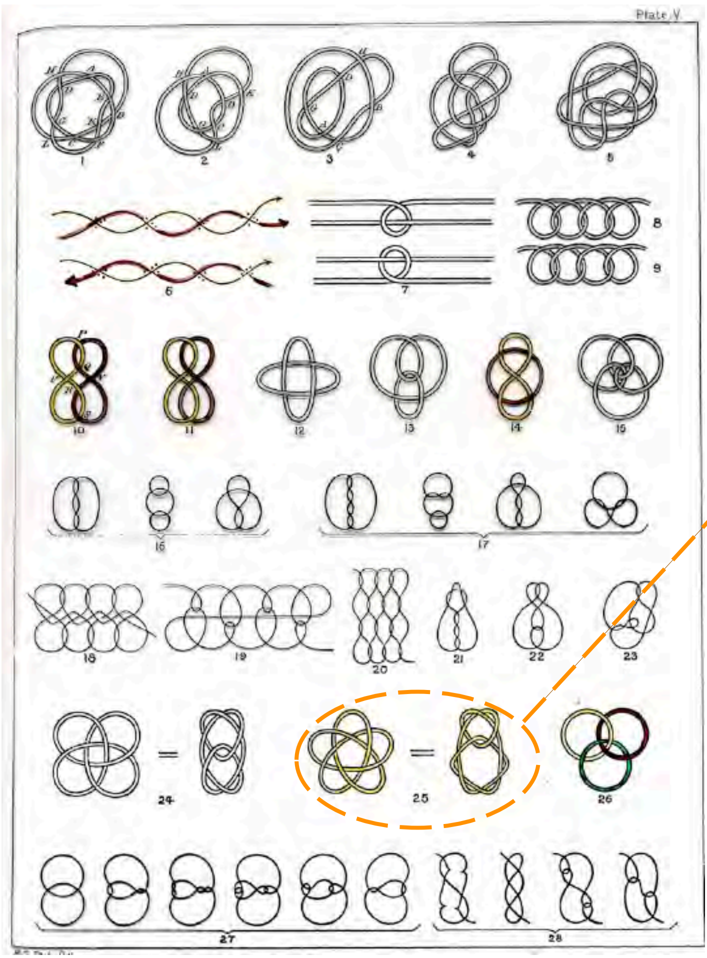
First results from Tait's tabulation

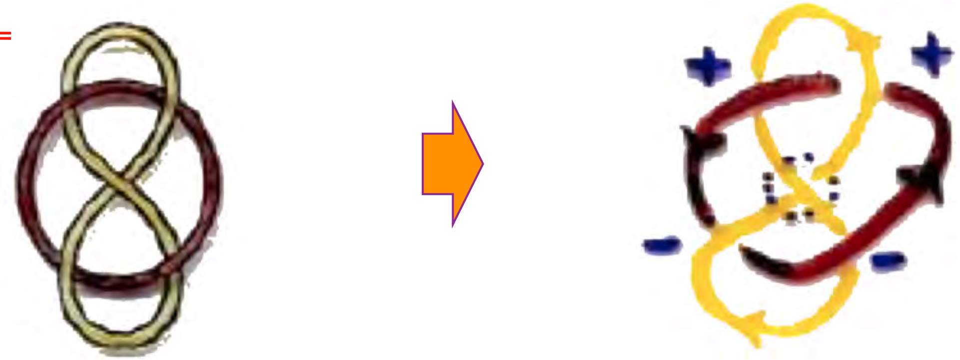
$c_{\min} = 5$, $Lk = 0$



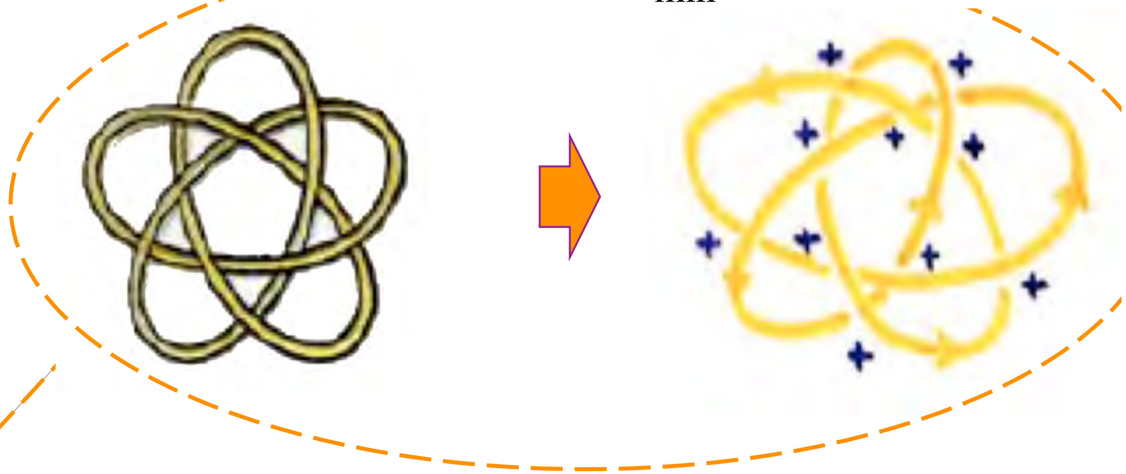


First results from Tait's tabulation



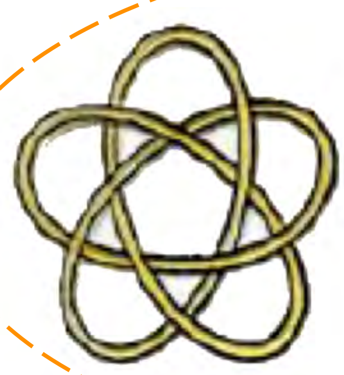


$c_{\min} = 5 \; , \quad Lk = 0$

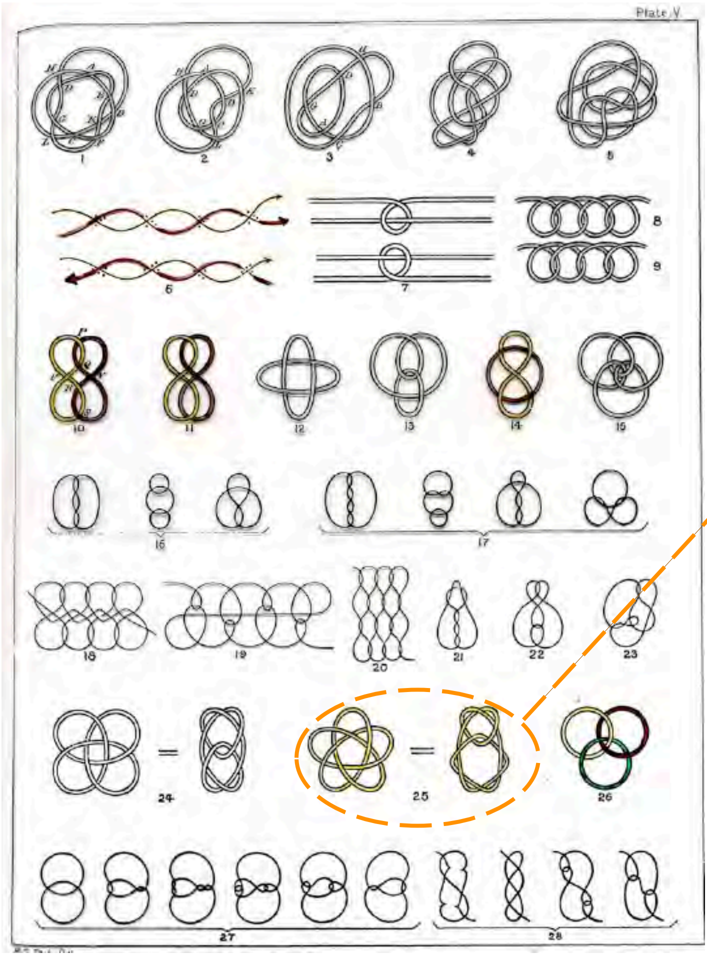


First results from Tait's tabulation

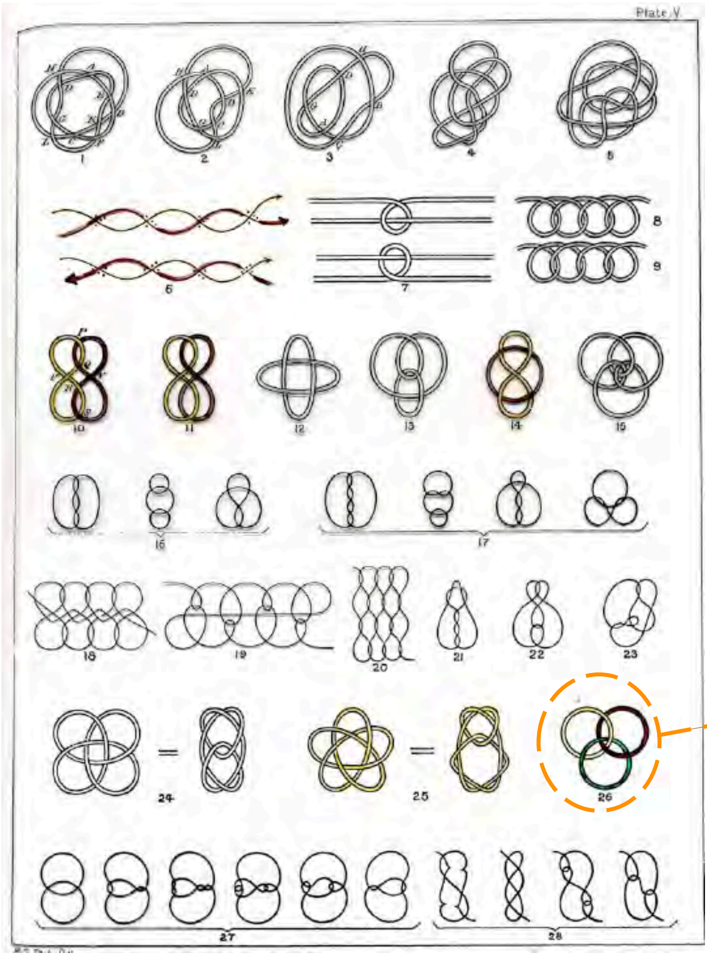
$c_{\min} = 5$, $Lk = 0$



$c_{\min} = 10$, $Lk = ?$



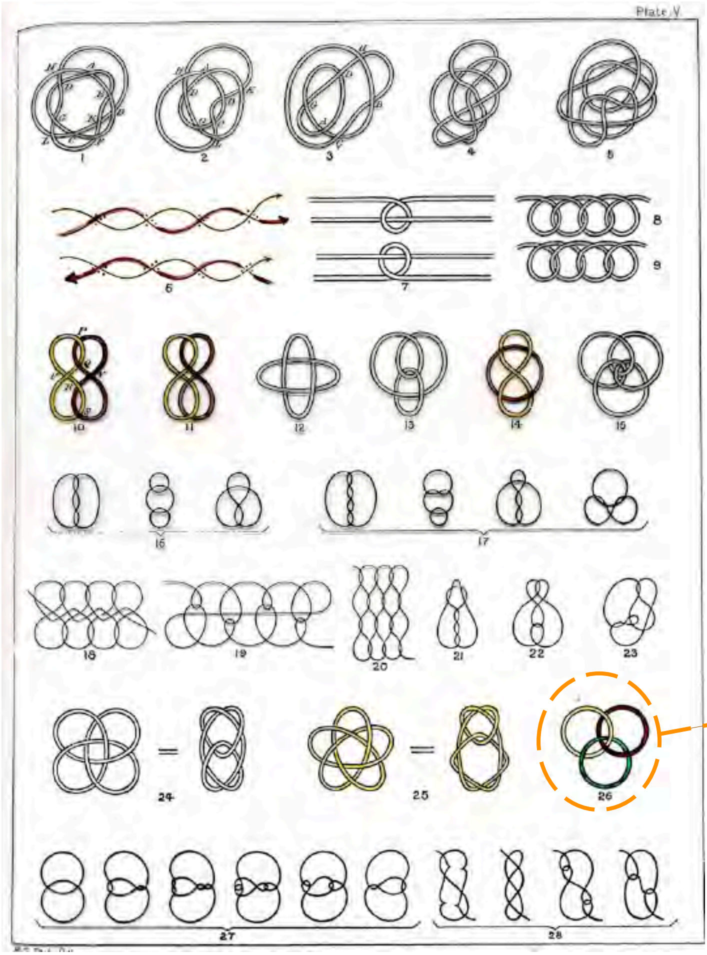
First results from Tait's tabulation



$c_{\min} = 5$, $Lk = 0$

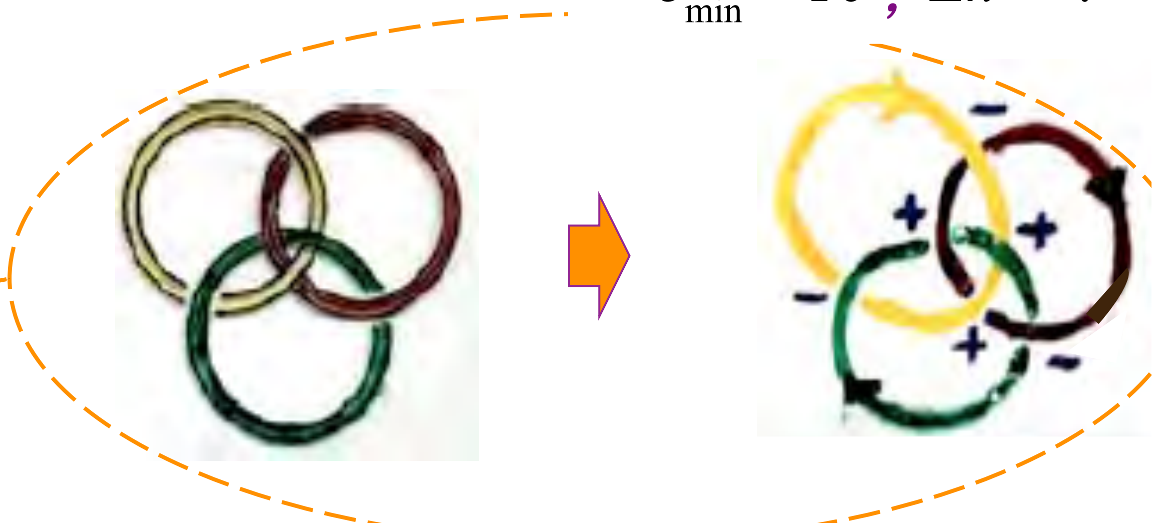
$c_{\min} = 10$, $Lk = ?$

First results from Tait's tabulation

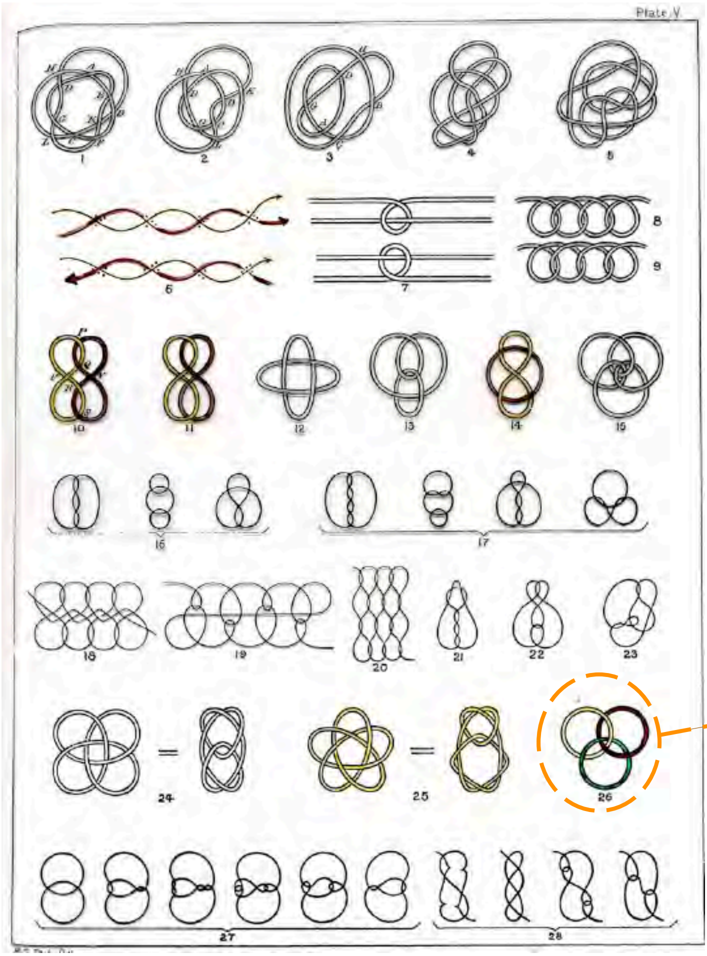


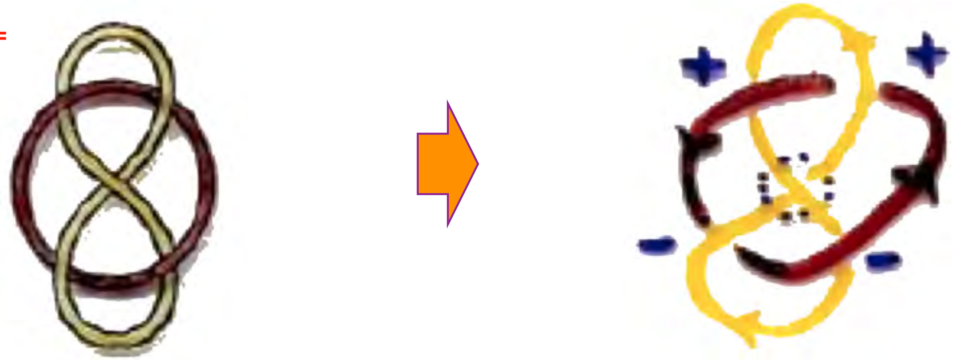
$c_{\text{min}} = 5$, $Lk = 0$

$c_{\text{min}} = 10$, $Lk = ?$

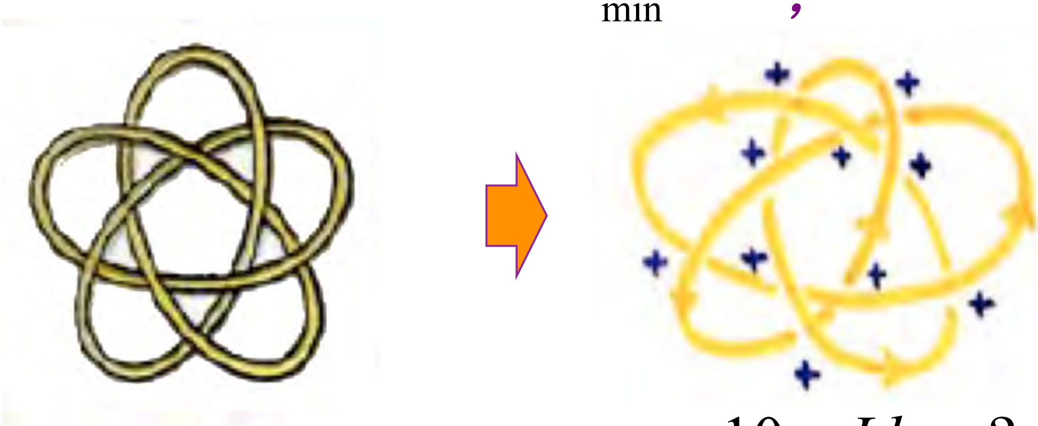


First results from Tait's tabulation

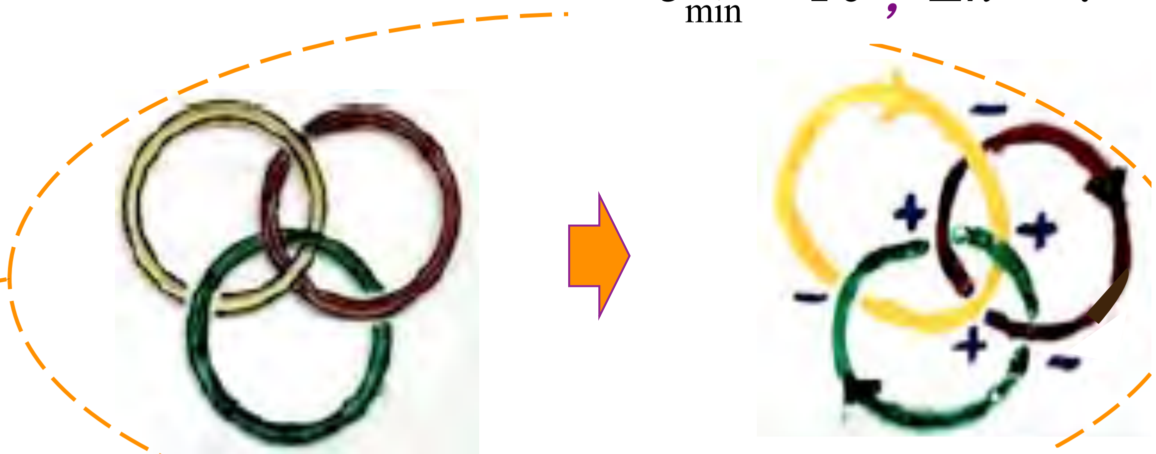




$c_{\min} = 5 \; , \quad Lk = 0$



$c_{\min} = 10 \; , \quad Lk = ?$



$c_{\min} = 6 \; , \quad Lk = 0$

Reidemeister's moves

- *Type I:*

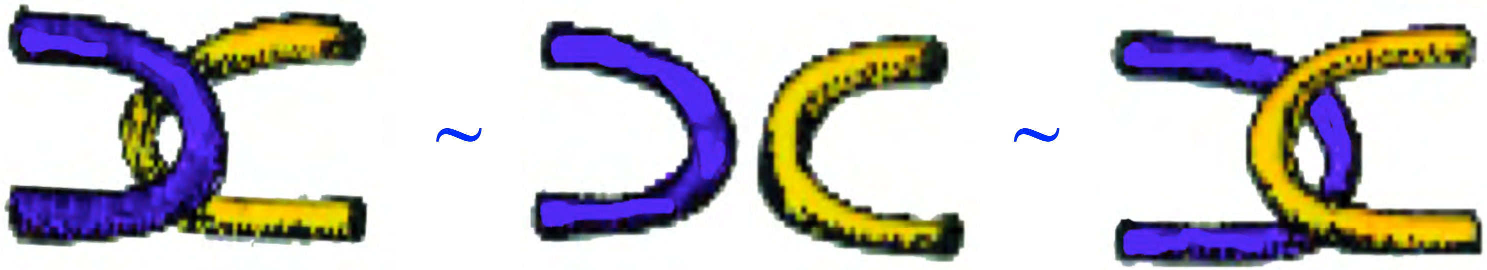
twist move

Reidemeister's moves

- **Type I:**

twist move

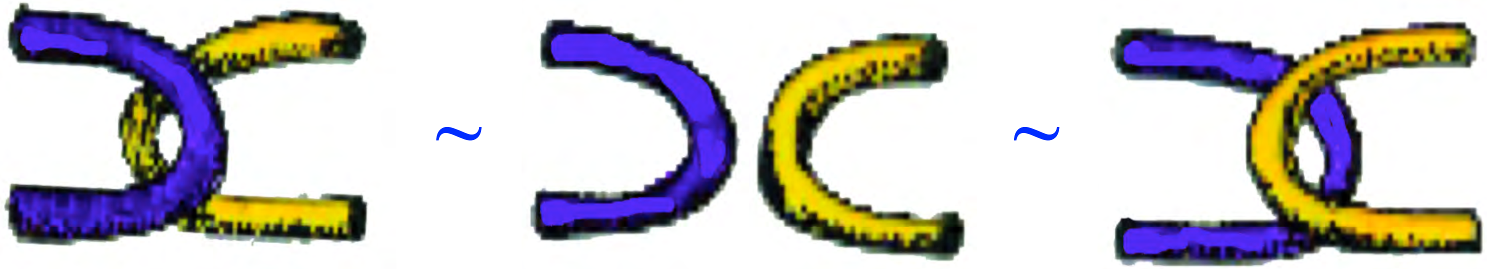
- **Type II:**



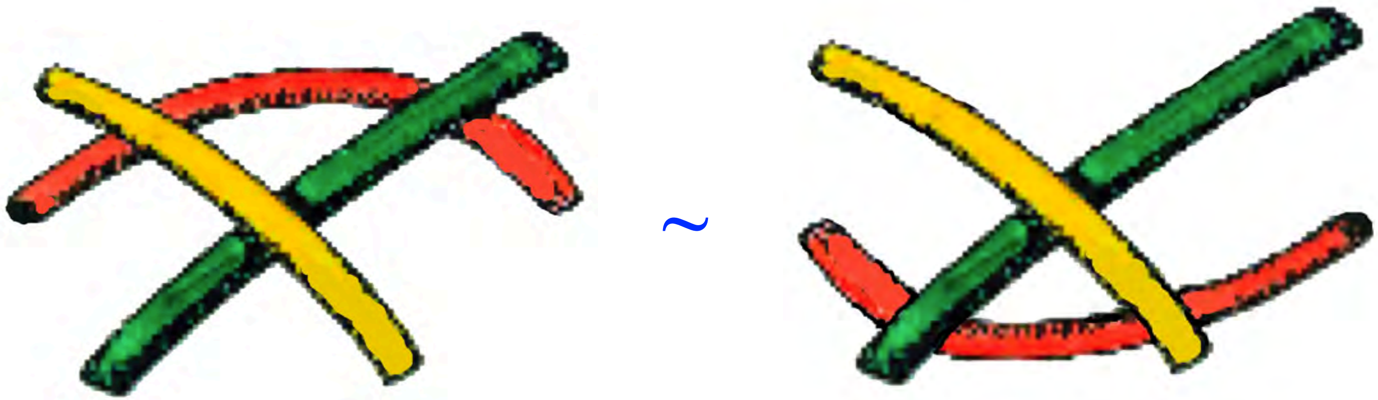
Reidemeister's moves

- ***Type I:***

- ***Type II:***



- ***Type III:***

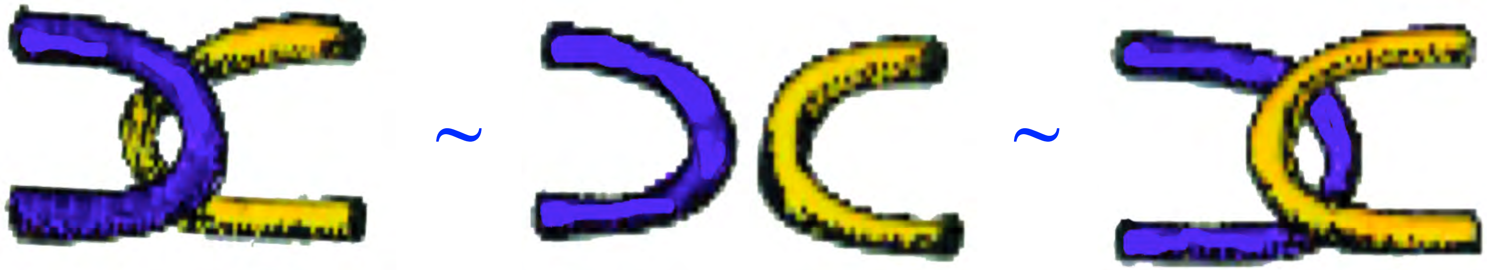


A remark: topology versus mechanics

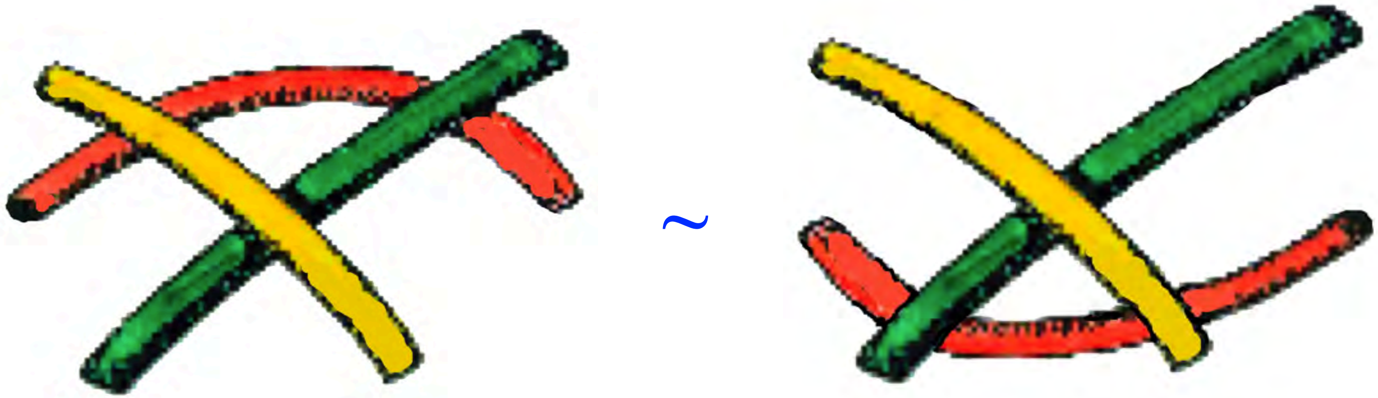
- ***Type I:***

twist move

- ***Type II:***



- ***Type III:***

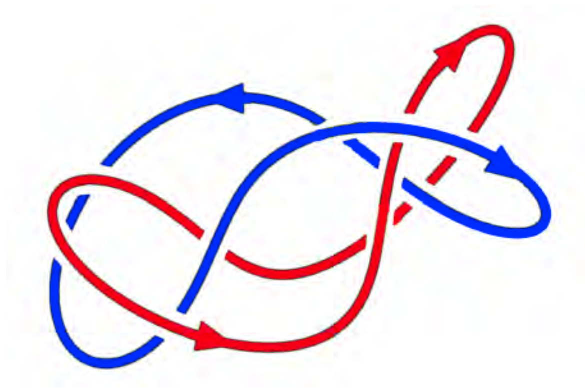


"flat topology"

Reidemeister's theorem

"Two knots/links are topologically equivalent – i.e. they represent the same knot/link type – if one can be transformed into the other by a finite sequence of Reidemeister's moves."

- *Example:*



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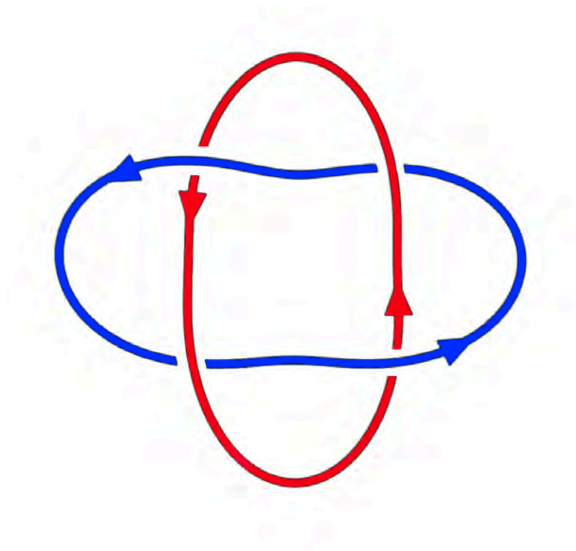
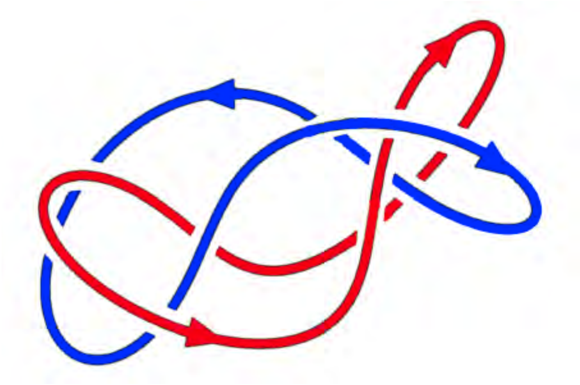
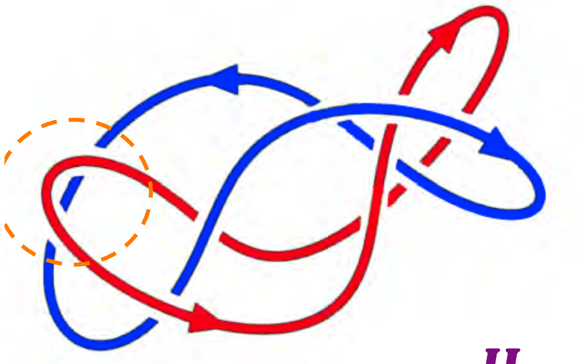


Reidemeister's moves in action

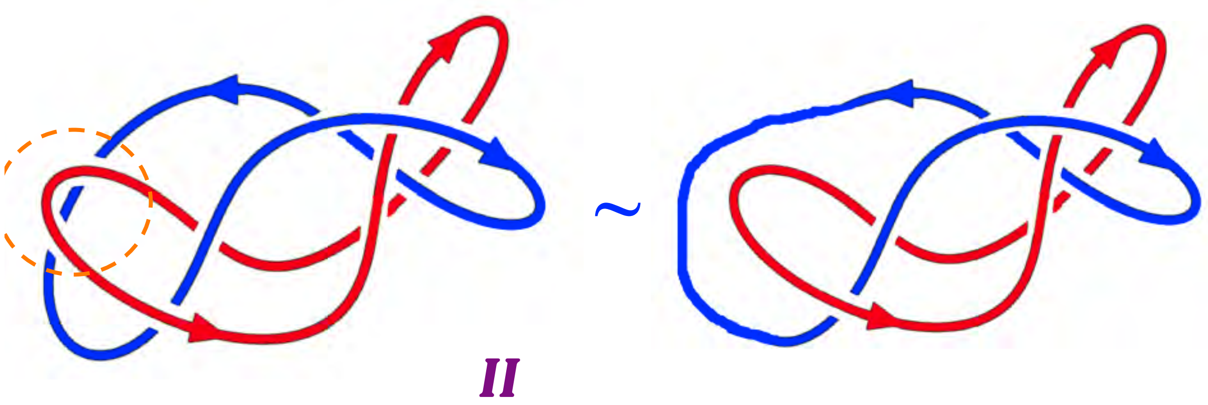


Reidemeister's moves in action

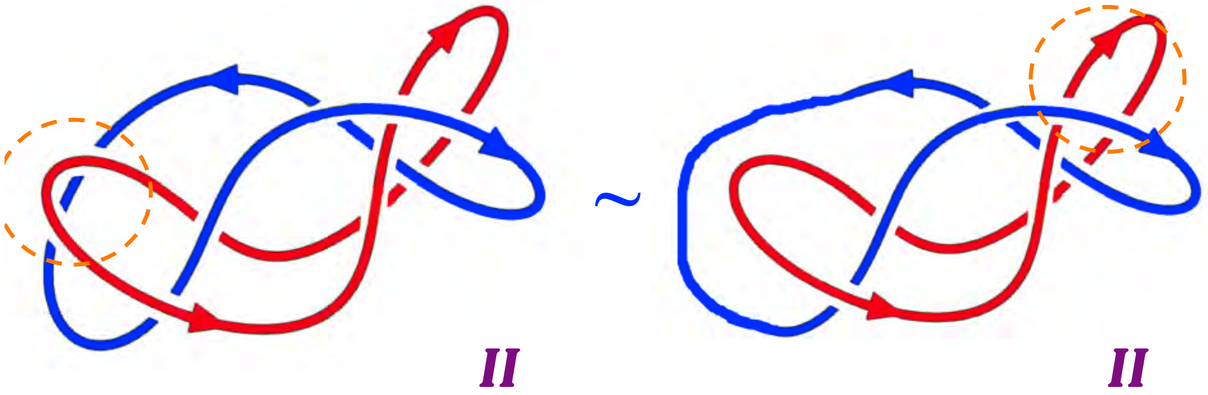


II

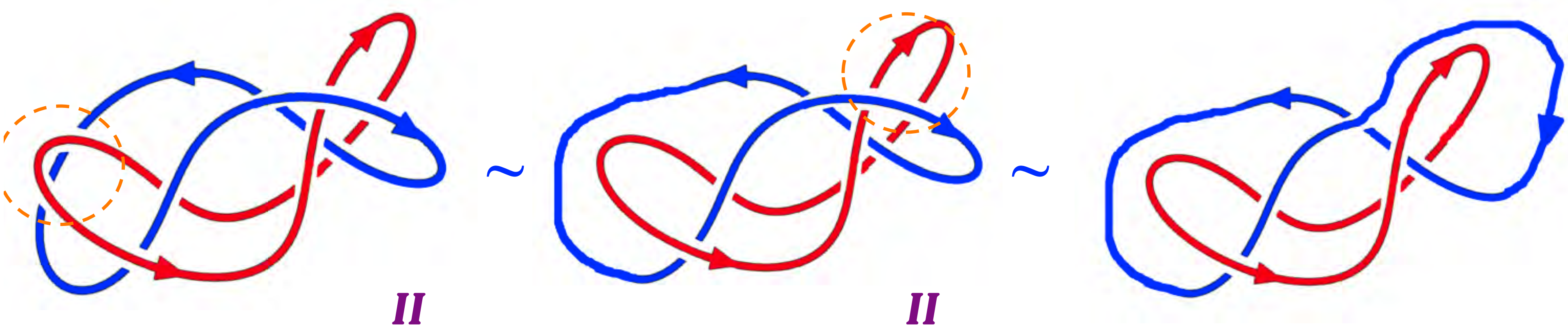
Reidemeister's moves in action



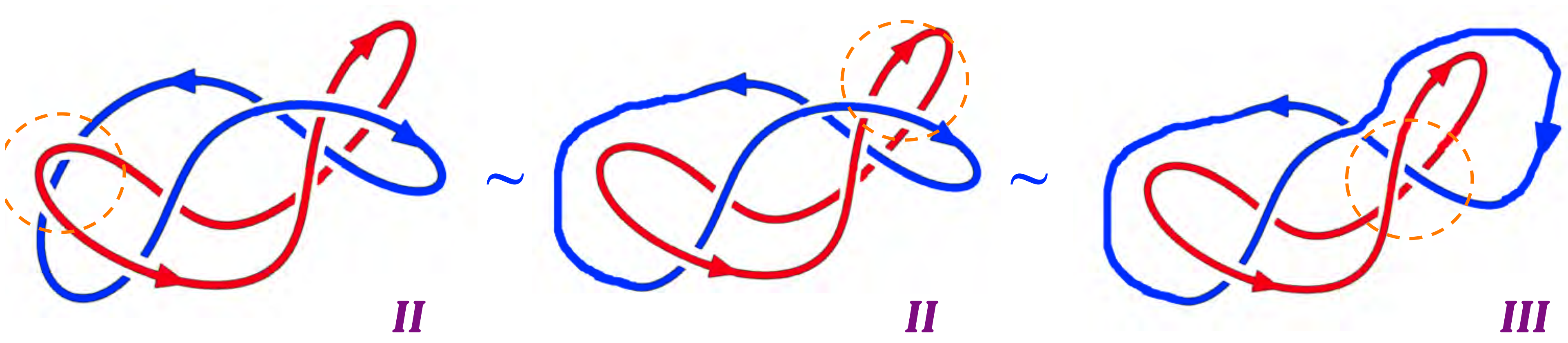
Reidemeister's moves in action



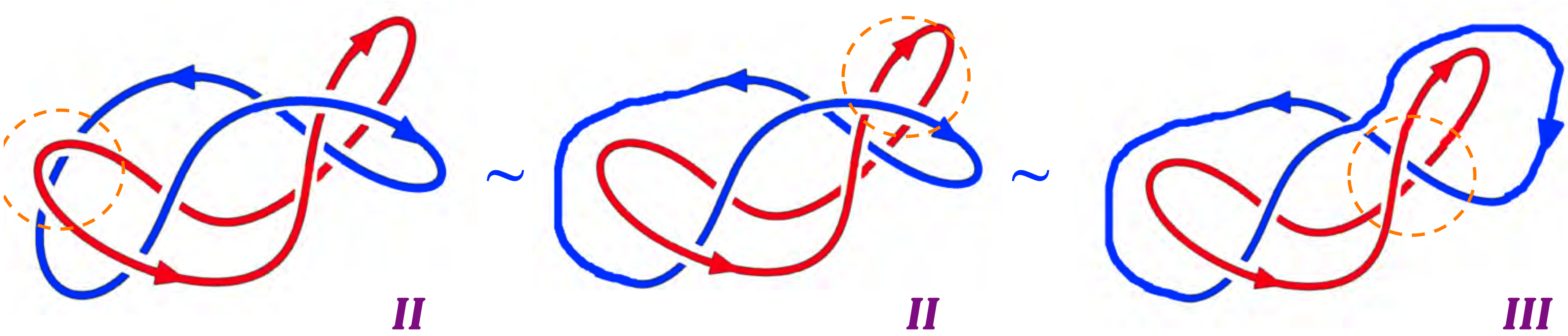
Reidemeister's moves in action

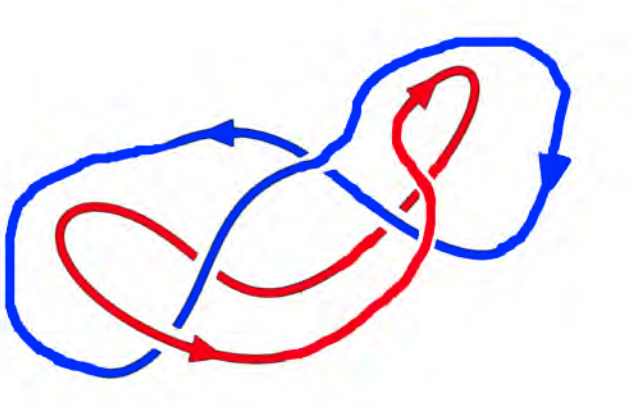


Reidemeister's moves in action

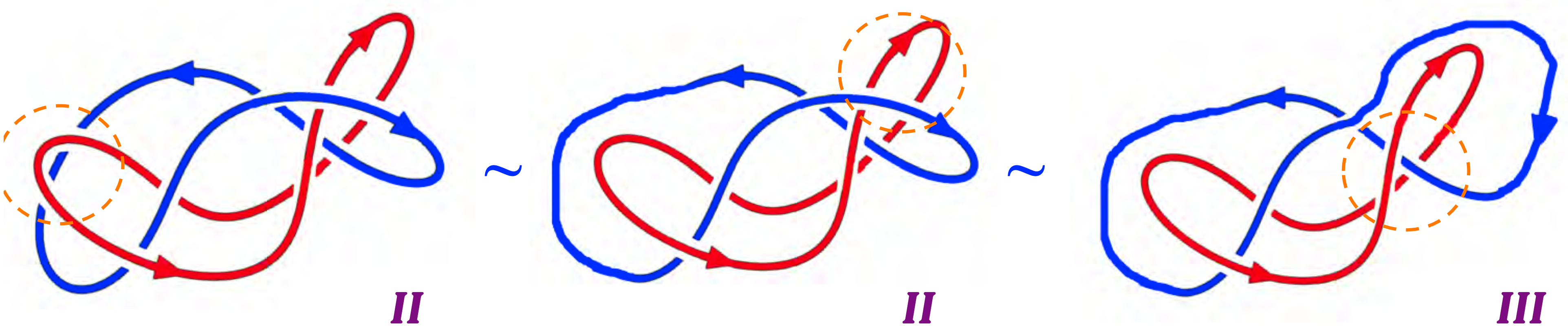


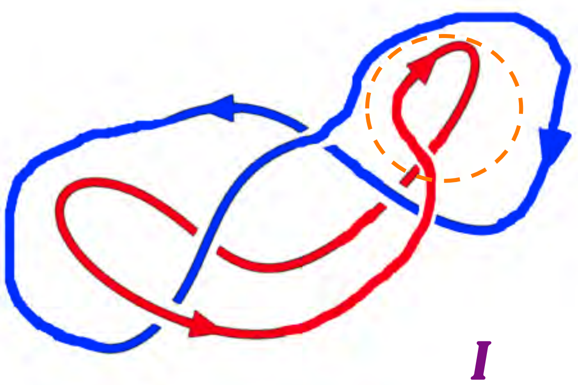
Reidemeister's moves in action



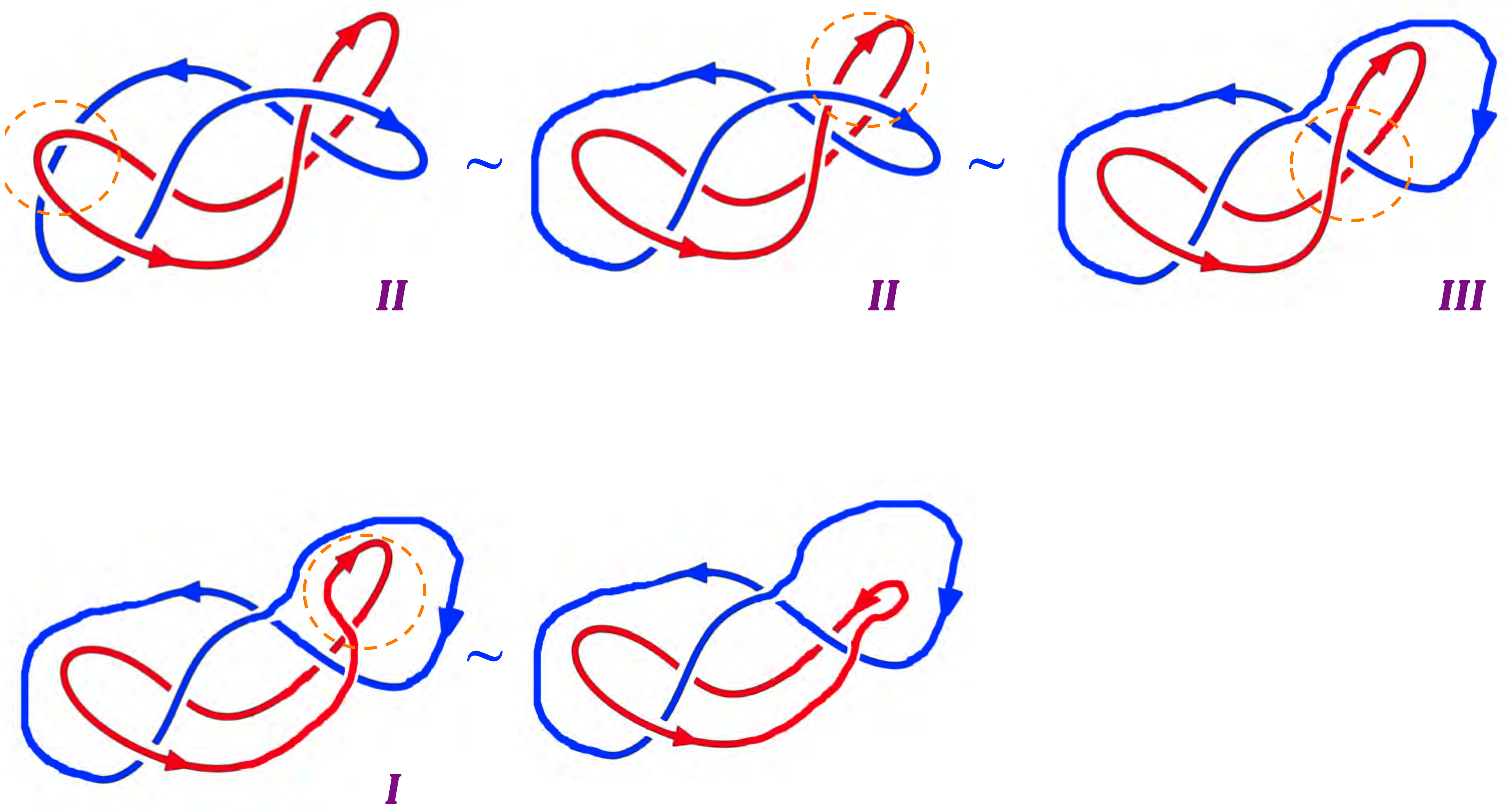


Reidemeister's moves in action

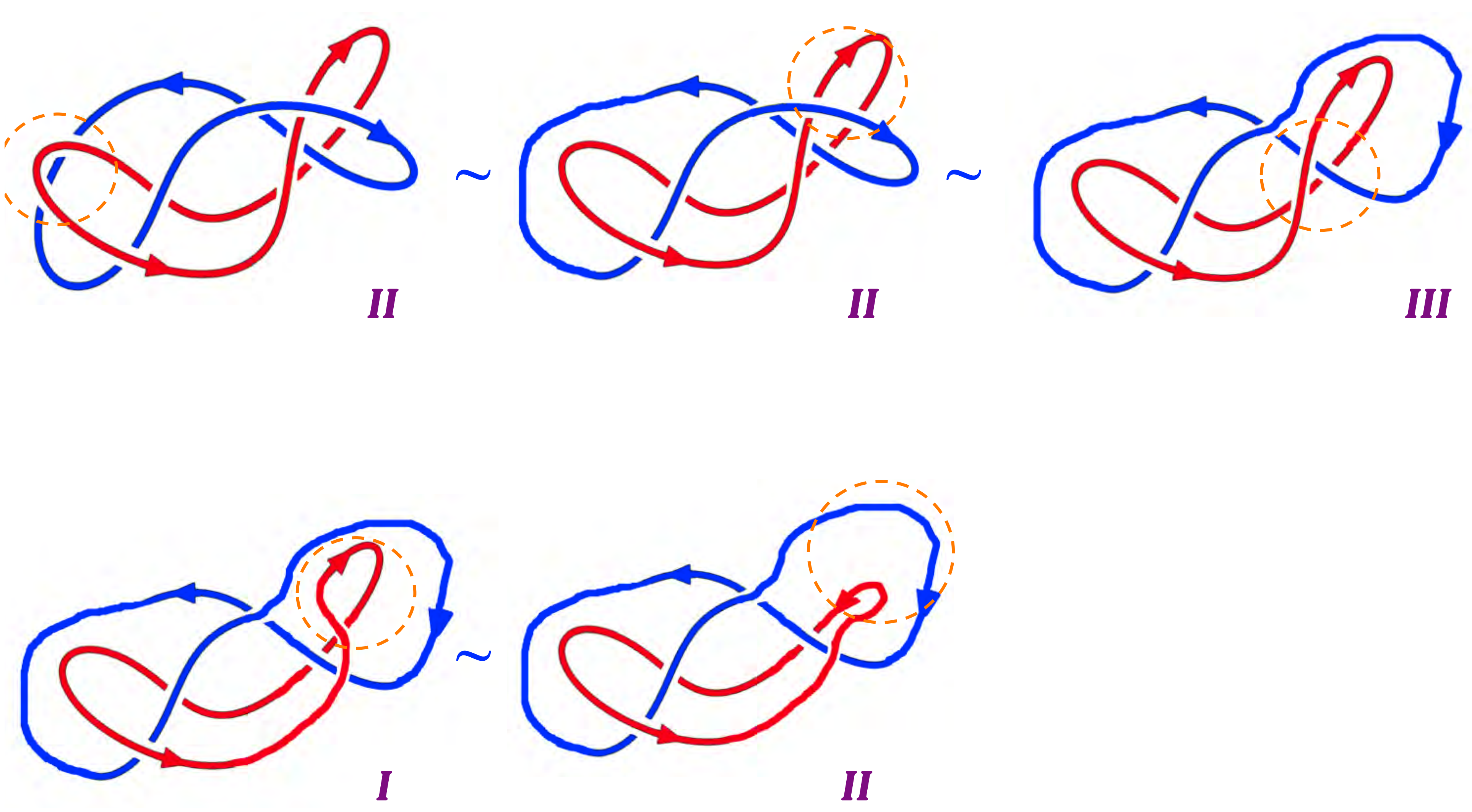




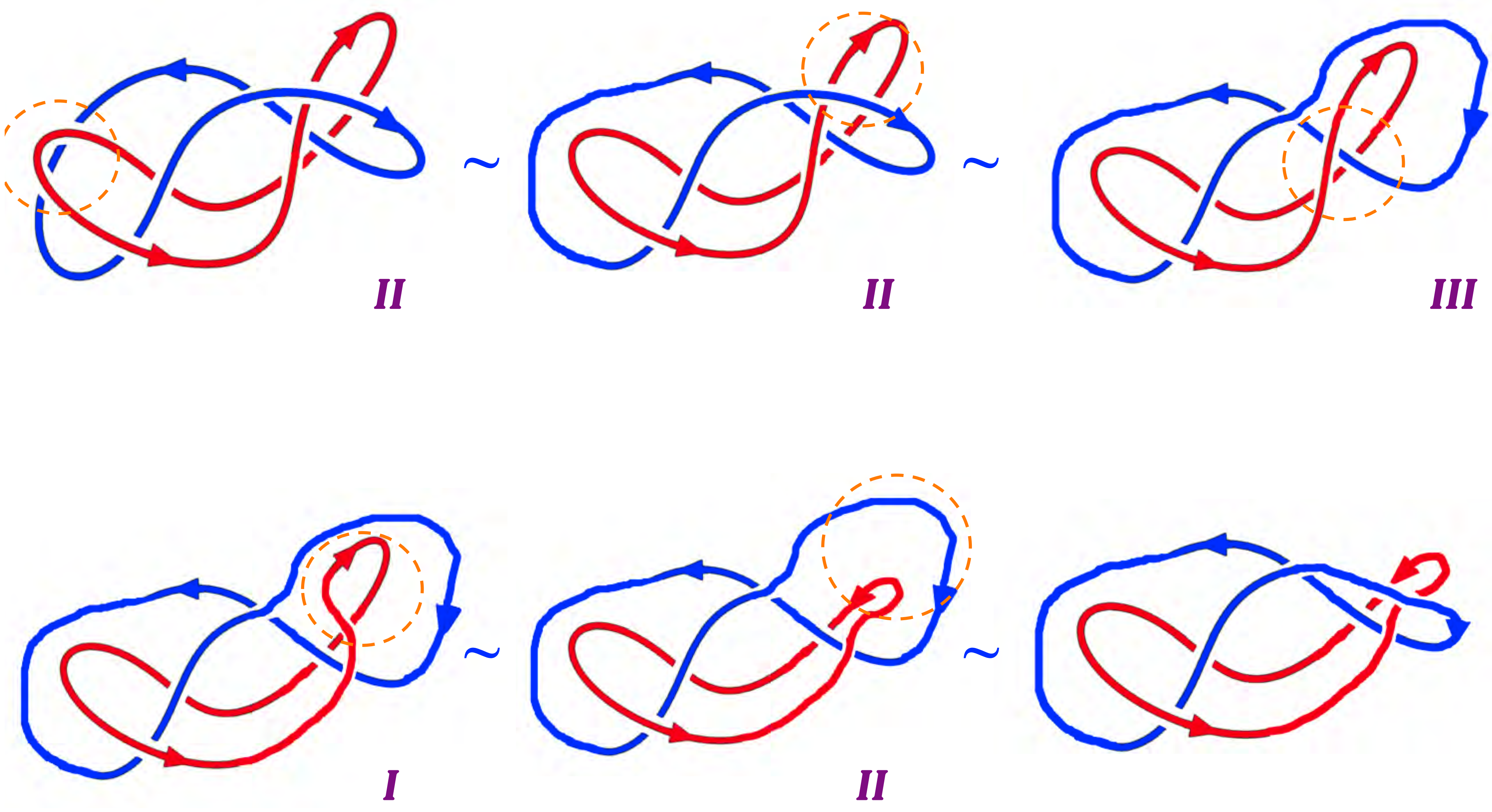
Reidemeister's moves in action



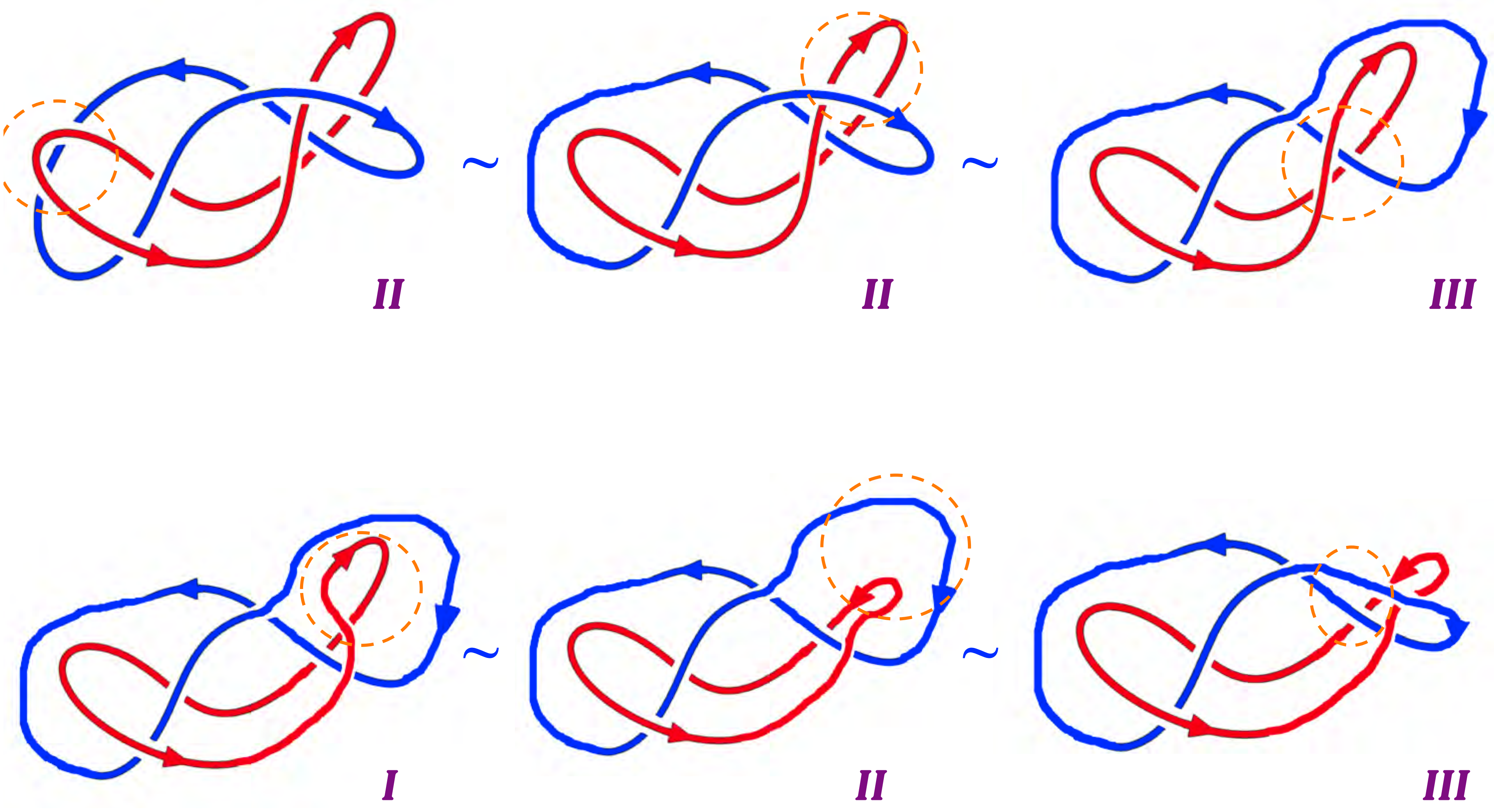
Reidemeister's moves in action



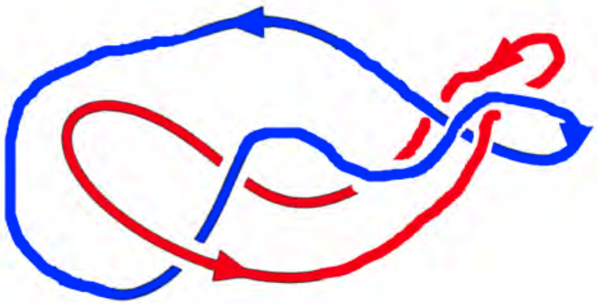
Reidemeister's moves in action



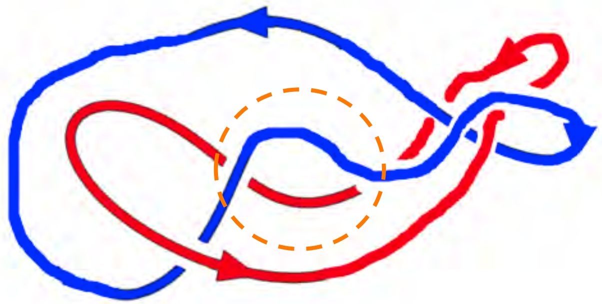
Reidemeister's moves in action



Reidemeister's moves in action (cont.)

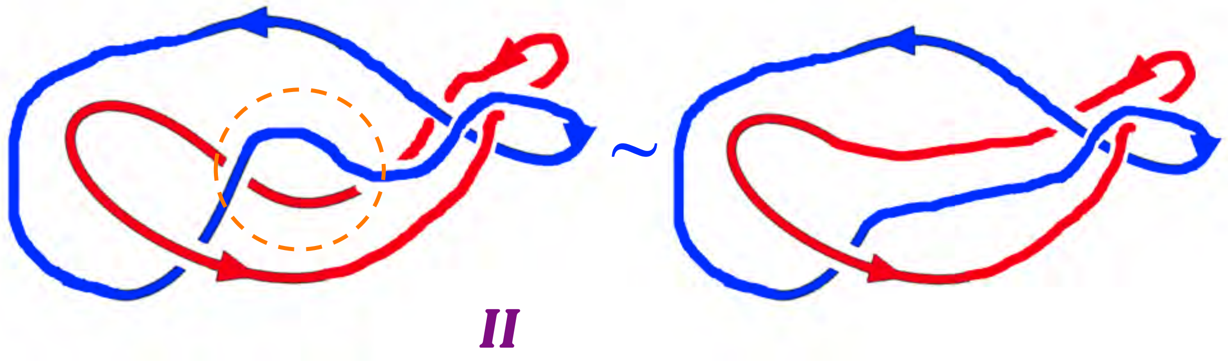


Reidemeister's moves in action (cont.)

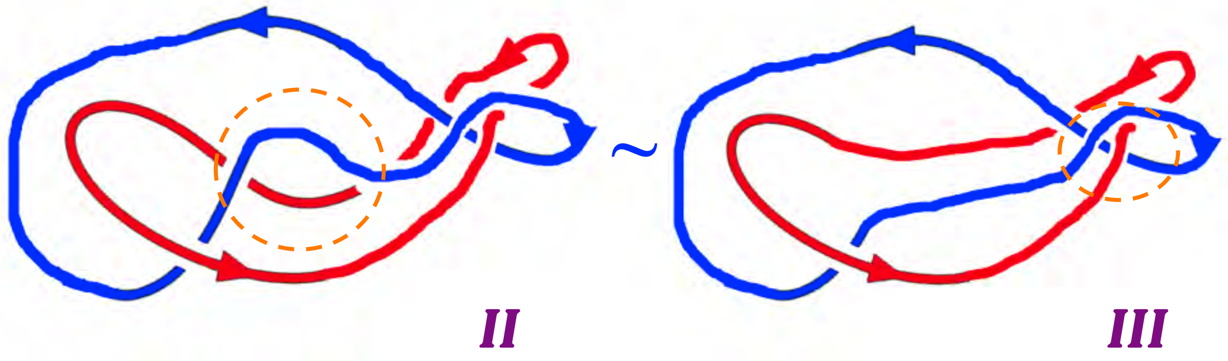


II

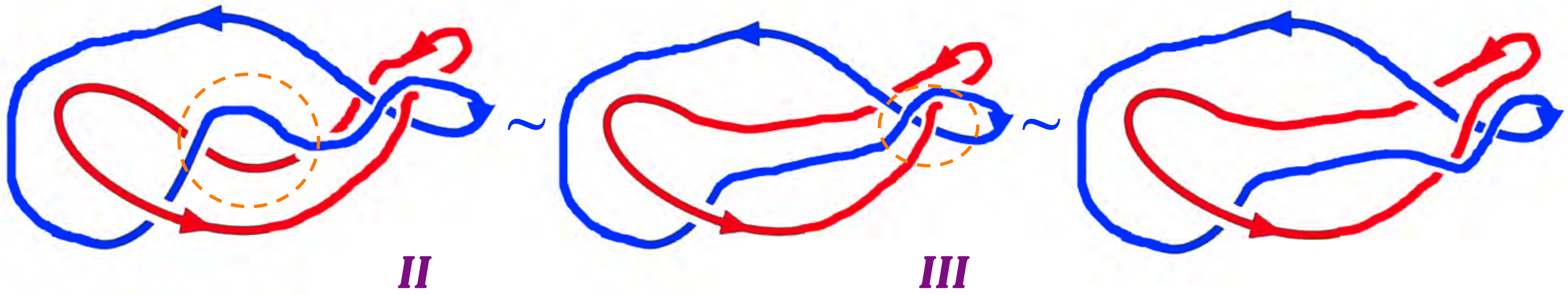
Reidemeister's moves in action (cont.)



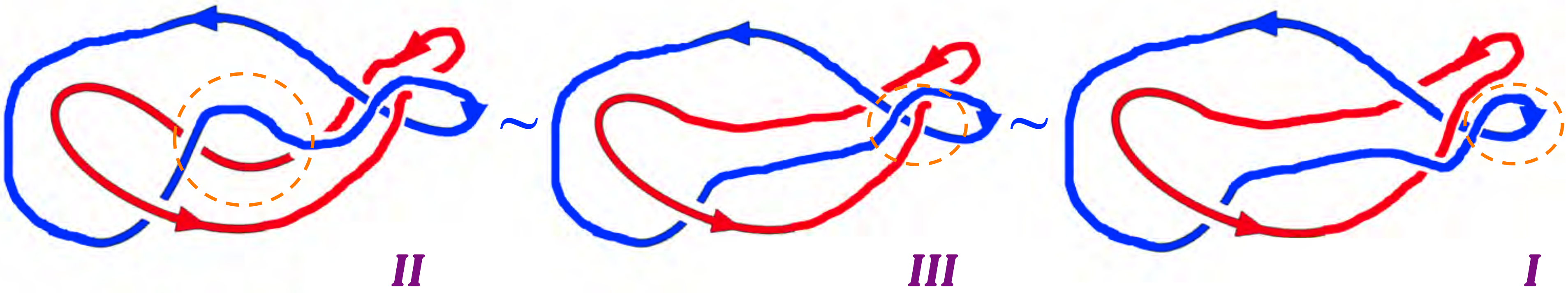
Reidemeister's moves in action (cont.)



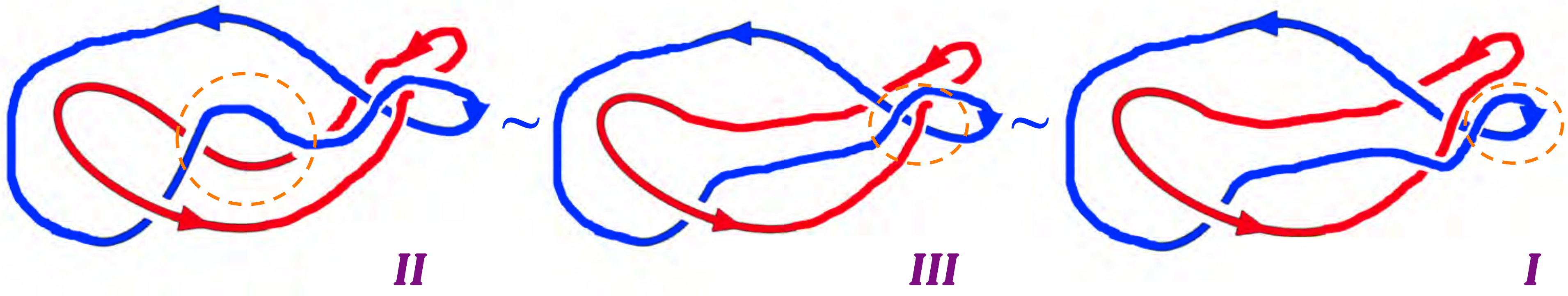
Reidemeister's moves in action (cont.)

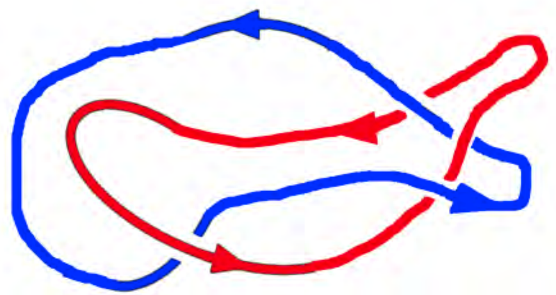


Reidemeister's moves in action (cont.)

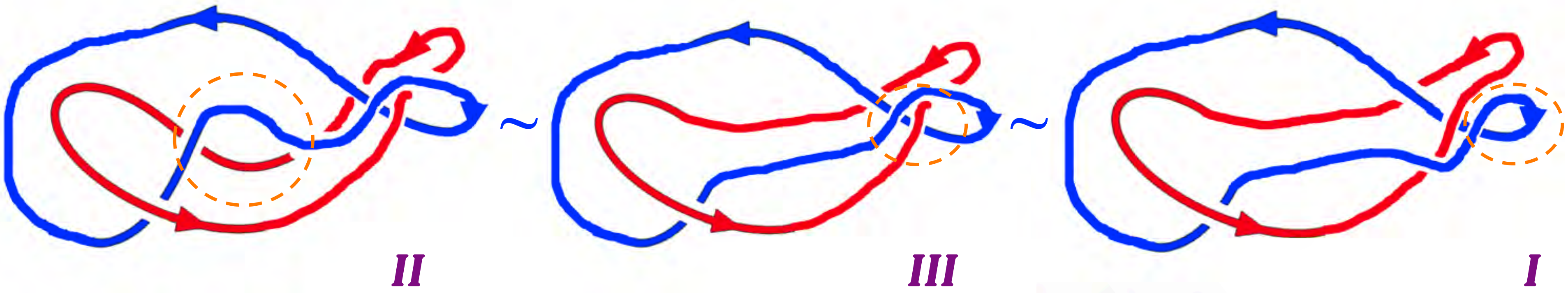


Reidemeister's moves in action (cont.)

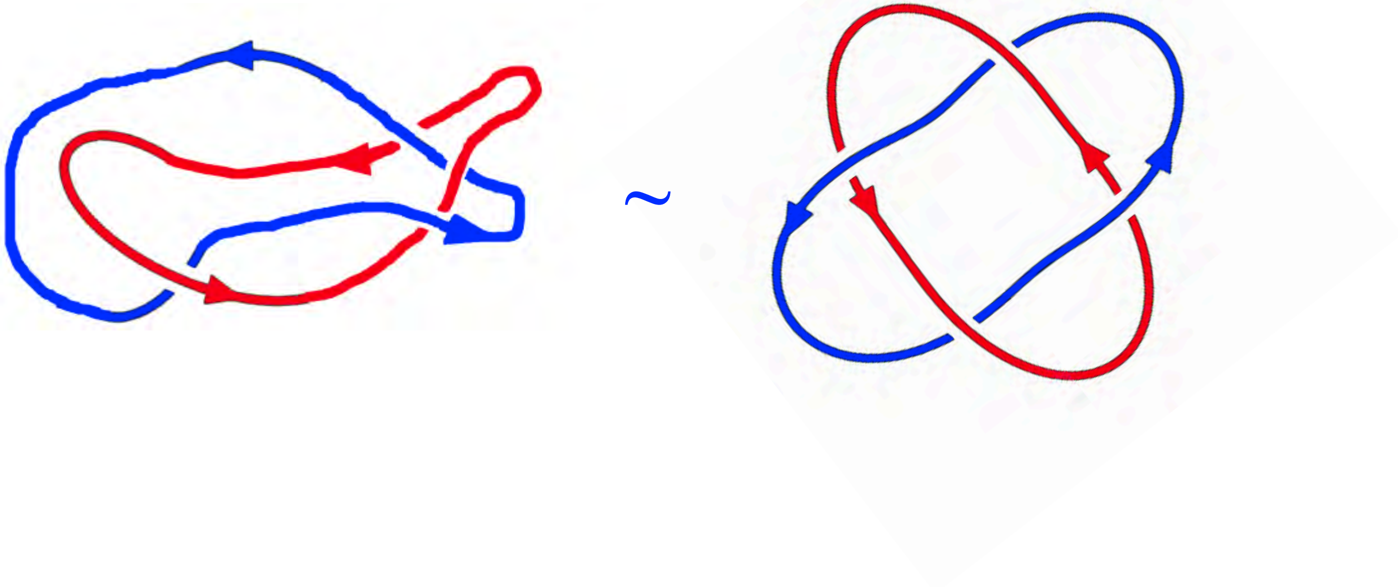




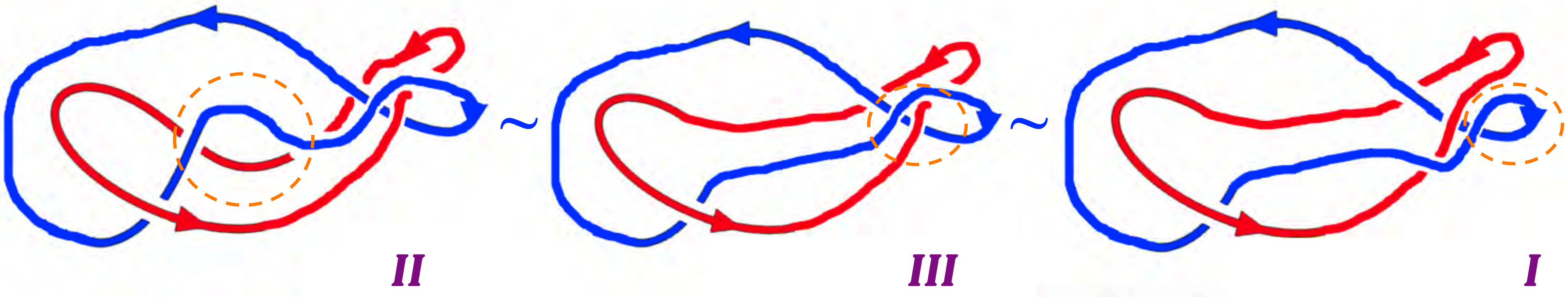
Reidemeister's moves in action (cont.)



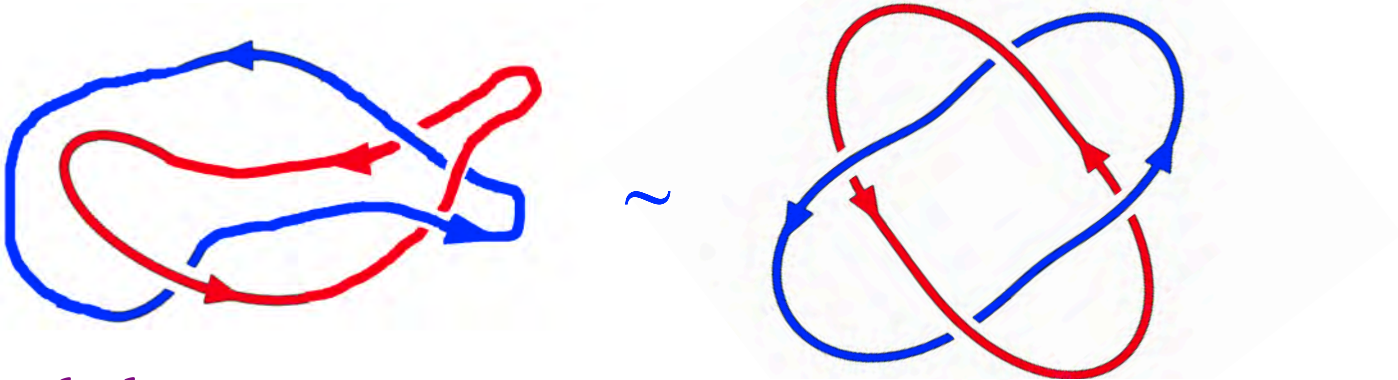
hence



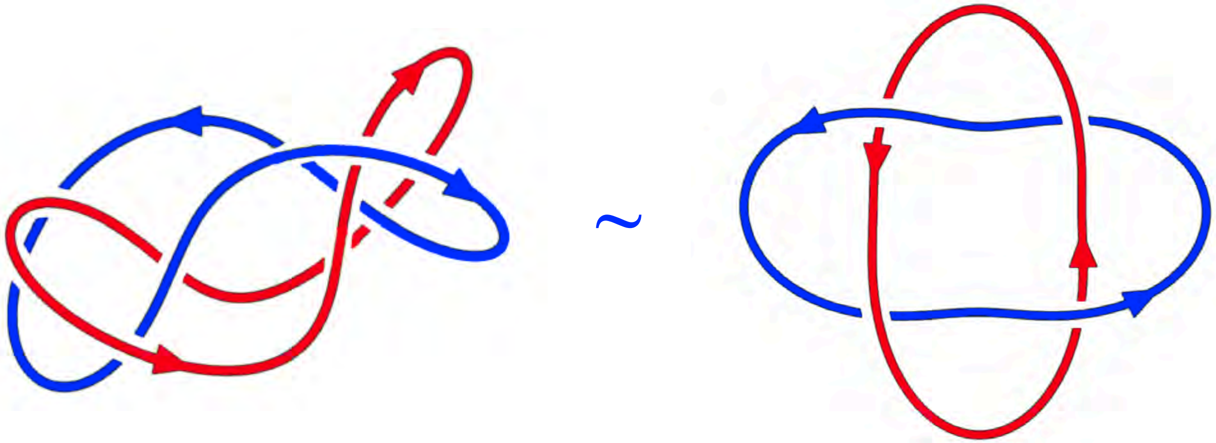
Reidemeister's moves in action (cont.)



hence



We have proved that



∎

End of Lecture 1

Lecture 2

- ***When topology meets geometry:***

  - Self-linking of a ribbon = Writhe + Twist

  - Compactification, relaxation and DNA topology

- ***From knots to braid presentations:***

  - bridge index and closed braids

  - braid index and Seifert surfaces

  - Plasma loops on the Sun

- ***The polynomial era:***

  - From Alexander to Jones and modern times

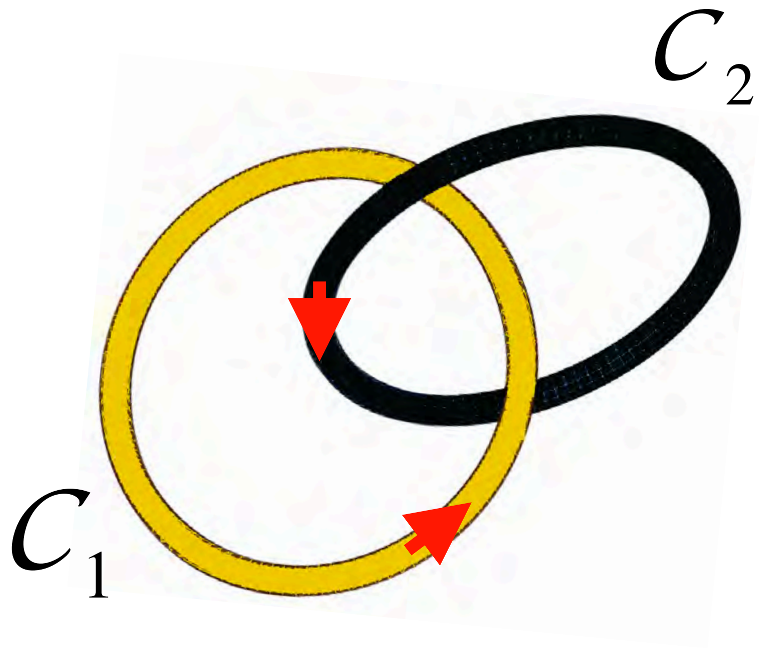
  - Jones' skein relations for knot polynomials

  - Examples of computation

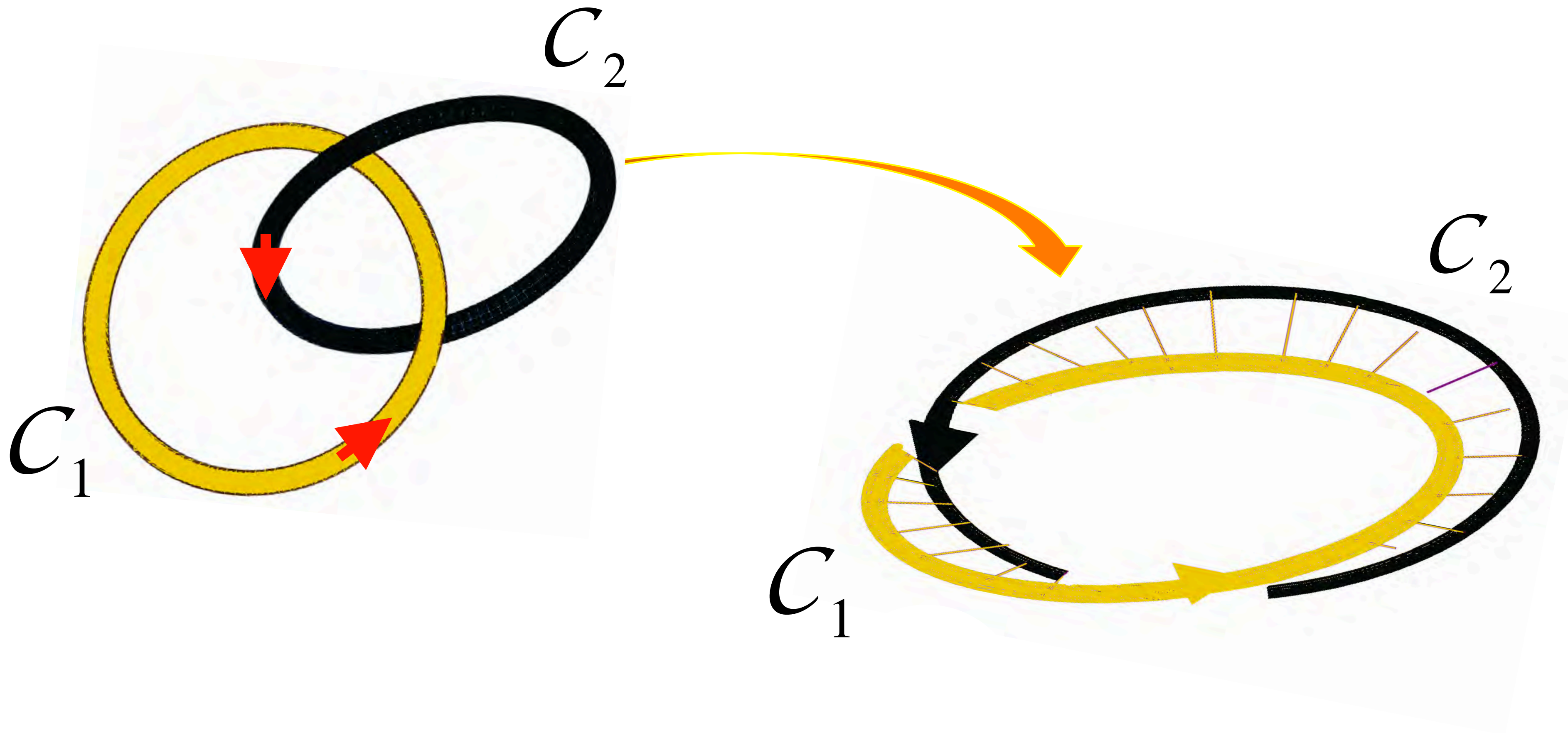
- ***Topics of current research:***

  - Vortex dynamics: HOMFLYPT best quantifier of topological complexity

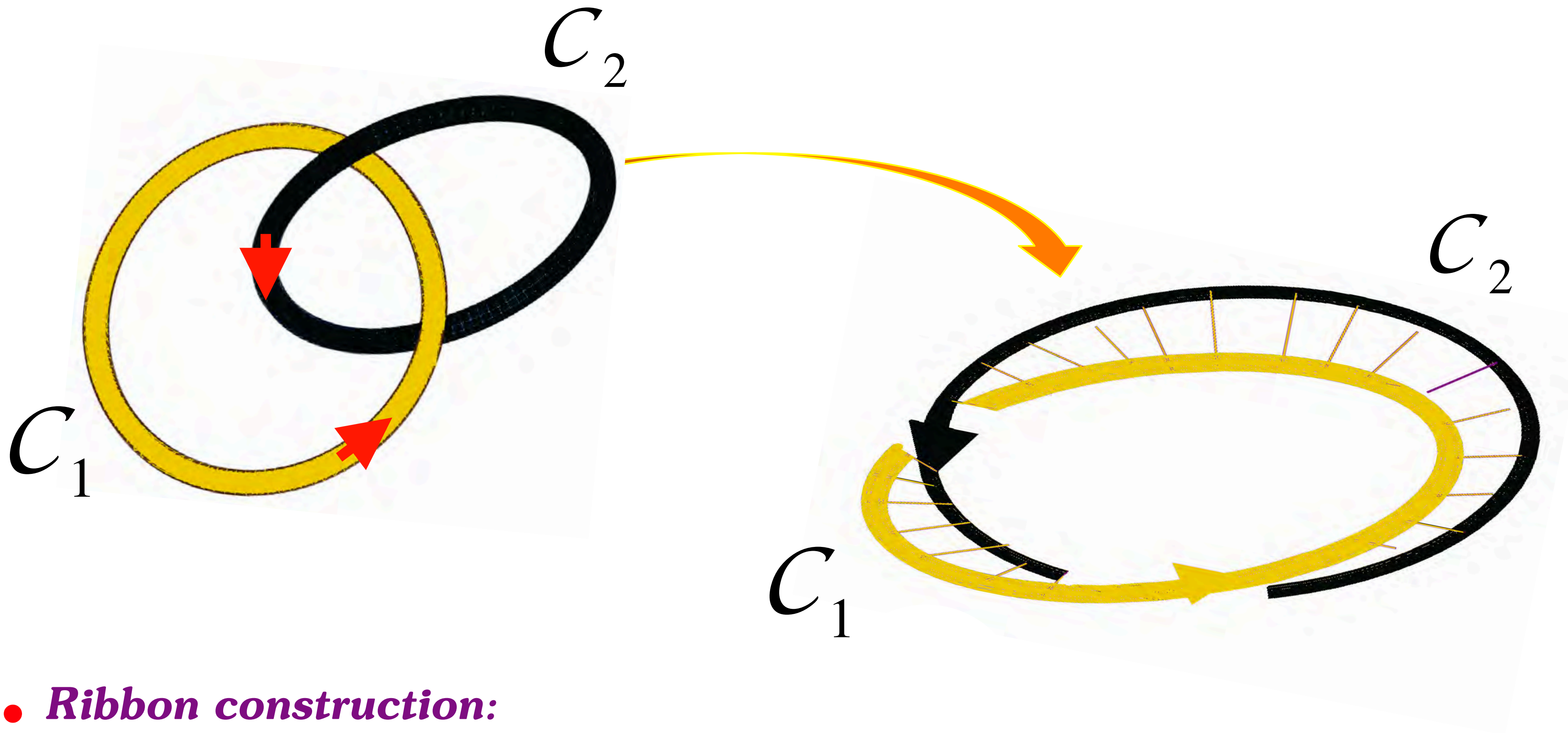
From Gauss linking number to self-linking number



From Gauss linking number to self-linking number



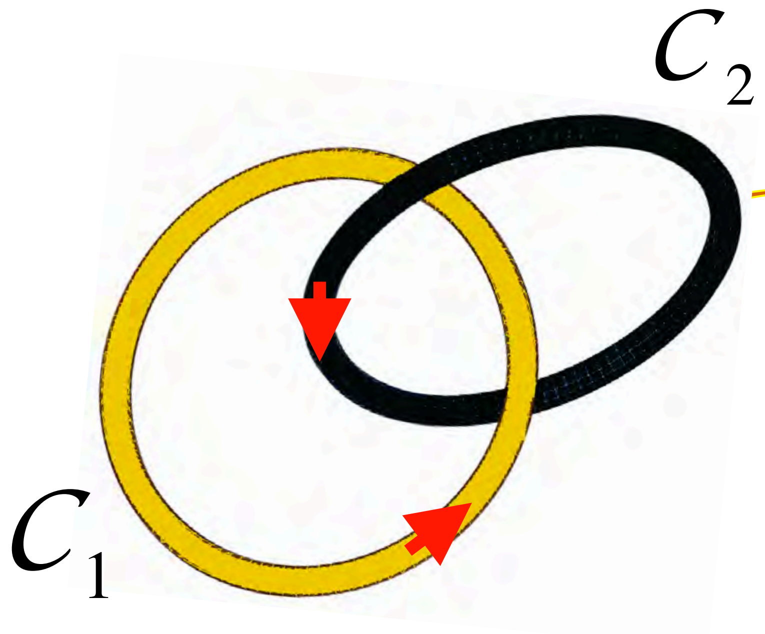
From Gauss linking number to self-linking number

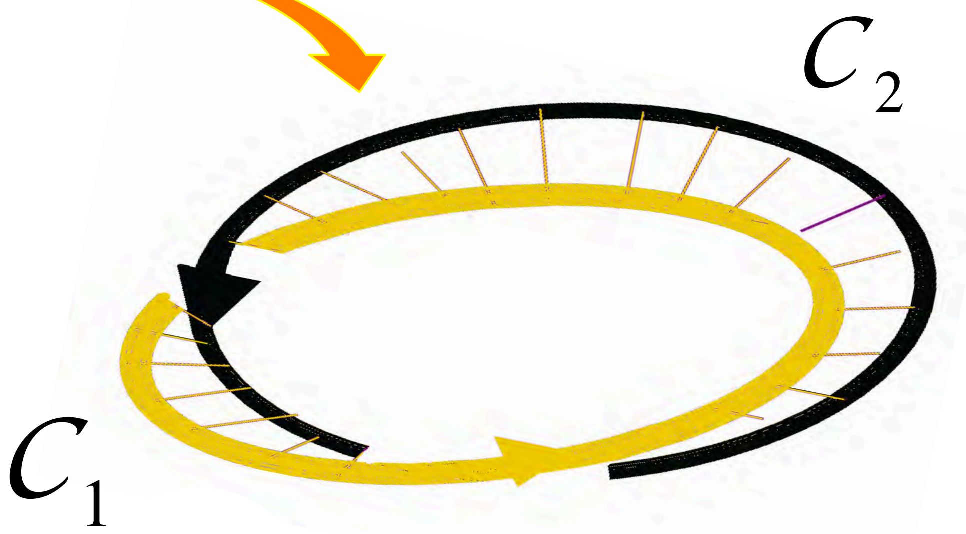


- ***Ribbon construction:***

$$\left.\begin{aligned} C_1 &\rightarrow C \\ C_2 &\rightarrow C^* \end{aligned}\right\} \mathcal{R}(C, C^*)$$

From Gauss linking number to self-linking number





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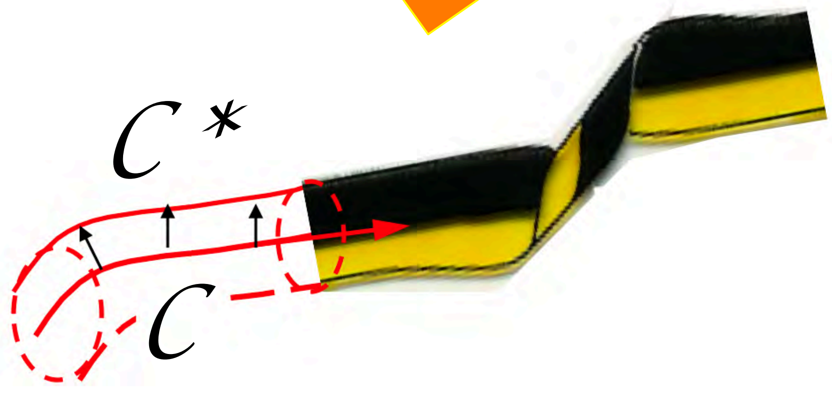


Self-linking number (Călugăreanu-White invariant)

- **Self-linking (number):**

$$SL = Wr + Tw$$

Self-linking number (Călugăreanu-White invariant)

- **Self-linking (number):**

$$SL = Wr + Tw$$

- **Writhe (writhing number):**

$$Wr = Wr(C)$$

Self-linking number (Călugăreanu-White invariant)

- ***Self-linking (number):***

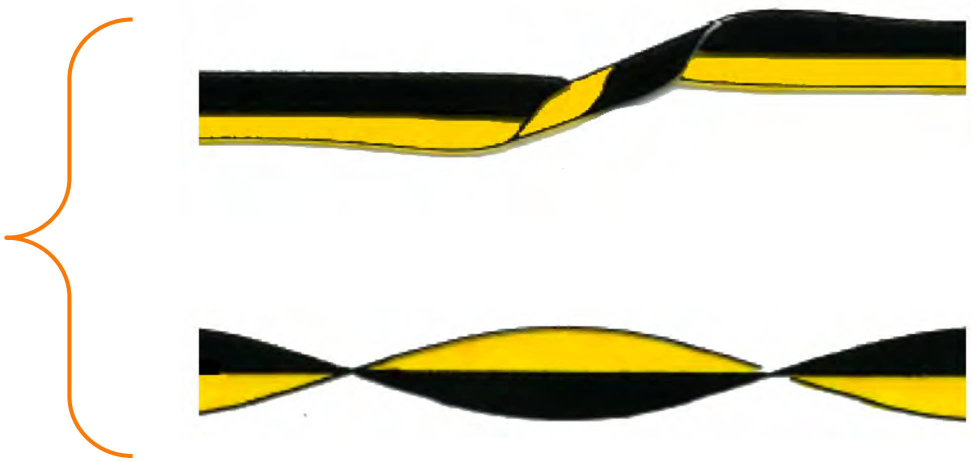
$$SL = Wr + Tw$$

- ***Writhe (writhing number):***

$$Wr = Wr(\mathcal{C})$$

- ***Twist (total twist number):***

$$Tw = Tw(\mathcal{R})$$



Properties of SL, Wr, and Tw

Properties of SL ***,*** Wr ***, and*** Tw

- ***Linking number*** $SL = SL(\mathcal{R})$ ***:***

    (i) $SL(\mathcal{R})$ ***is a topological invariant of the ribbon;***

    (ii) it is an integer;

    (iii) under cross-switching ($\mp \longrightarrow \pm$)***:*** $\Delta SL = \pm 2$ ***.***

Properties of SL , Wr , and Tw

- **Linking number** $SL = SL(\mathcal{R})$ **:**

  (i) $SL(\mathcal{R})$ ***is a topological invariant of the ribbon;***

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- **Writhing number** $Wr = Wr(C)$ **:**

  (i) $Wr(C)$ ***is a geometric measure of the curve*** C ***;***

  (ii) it is a conformational invariant;

  (iii) under cross-switching ($\mp \longrightarrow \pm$)**:** $\Delta SL = \pm 2$ **.**

Properties of SL ***,*** Wr ***, and*** Tw

- ***Linking number*** $SL = SL(\mathcal{R})$ ***:***
  - ***(i)*** $SL(\mathcal{R})$ ***is a topological invariant of the ribbon;***
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- ***Writhing number*** $Wr = Wr(\mathcal{C})$ ***:***
  - ***(i)*** $Wr(\mathcal{C})$ ***is a geometric measure of the curve*** $\mathcal{C}$ ***;***
  - ***(ii) it is a conformational invariant;***
  - ***(iii) under cross-switching*** $(\mp \rightarrow \pm)$***:*** $\Delta SL = \pm 2$ ***.***

- ***Total twist number*** $Tw = Tw(\mathcal{R})$ ***:***
  - ***(i)*** $Tw(\mathcal{R})$ ***is a geometric measure of the ribbon*** $\mathcal{R}(\mathcal{C}, \mathcal{C}^*)$ ***;***
  - ***(ii) it is a conformational invarant;***
  - ***(iii) it is additive:*** $Tw(\mathcal{A}) + Tw(\mathcal{B}) = Tw(\mathcal{A} + \mathcal{B})$ ***.***

Zero-framing

$$SL = Wr + Tw = 0 \quad \Leftrightarrow \quad Wr = -Tw$$

Zero-framing

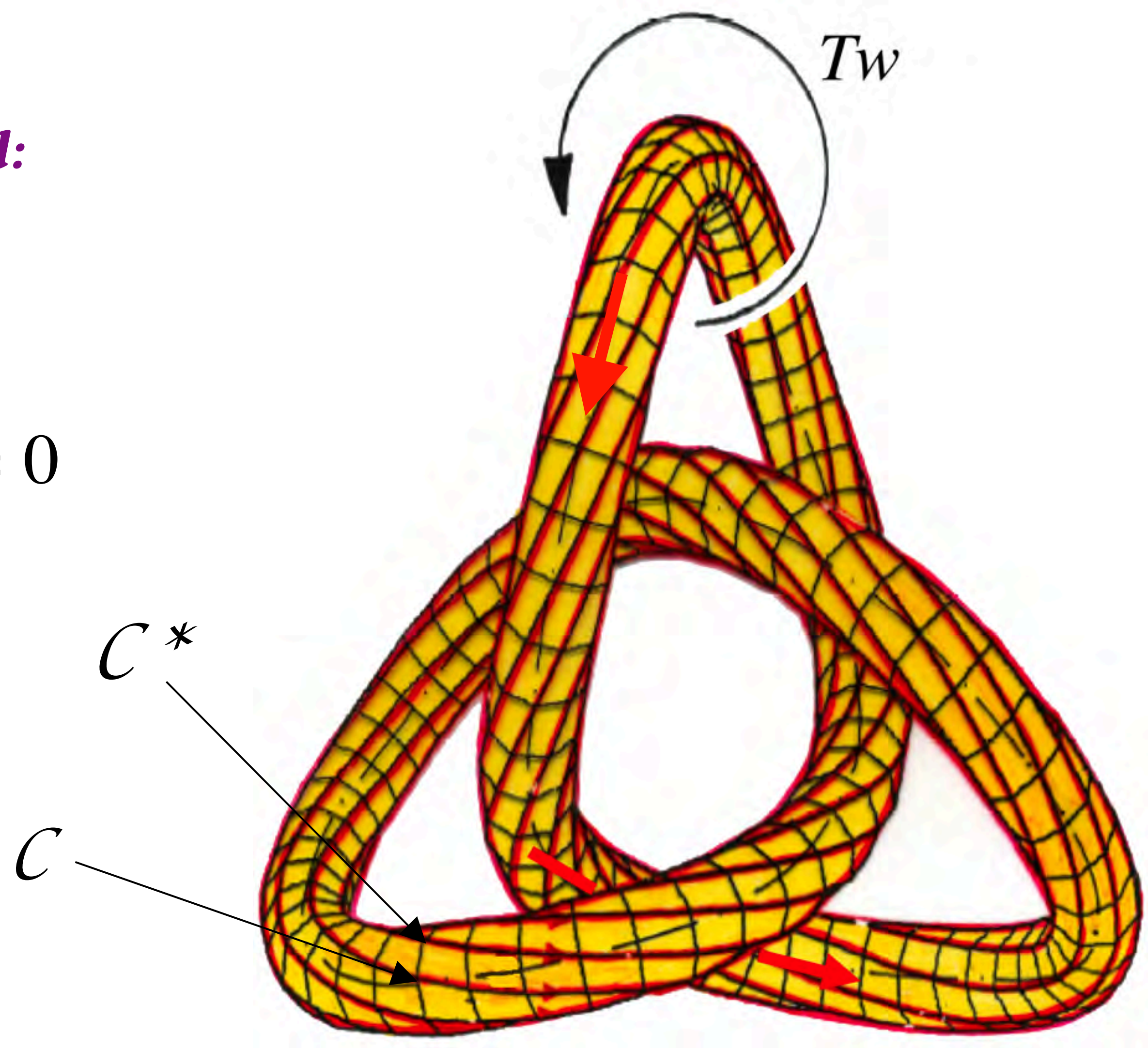
$$SL = Wr + Tw = 0 \quad \Leftrightarrow \quad Wr = -Tw$$

- ***Zero-framed trefoil:***

  $Wr = -3$

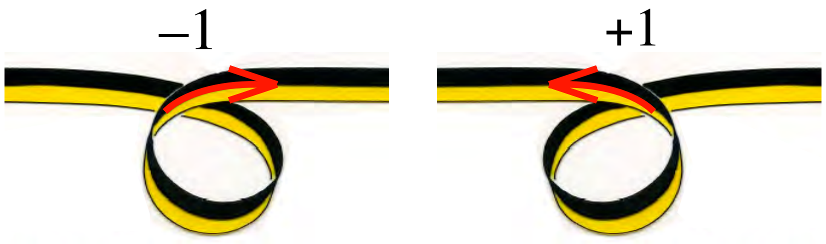
  $Tw = +3$

  $SL = Wr + Tw = 0$

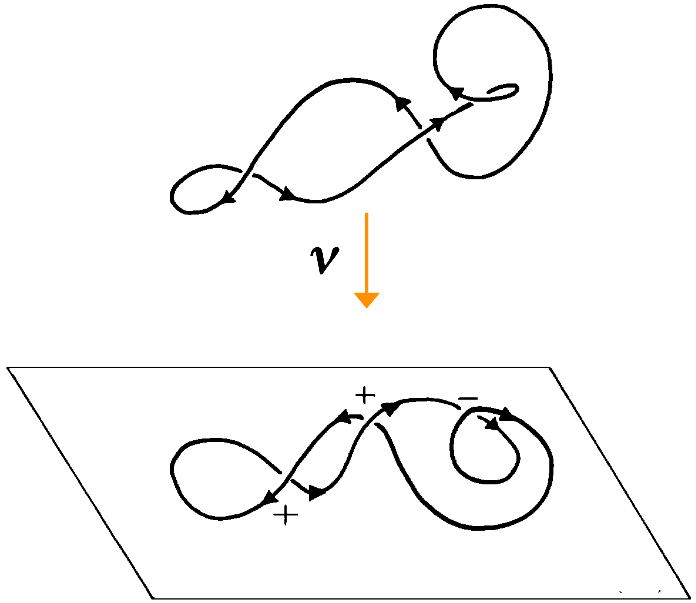


Interpretation of writhe in terms of signed crossings

Crossing sign convention again:

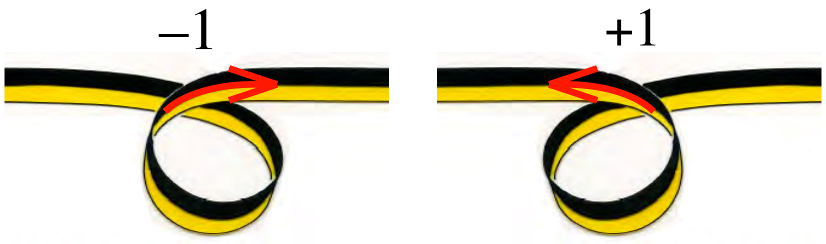


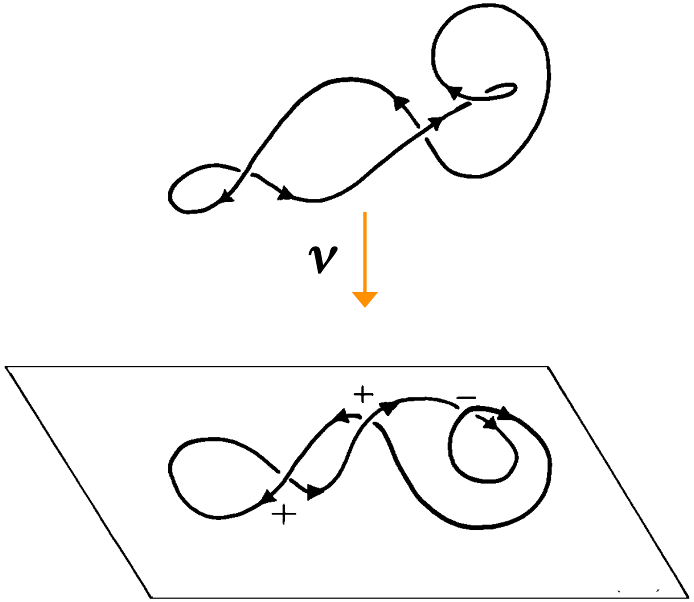
$$Wr = \left\langle \sum_r \varepsilon_r \right\rangle$$



Interpretation of writhe in terms of signed crossings

Crossing sign convention again:





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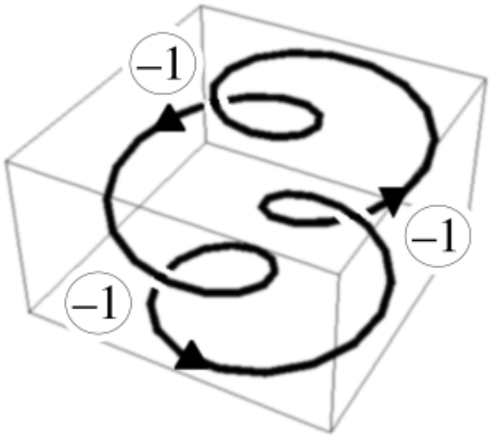
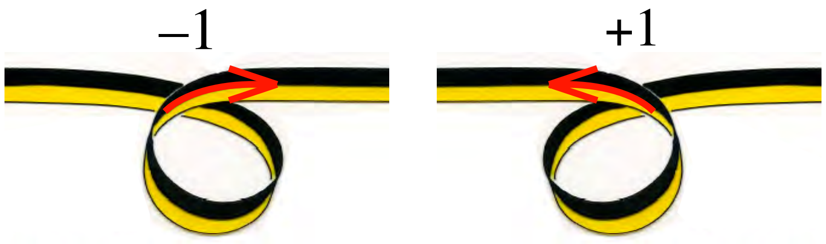
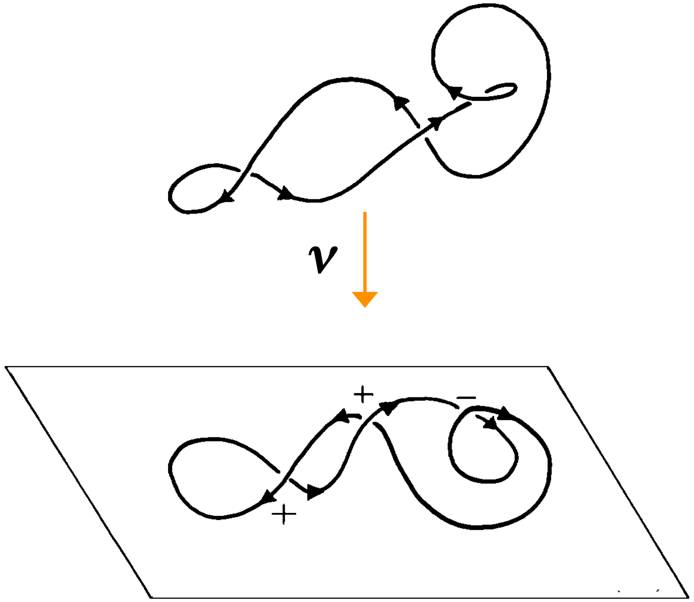


$Wr = -3$

Interpretation of writhe in terms of signed crossings

Crossing sign convention again:





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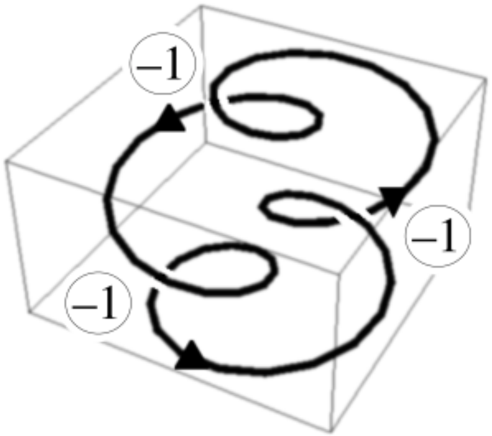
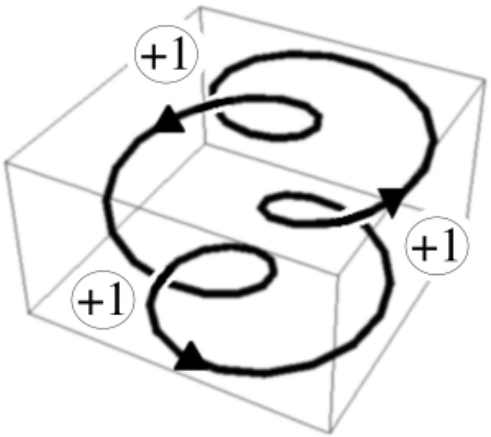


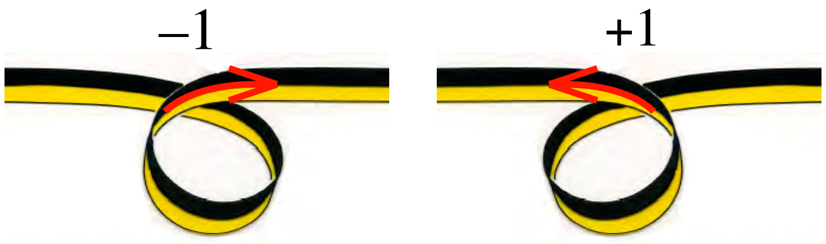
$Wr = -3$

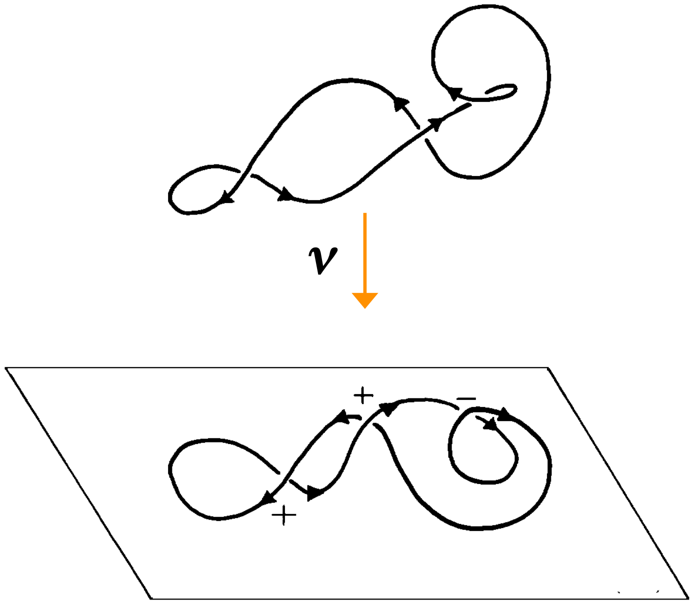


$Wr = +3$

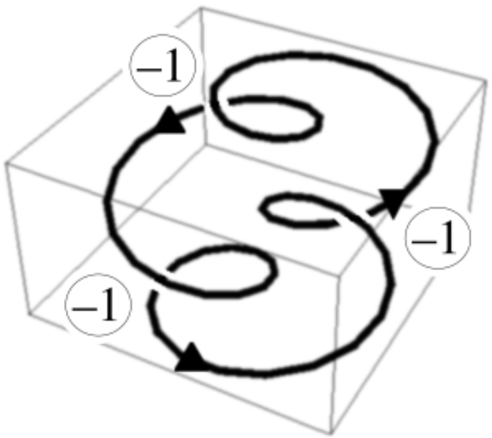
Interpretation of writhe in terms of signed crossings

Crossing sign convention again:

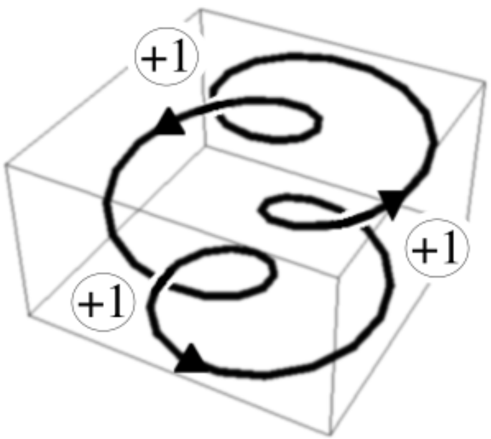




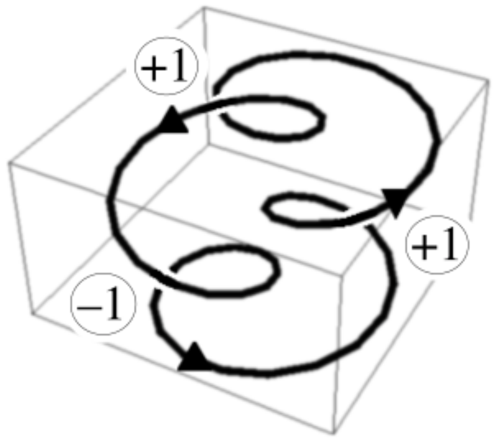
$$Wr = \left\langle \sum_r \varepsilon_r \right\rangle$$



$Wr = -3$

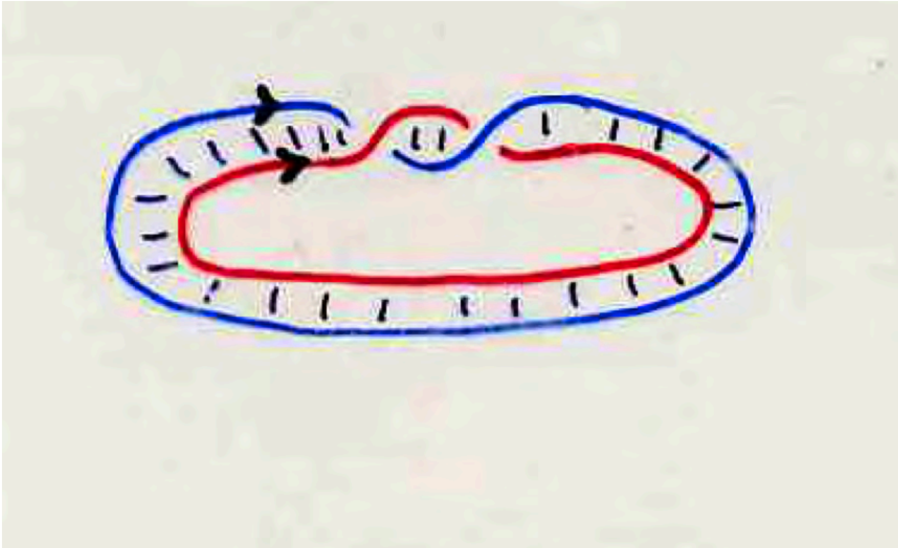


$Wr = +3$



$Wr = +1$

A simple example

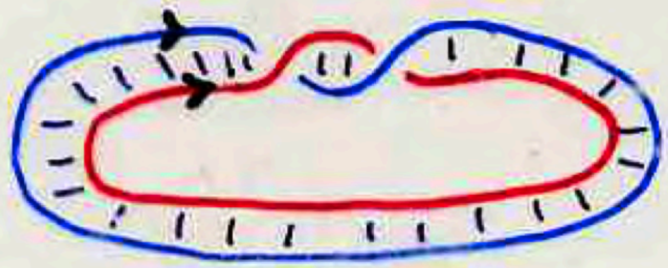


$$Wr = -0$$

$$Tw = -1$$

$$SL = -1$$

A simple example



$Wr = -0$

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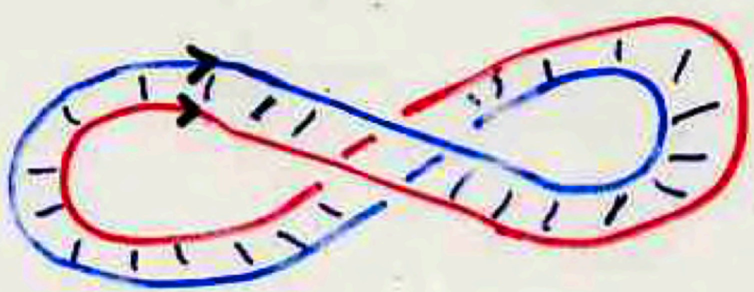
$SL = -1$



$Wr = -1$

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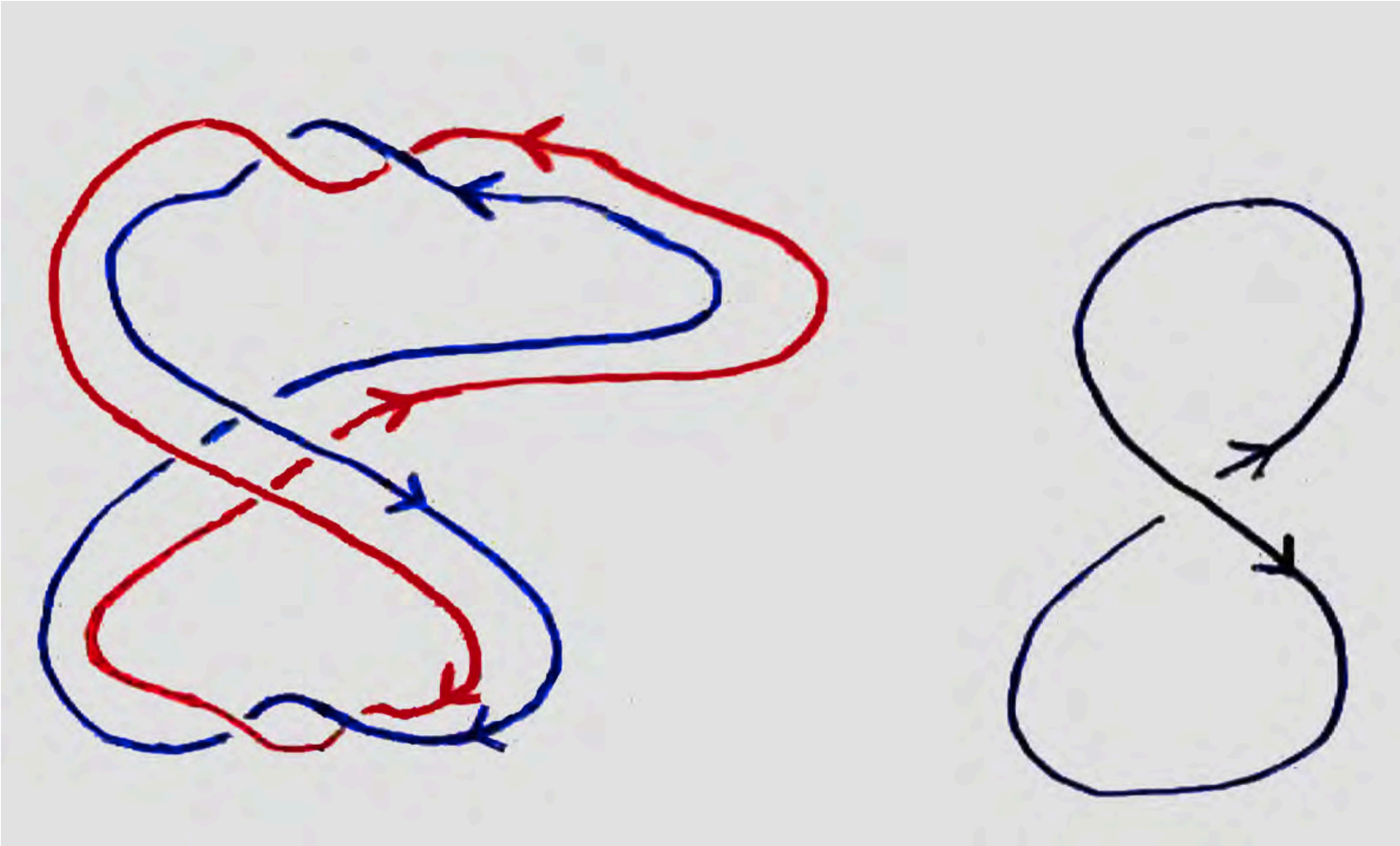
$SL = -1$

Let's do some homework



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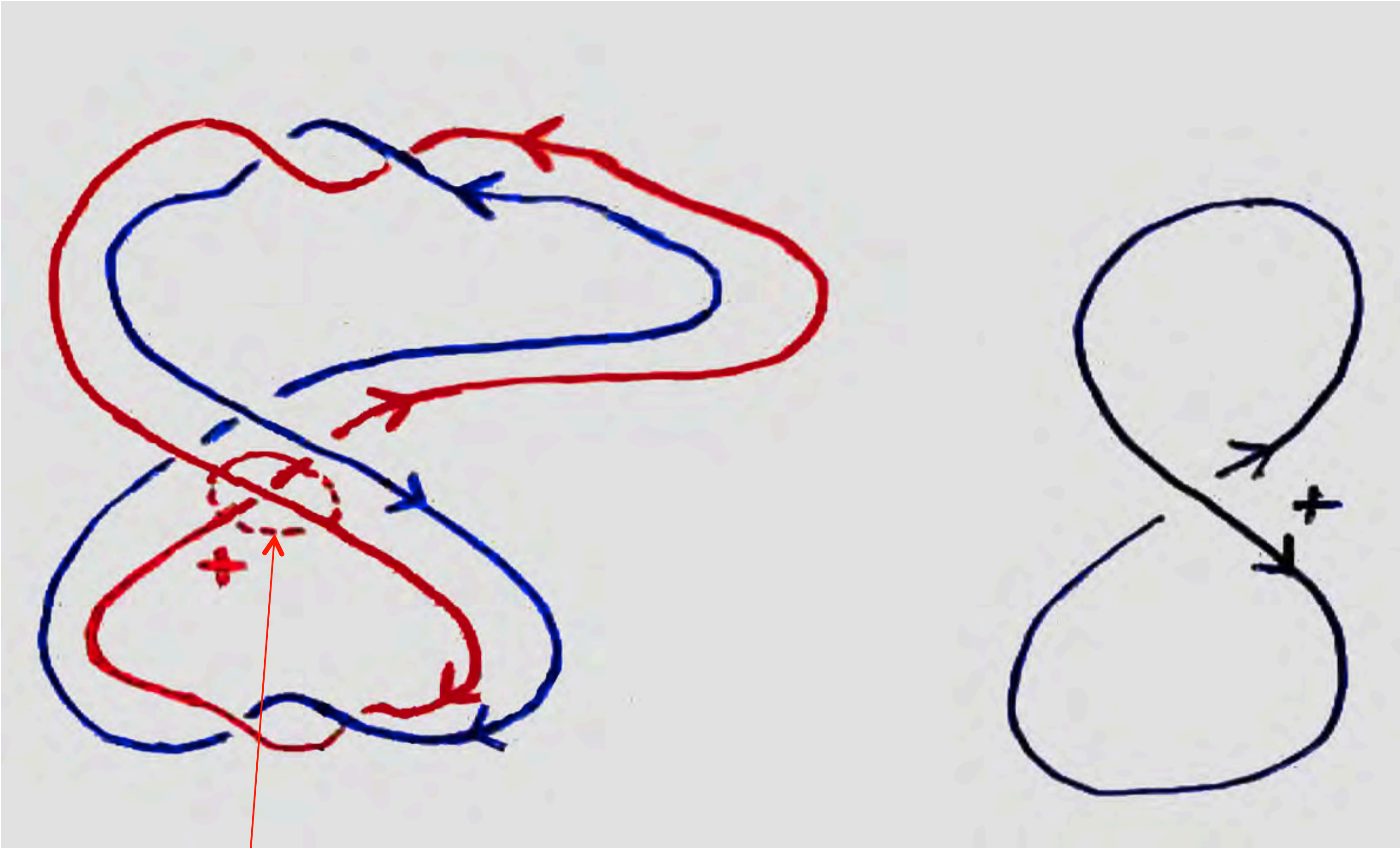

$Wr = +1$

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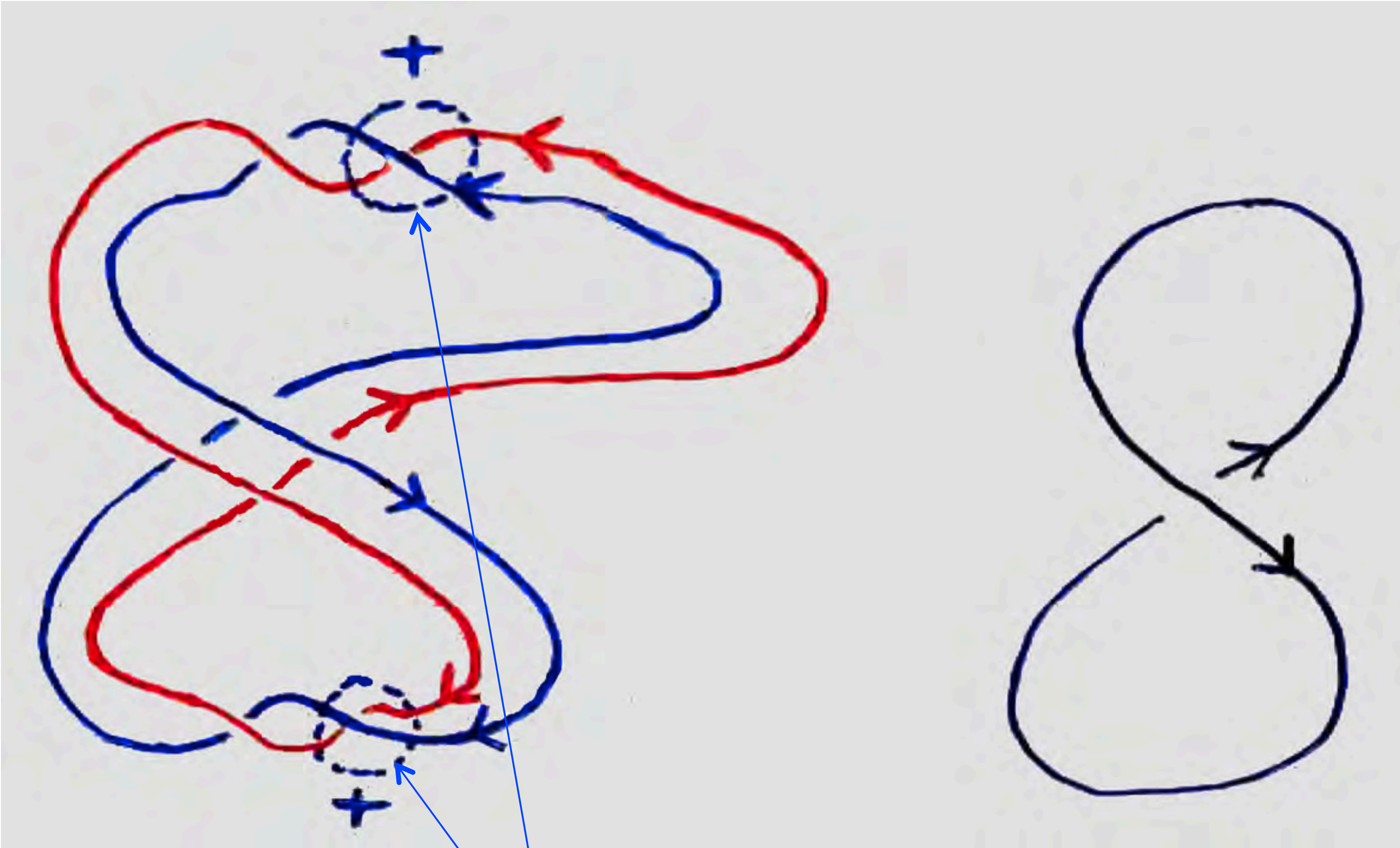
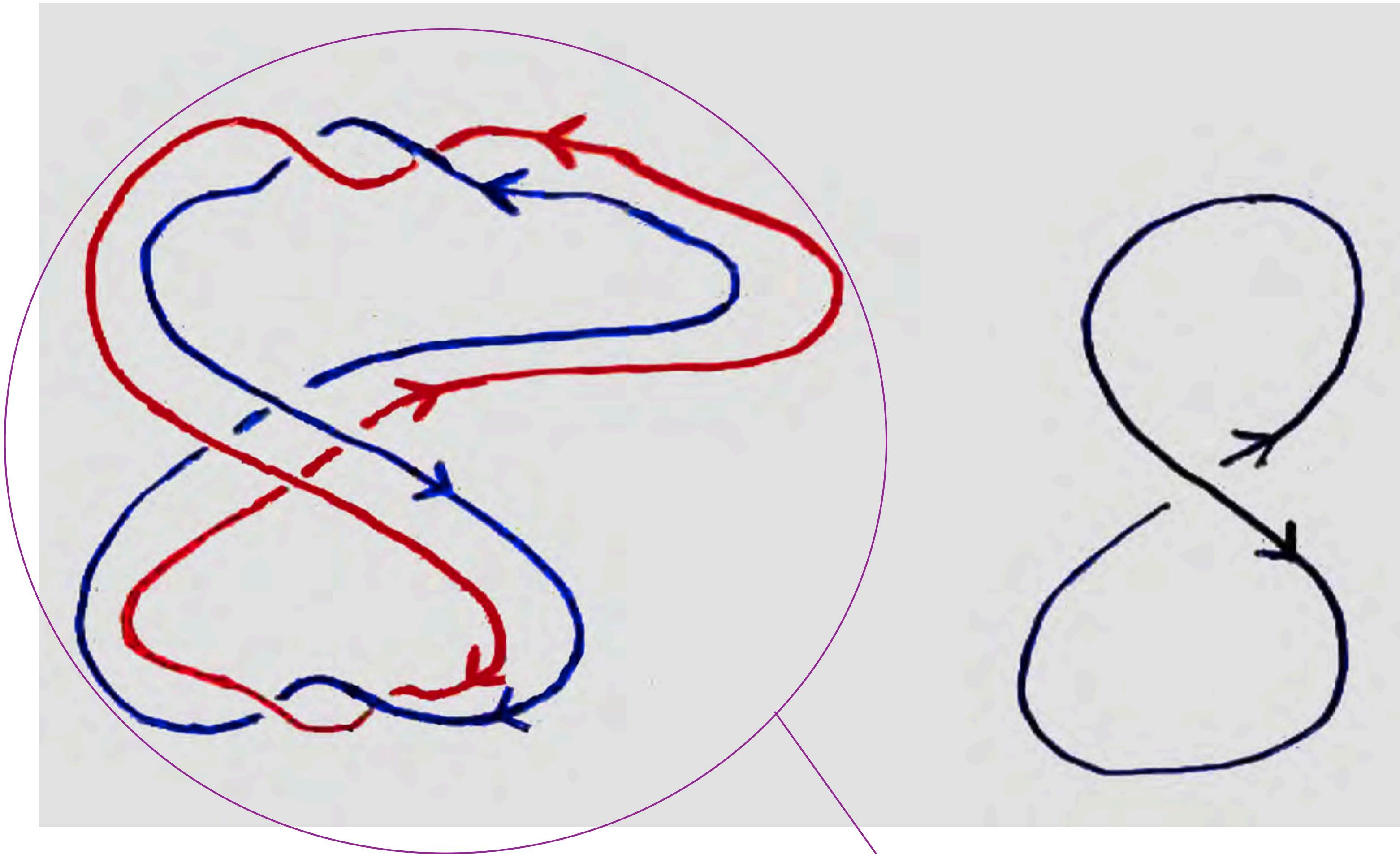


$Wr = +1$, $\mathrm{Tw} = +2$

Let's do some homework

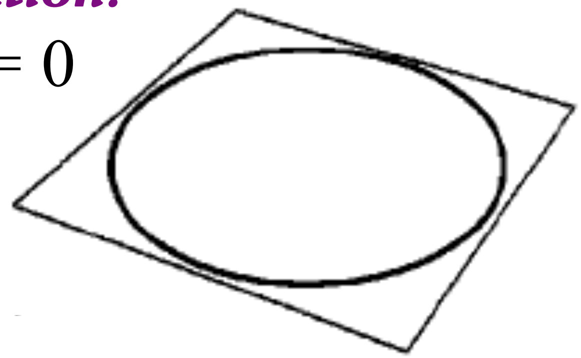


$Wr = +1$, $\mathrm{Tw} = +2 \Rightarrow SL = +3$

First remark: compactification in space

initial configuration:

$SL = Tw,\ Wr = 0$



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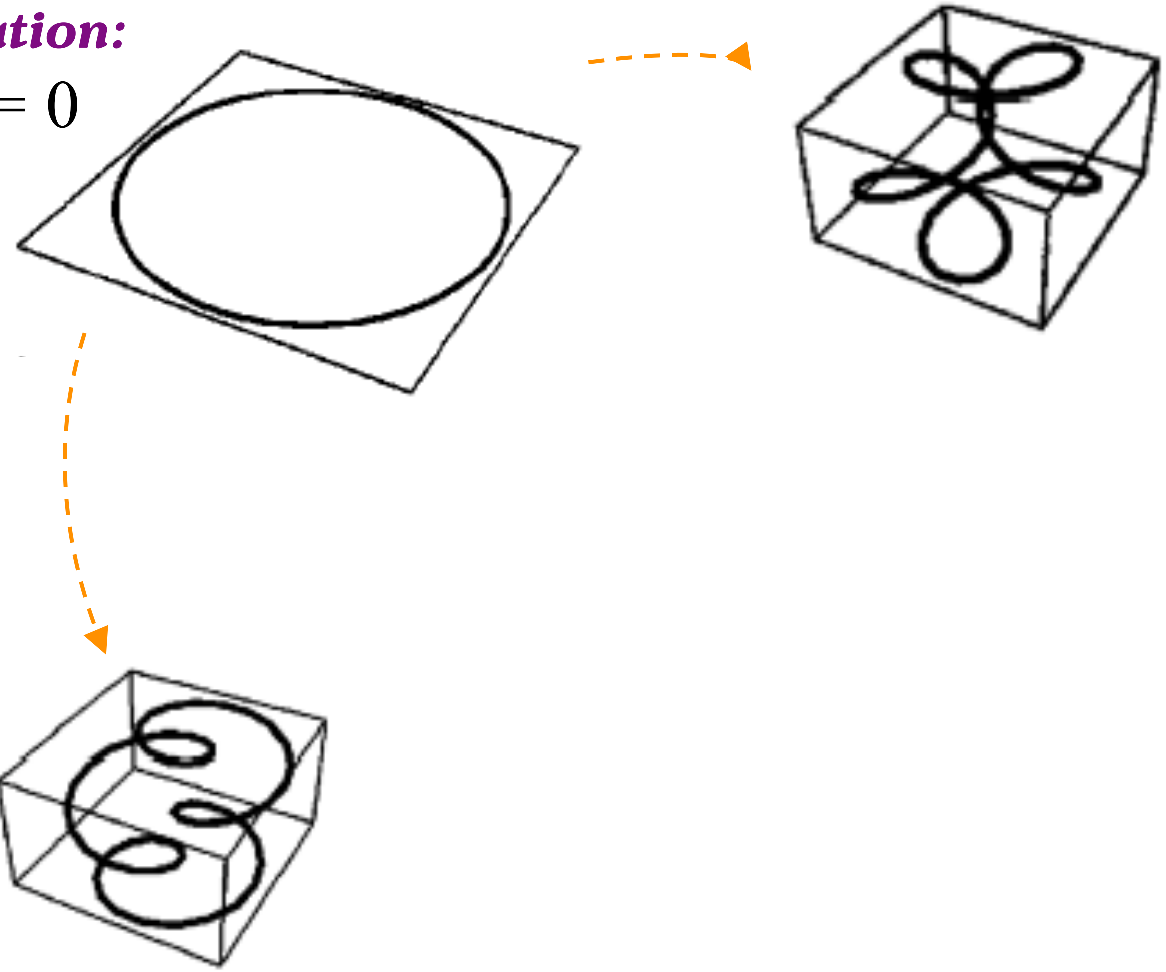

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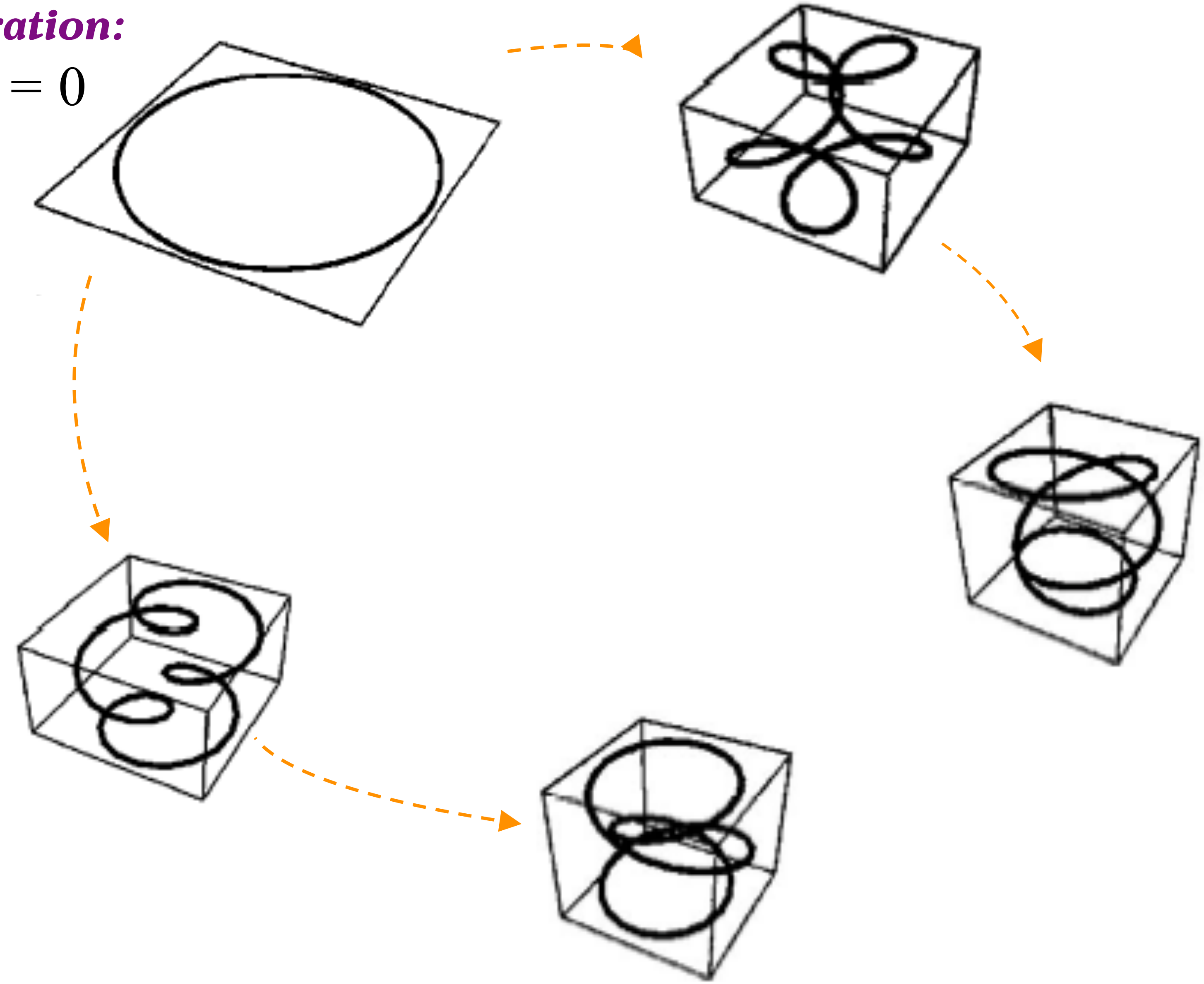

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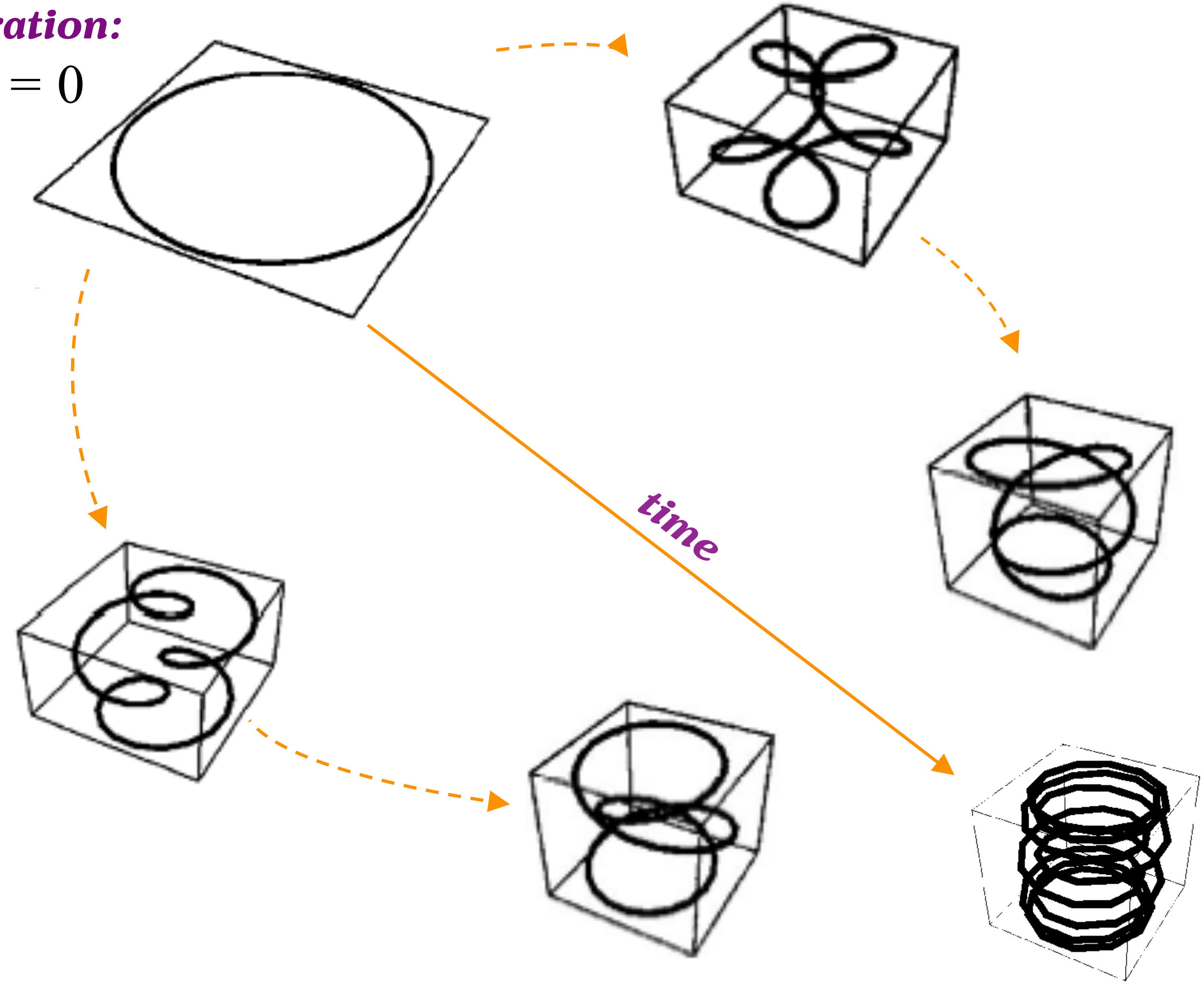

final configuration

First remark: compactification in space

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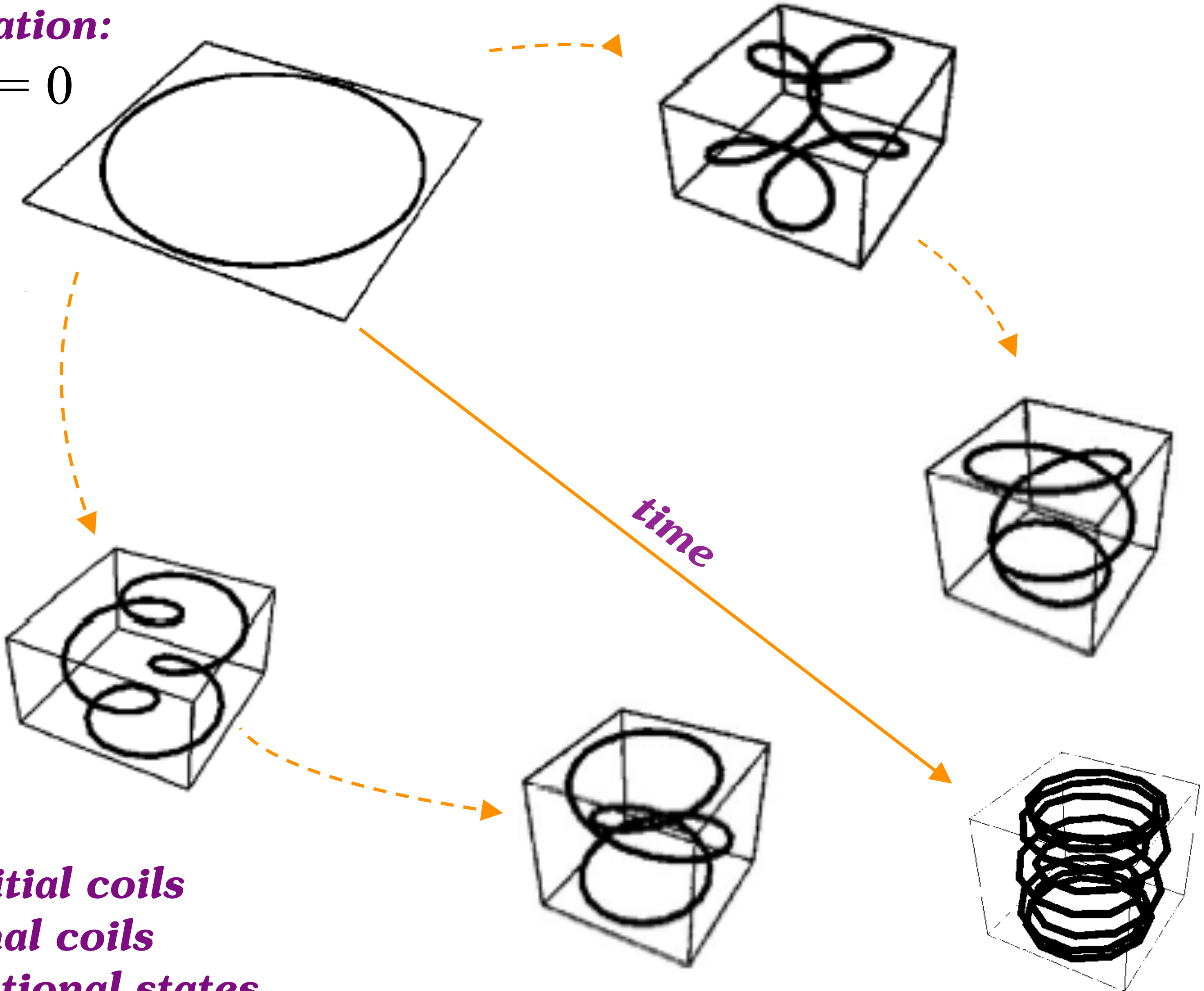
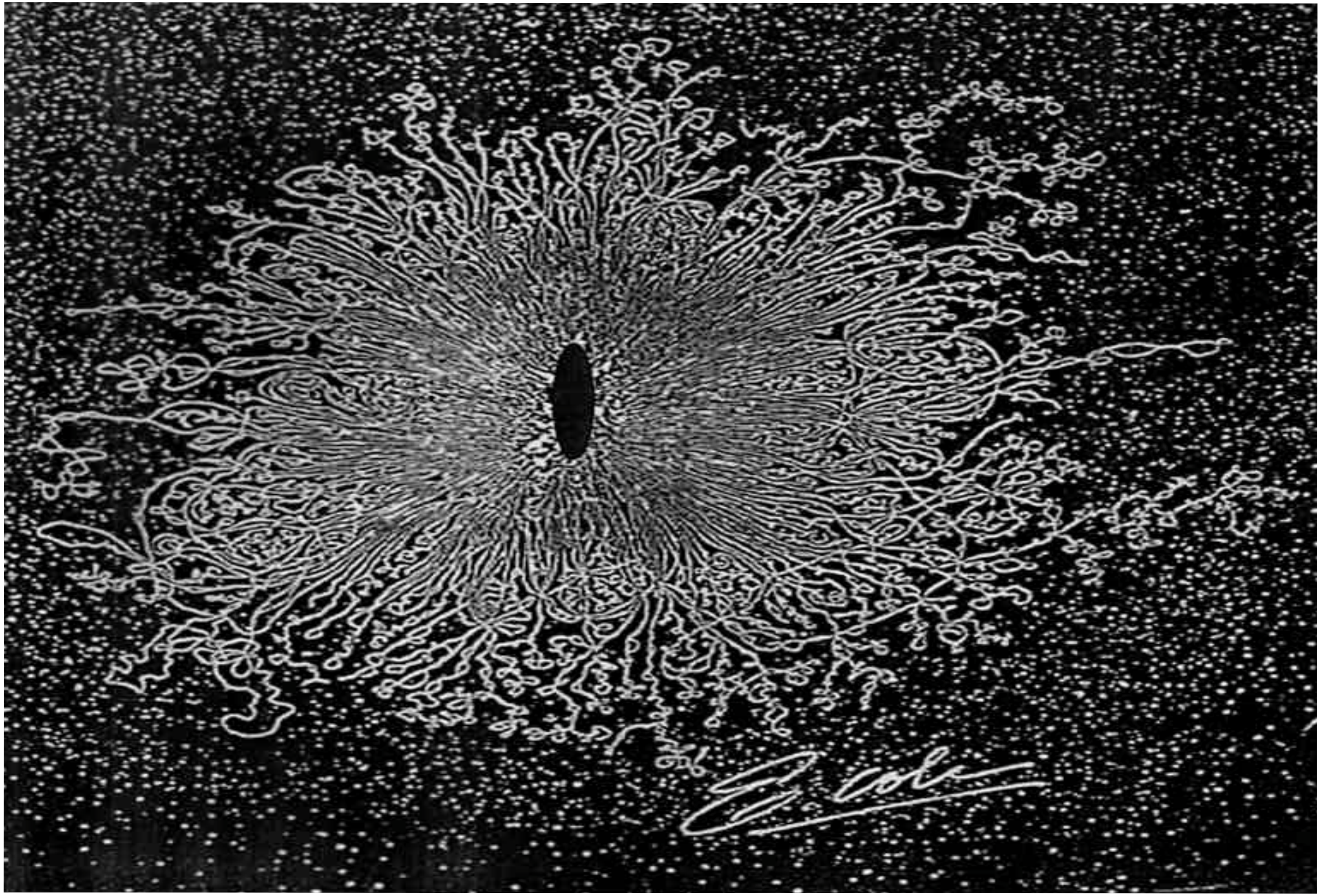


final configuration

- ***number of initial coils***
- ***number of final coils***
- ***type of transitional states***
- ***writhing rates and energetics***

DNA is crowded in the cell



Second remark: relaxation of elastic energy

- ***Supercoiling as transition from*** Tw ***to*** Wr***, under conservation of*** SL ***:***

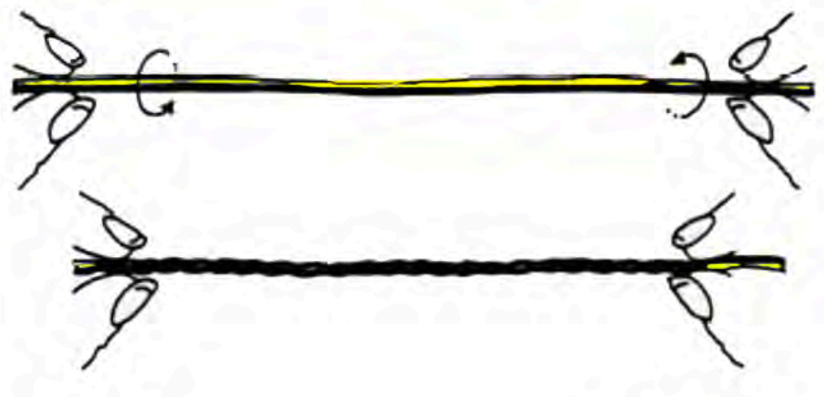
Second remark: relaxation of elastic energy

- *Supercoiling as transition from* Tw *to* Wr*, under conservation of* SL *:*

  (i) keep straight and twist,

Second remark: relaxation of elastic energy

- *Supercoiling as transition from* Tw *to* Wr*, under conservation of* SL *:*

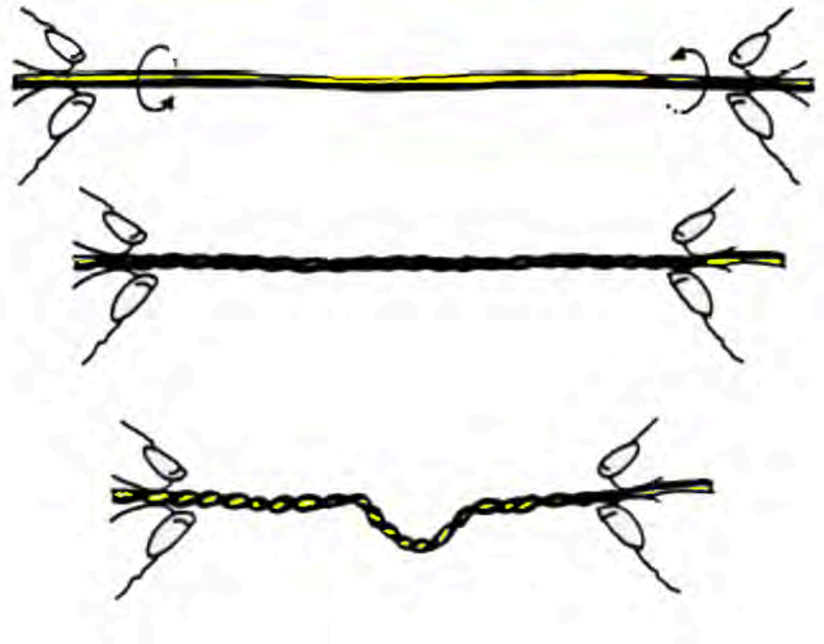


(i) keep straight and twist,

(ii) pull tight,

Second remark: relaxation of elastic energy

- Supercoiling as transition from Tw to Wr, under conservation of SL :



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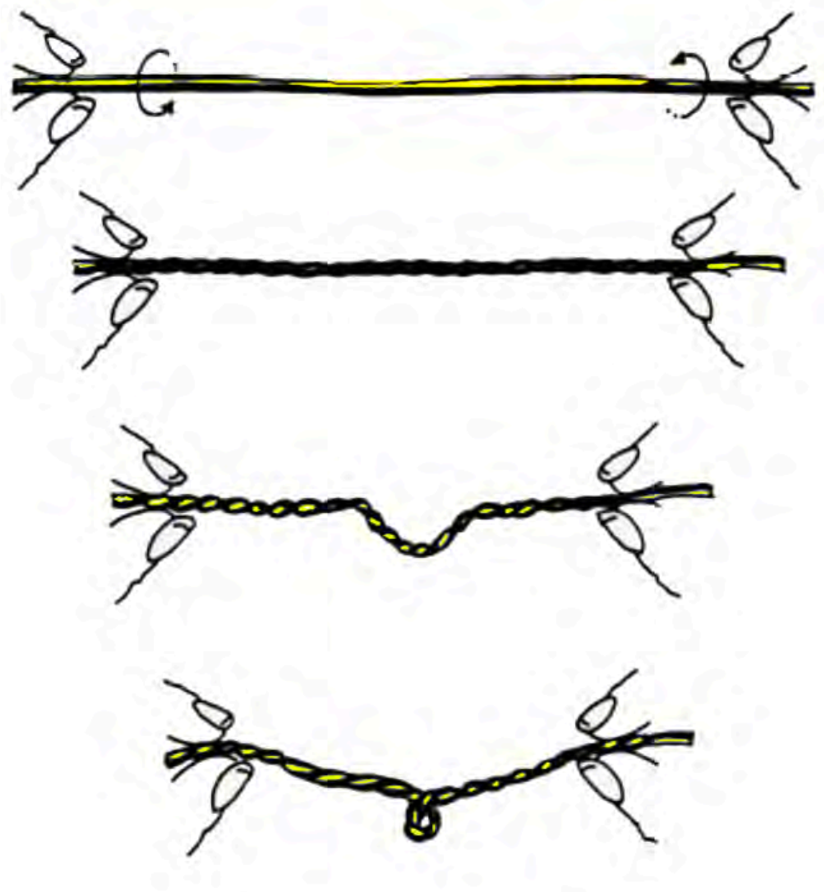

(i) keep straight and twist,

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(iv) … and schrink,

Second remark: relaxation of elastic energy

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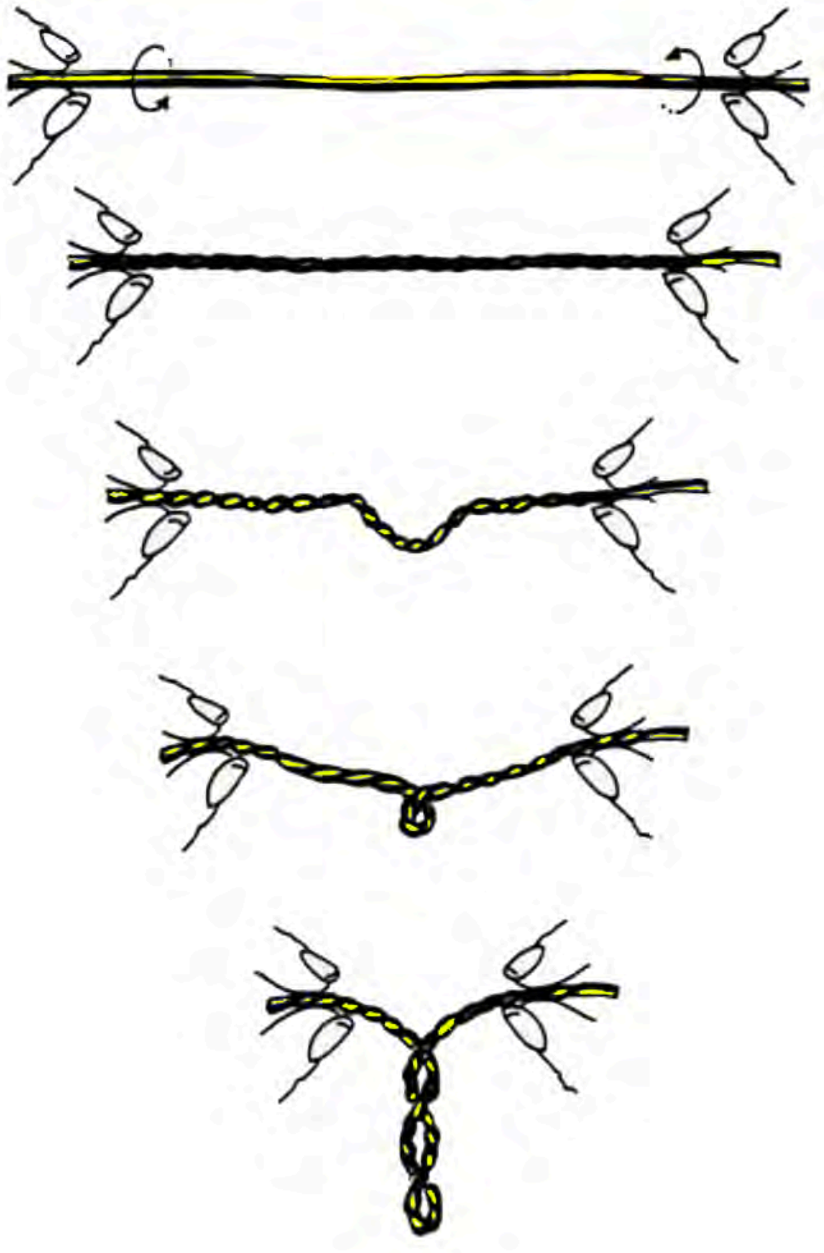

(i) **keep straight and twist,**

(ii) **pull tight,**

(iii) **let schrink,**

(iv) **… and schrink,**

(v) **… and schrink again!**

Second remark: relaxation of elastic energy

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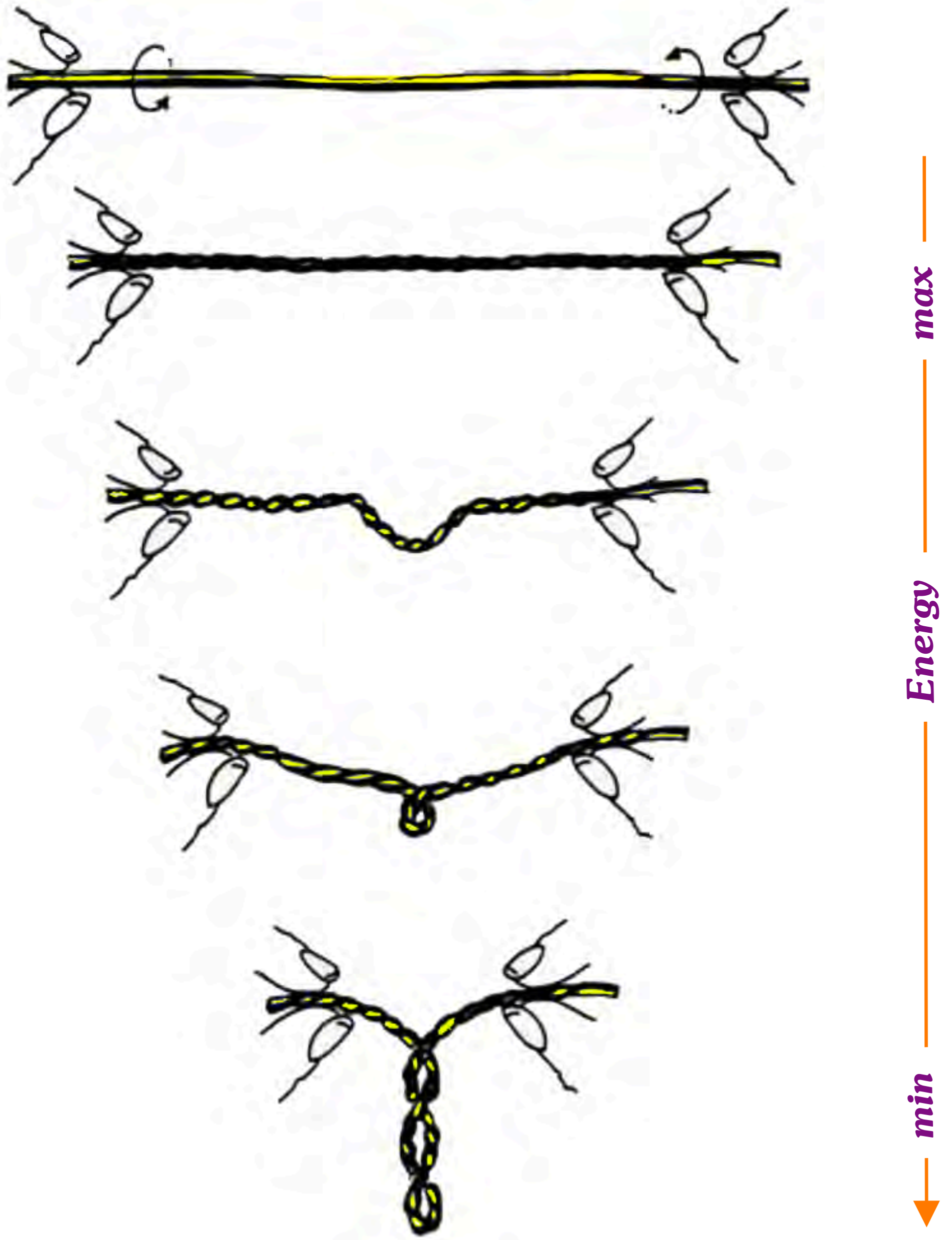
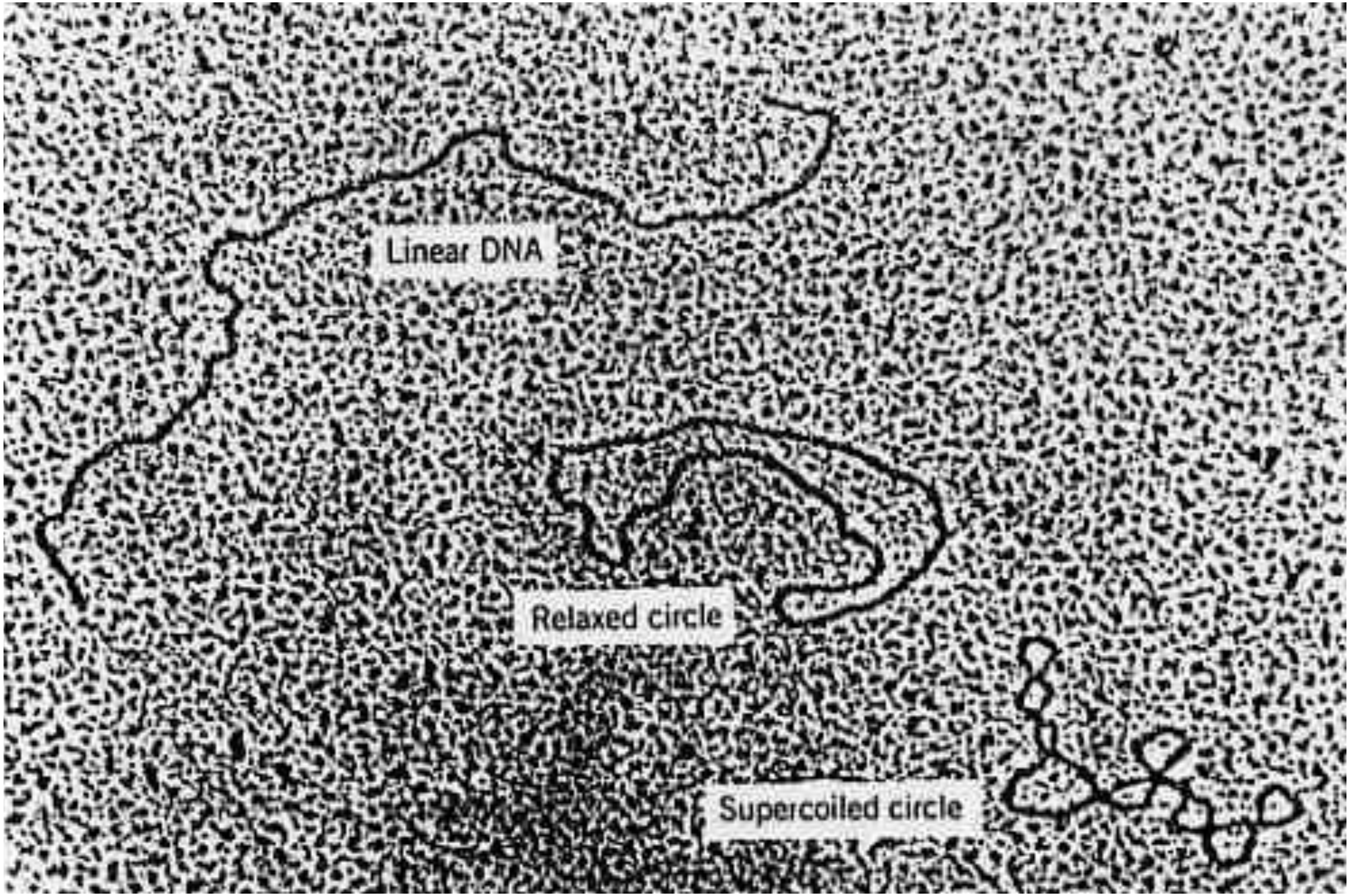
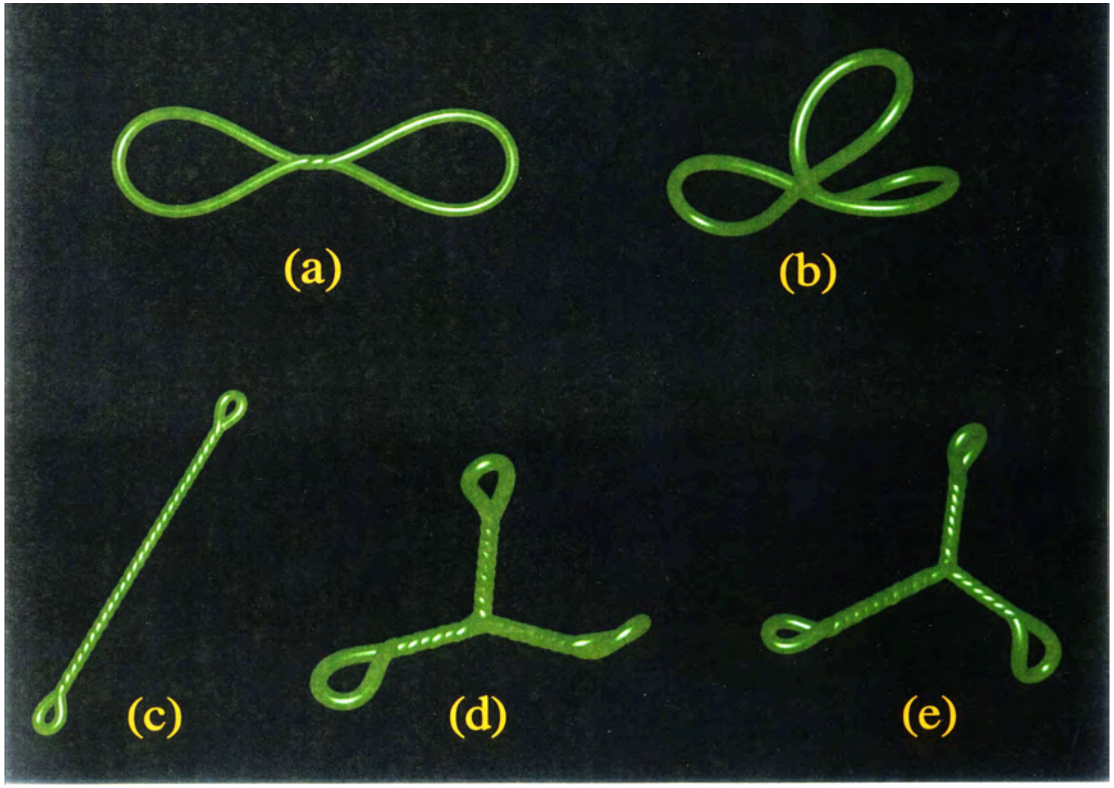


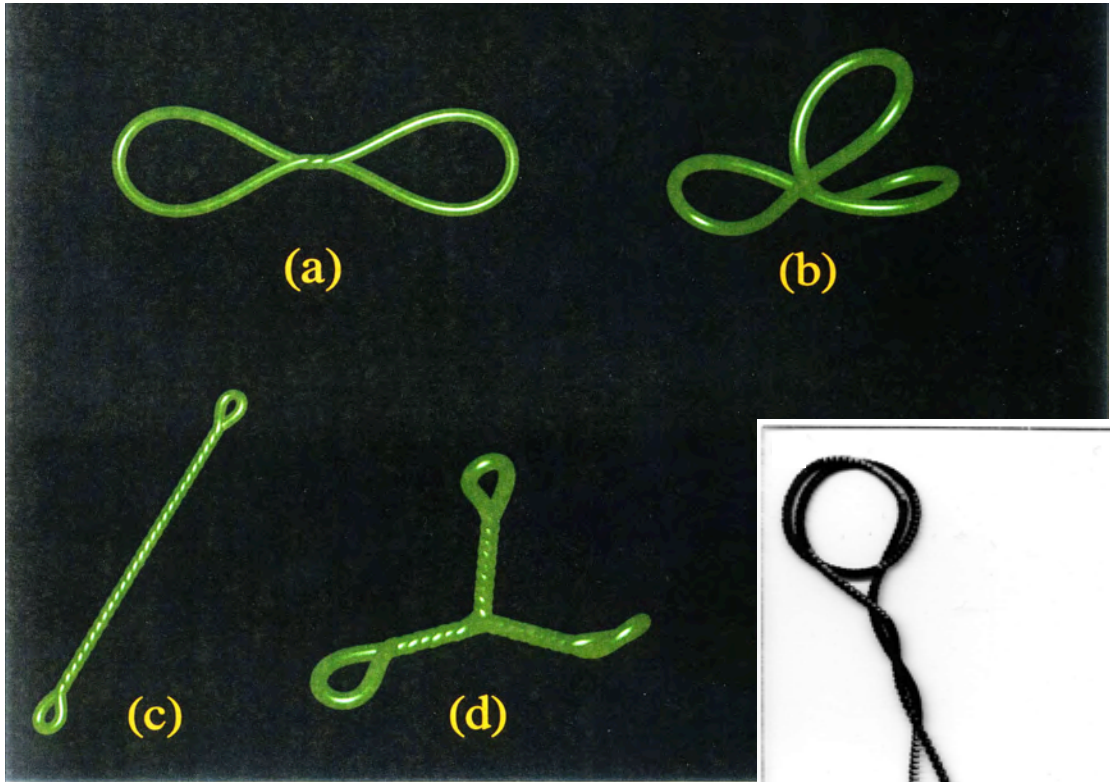
Linear, relaxed and supercoiled DNA

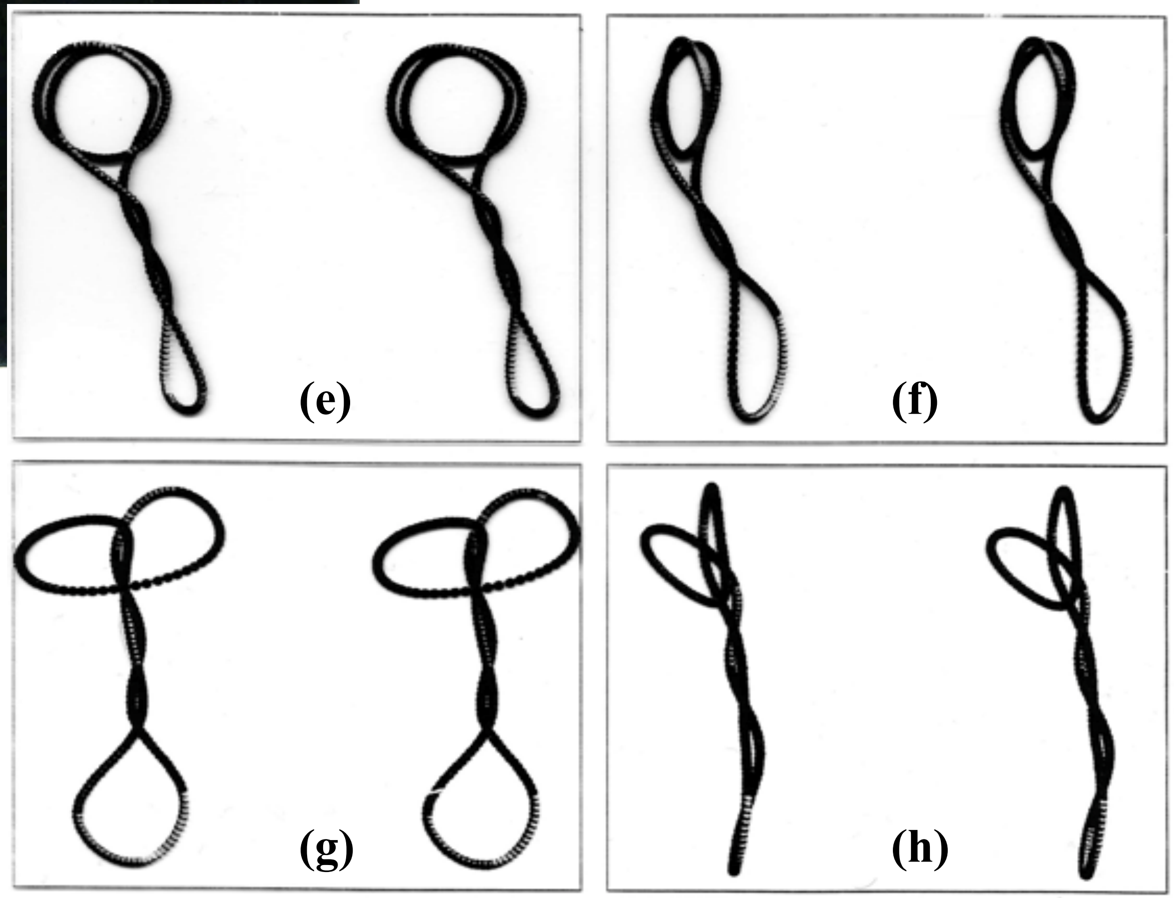


Applications in fiber technology

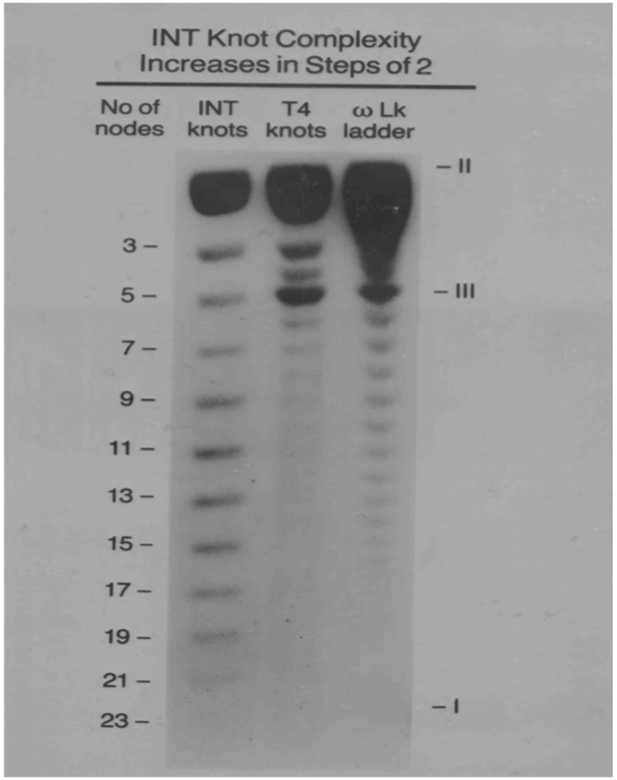


Applications in fiber technology



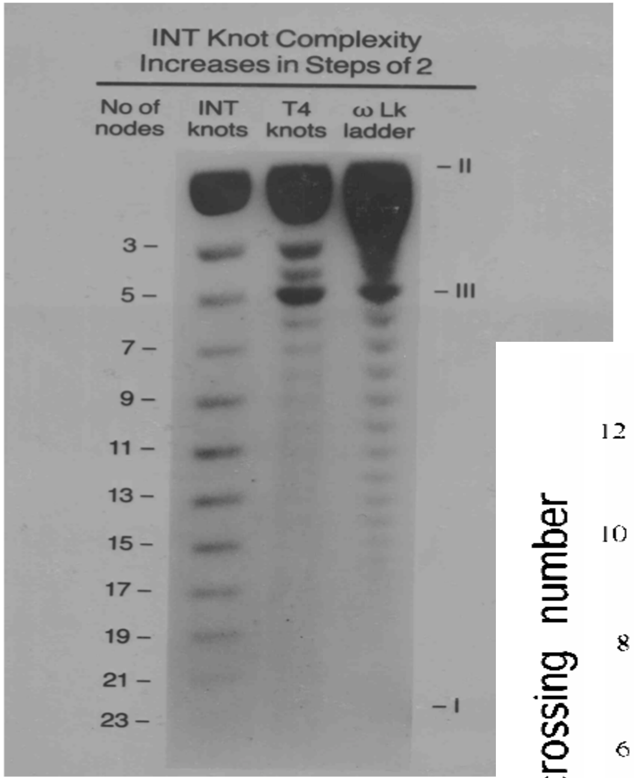


In search of DNA knots



Gel electrophoresis

In search of DNA knots



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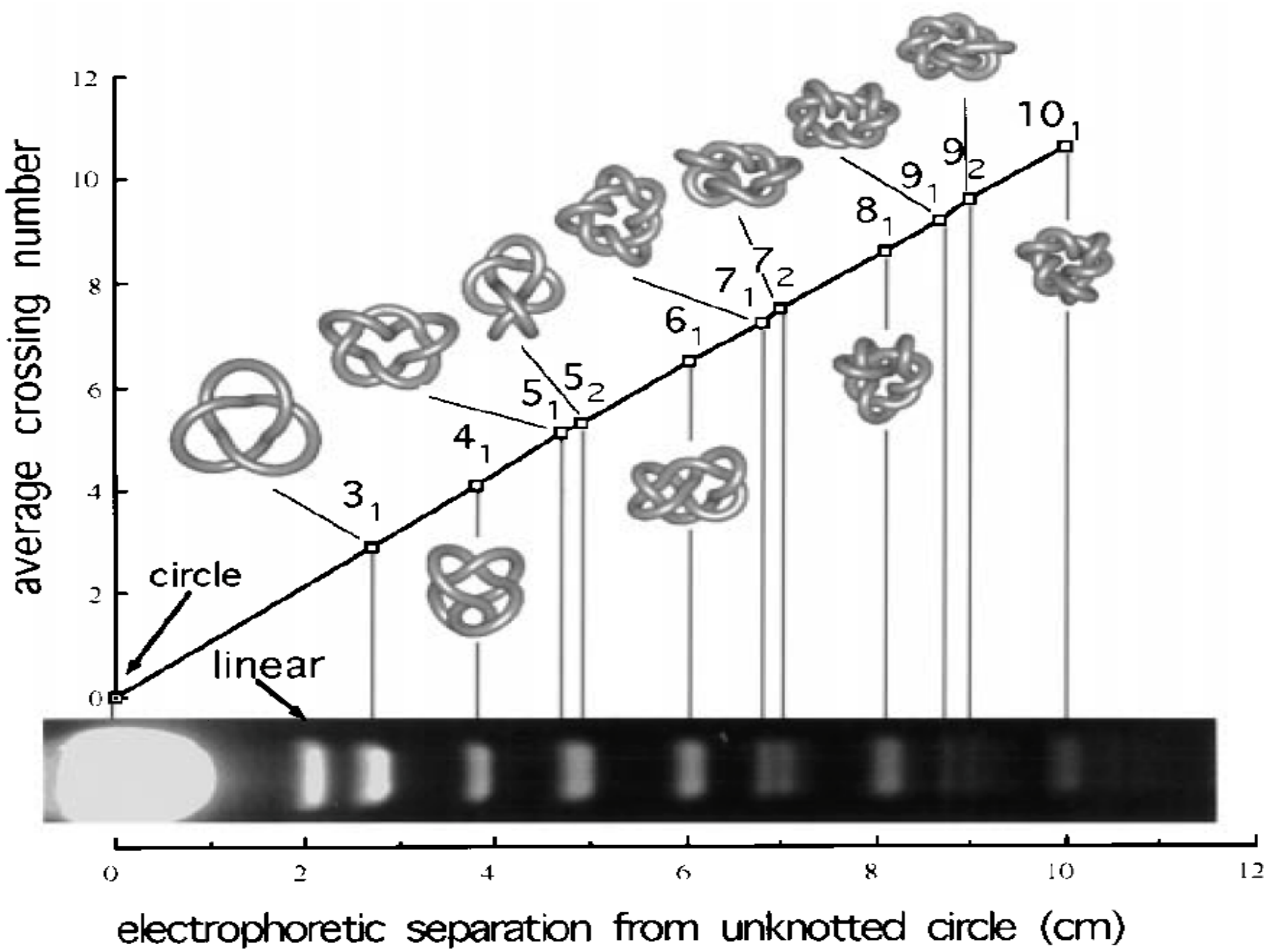

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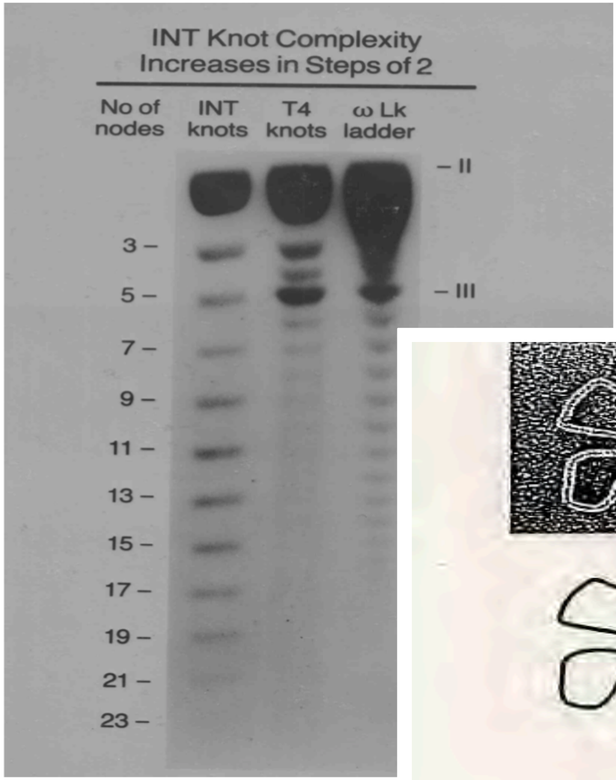

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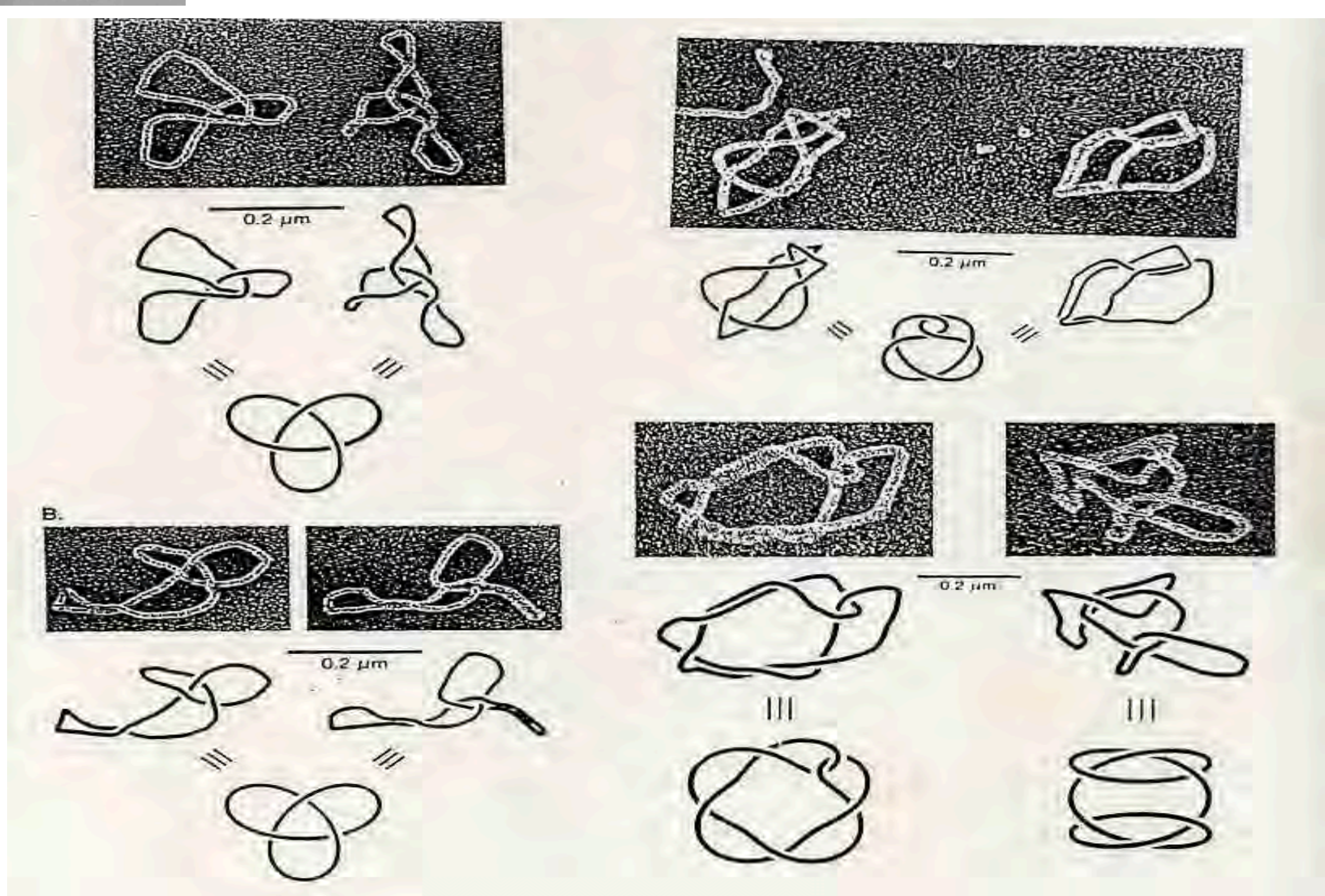

From knots to braids

- Bridge index (invariant) $b(K)$: minimal number of bridges ("maxima") required in all the possible bridge representation of the knot.

From knots to braids

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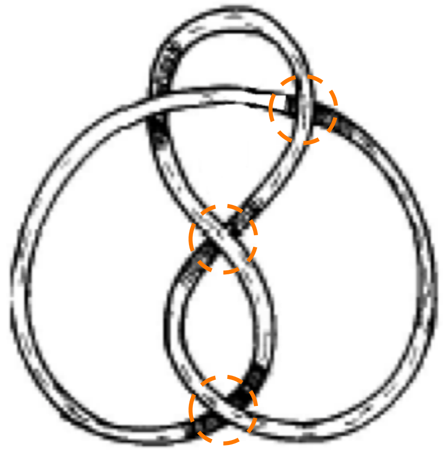
$b(F^8) = 3$



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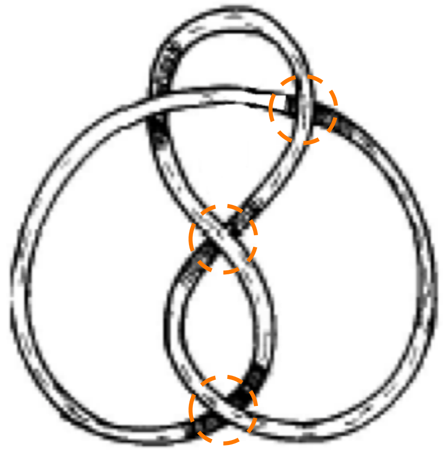

- ***Figure-of-8 in braid presentation:***

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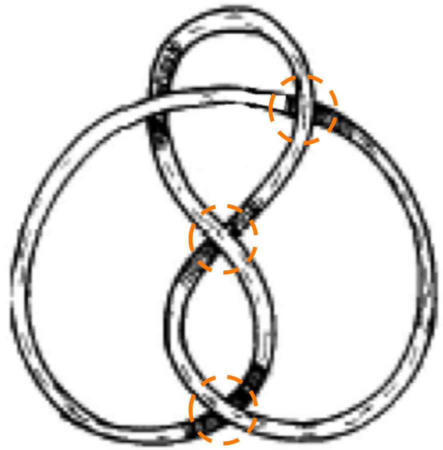

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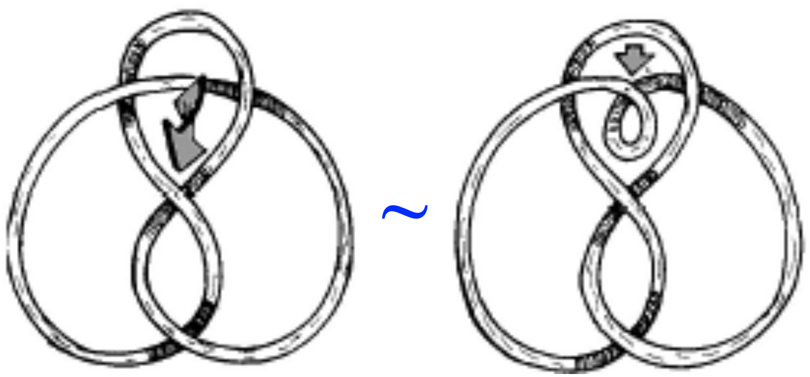

From knots to braids

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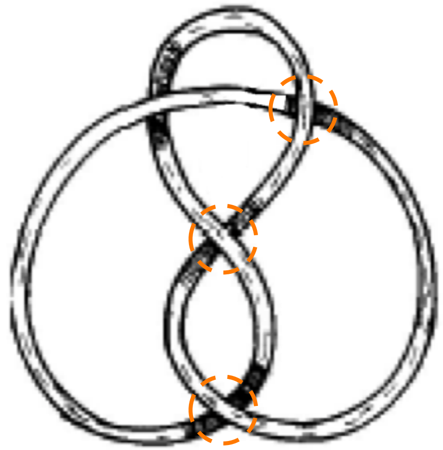

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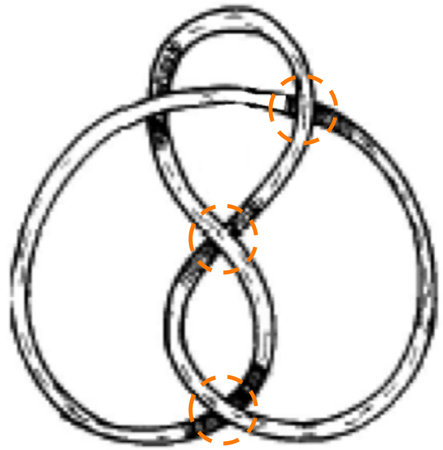

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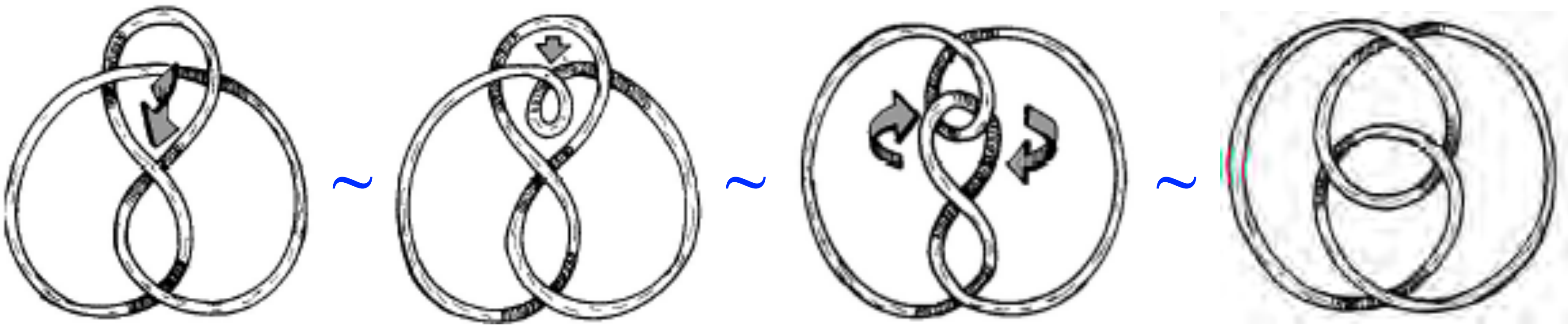

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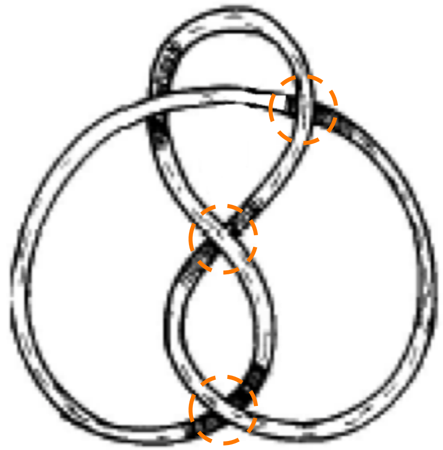

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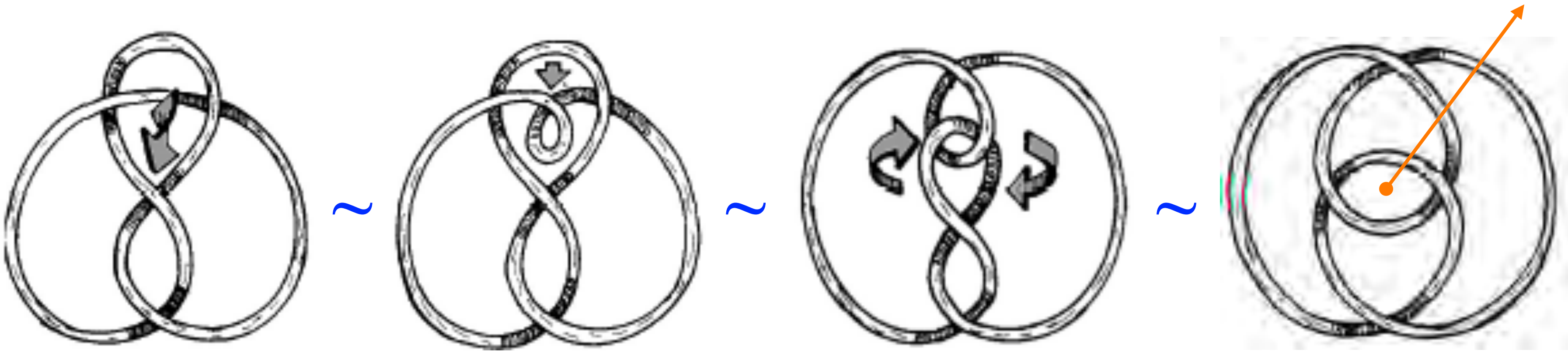

- ***Braid index (invariant)*** $\beta(K)$***: minimal number of strings in braid form.***

From knots to braids

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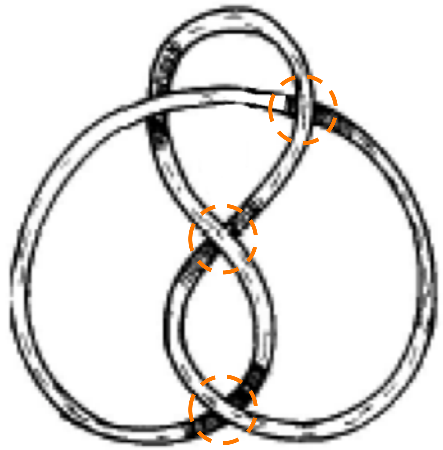

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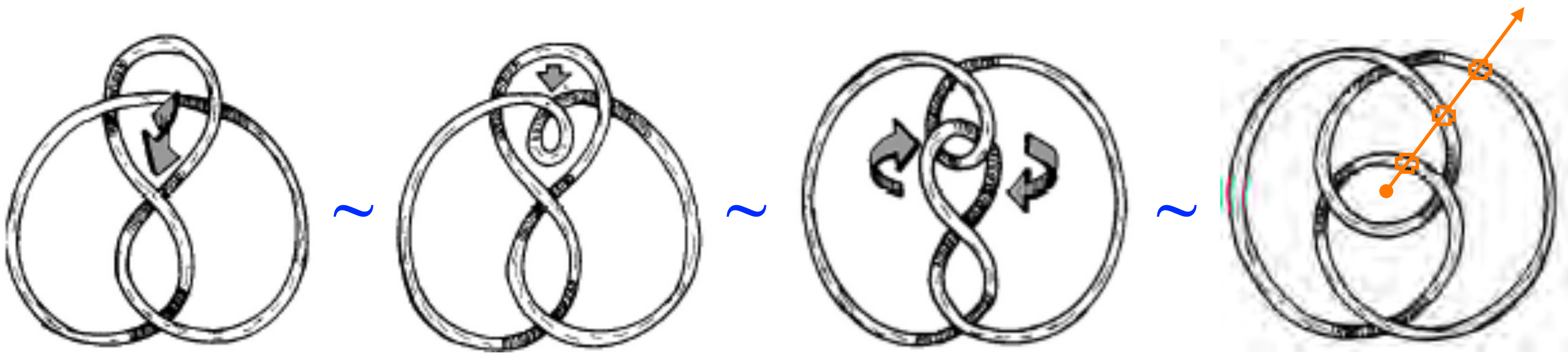

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From knots to braids

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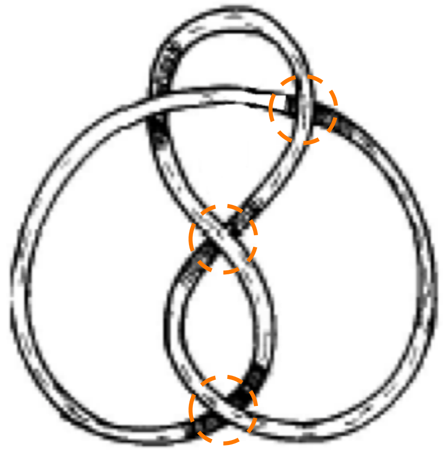

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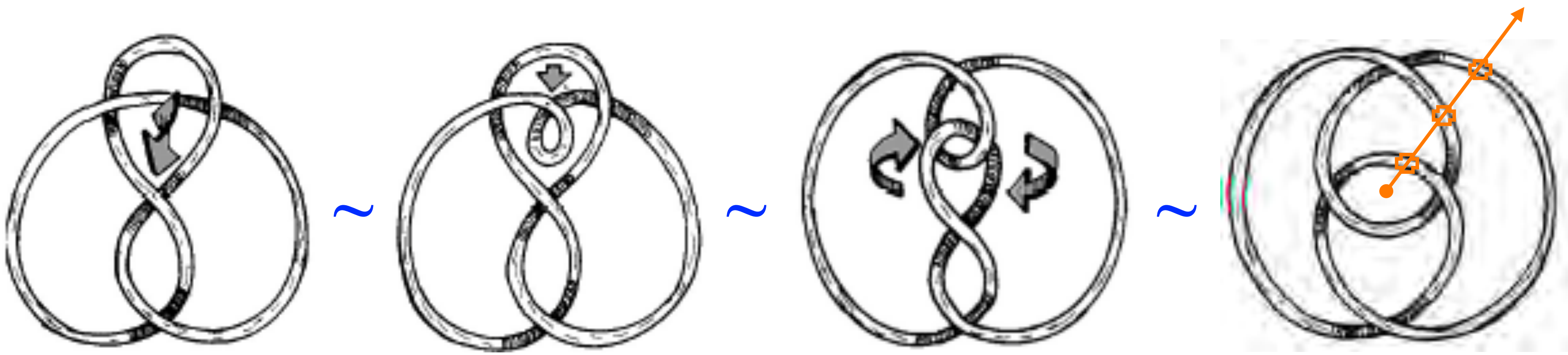

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- **Braid index (invariant)** $\beta(K)$**: minimal number of strings in braid form.**

$$b(K) \leq \beta(K)$$

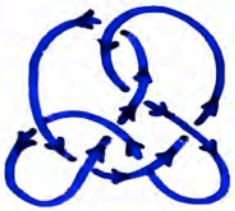
Some exercises!

$$K = ?$$

$$c_{\min}(K) = ?$$

$$Wr(K) = ?$$

Some exercises!



$$K = ?$$

$$c_{\min}\left(K\right) = ?$$

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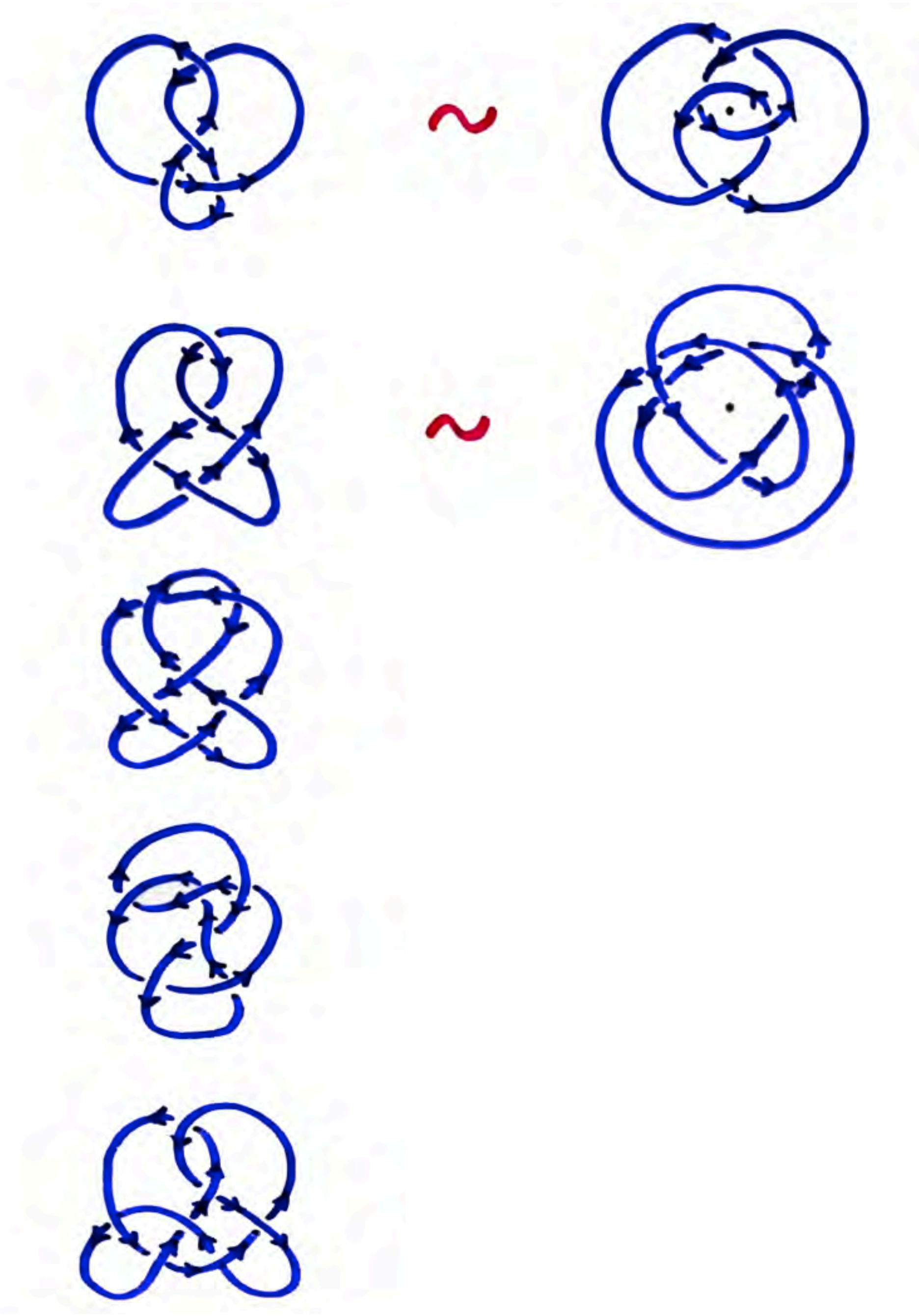

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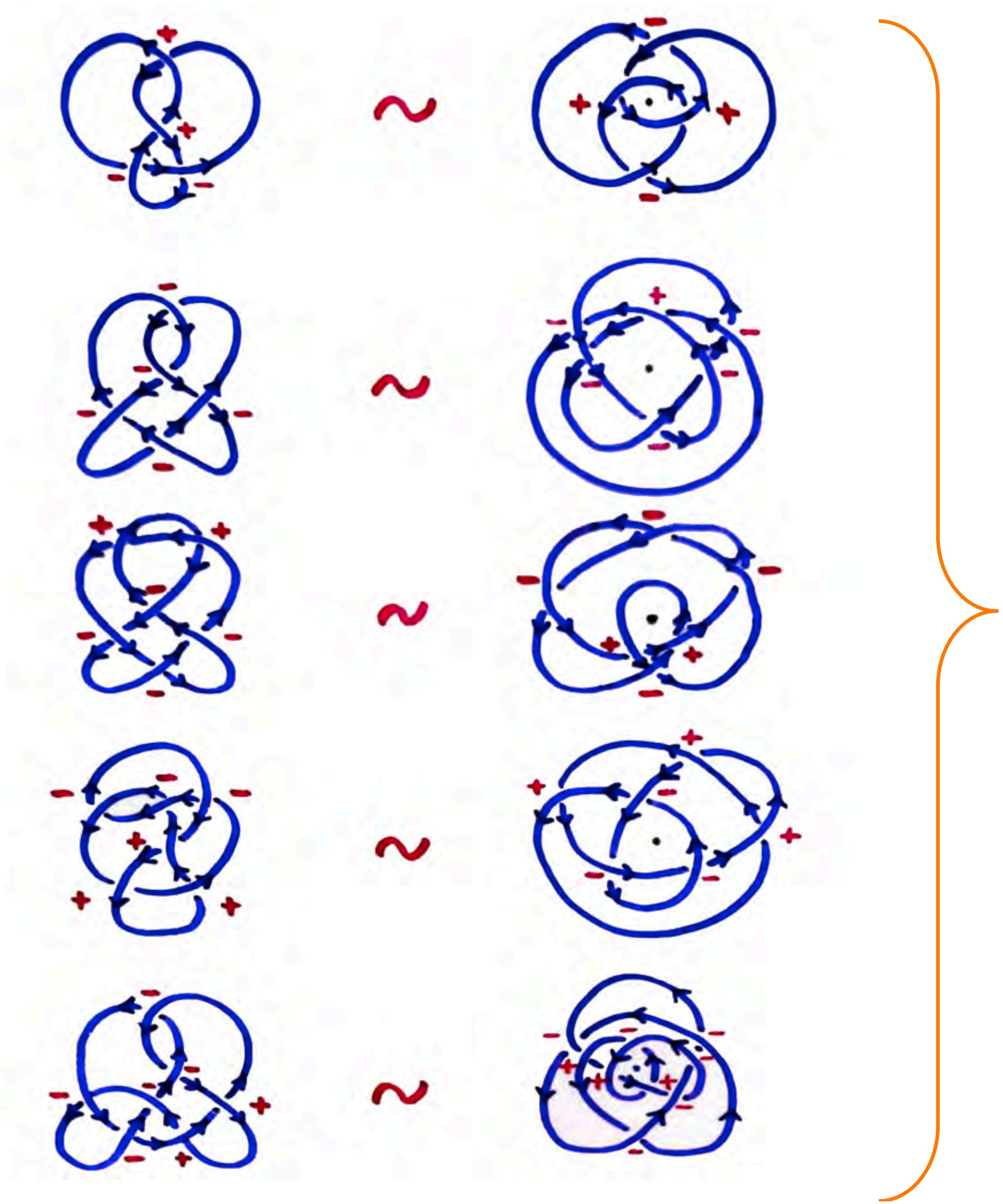

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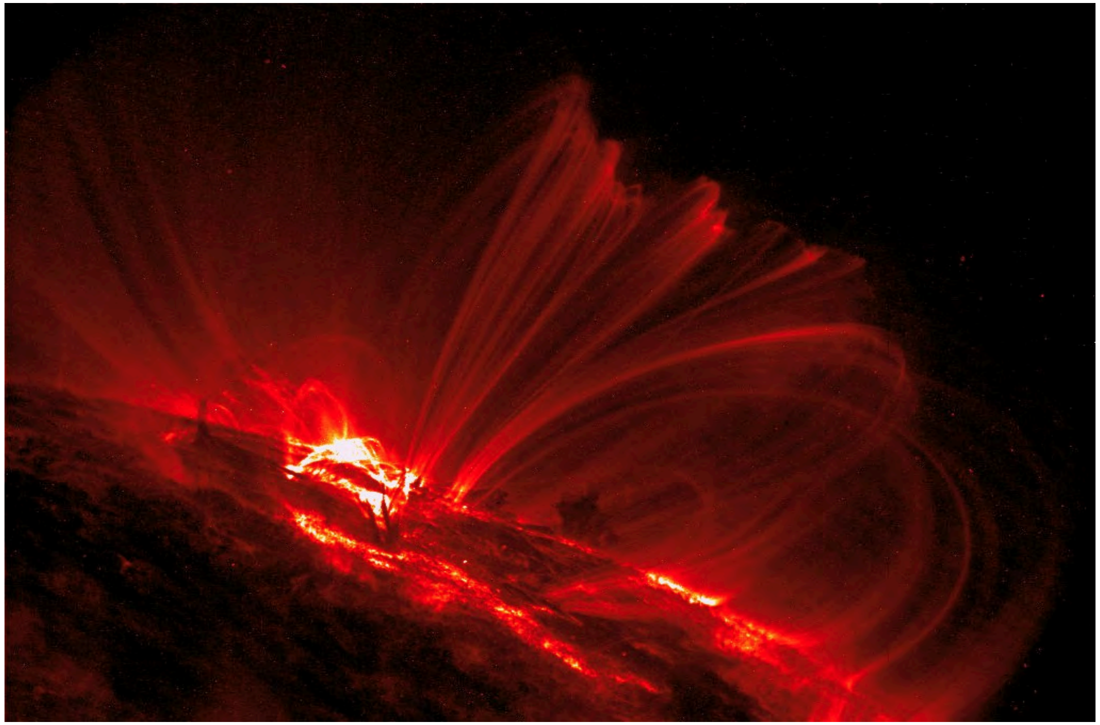
Plasma loops on the Sun



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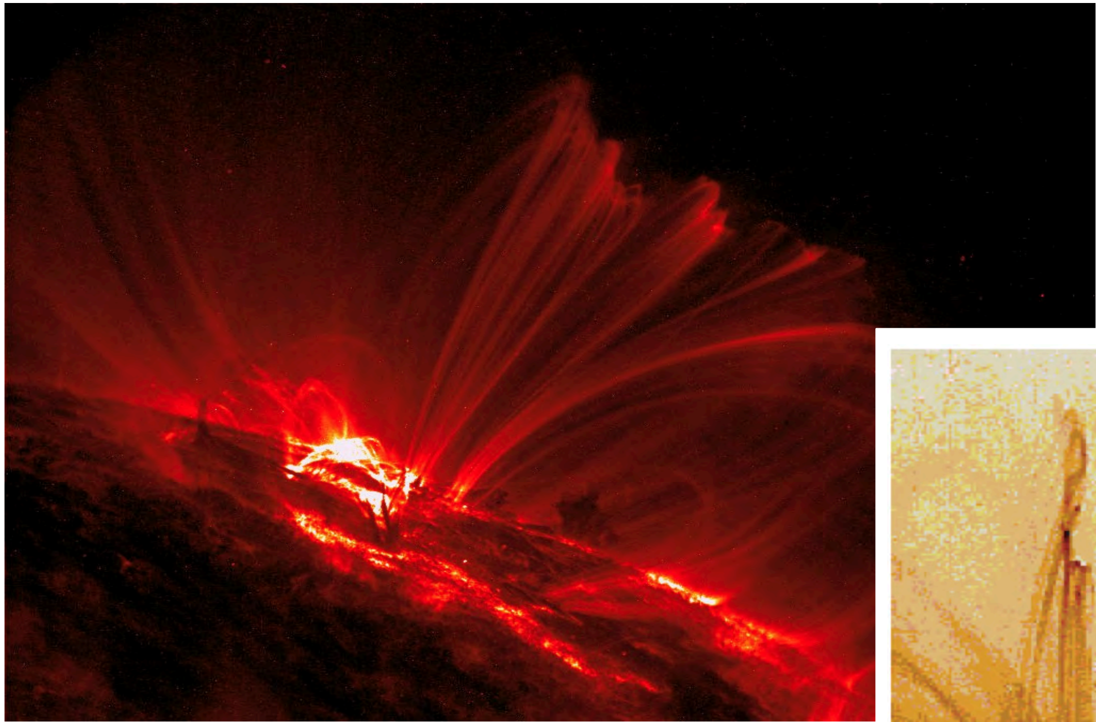

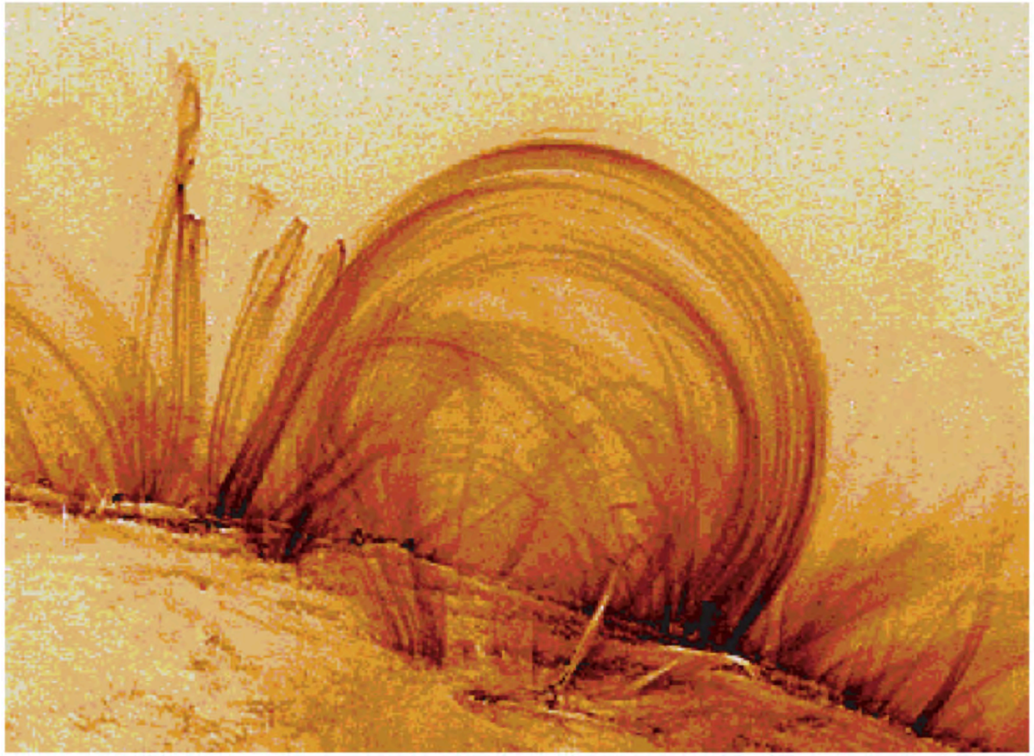

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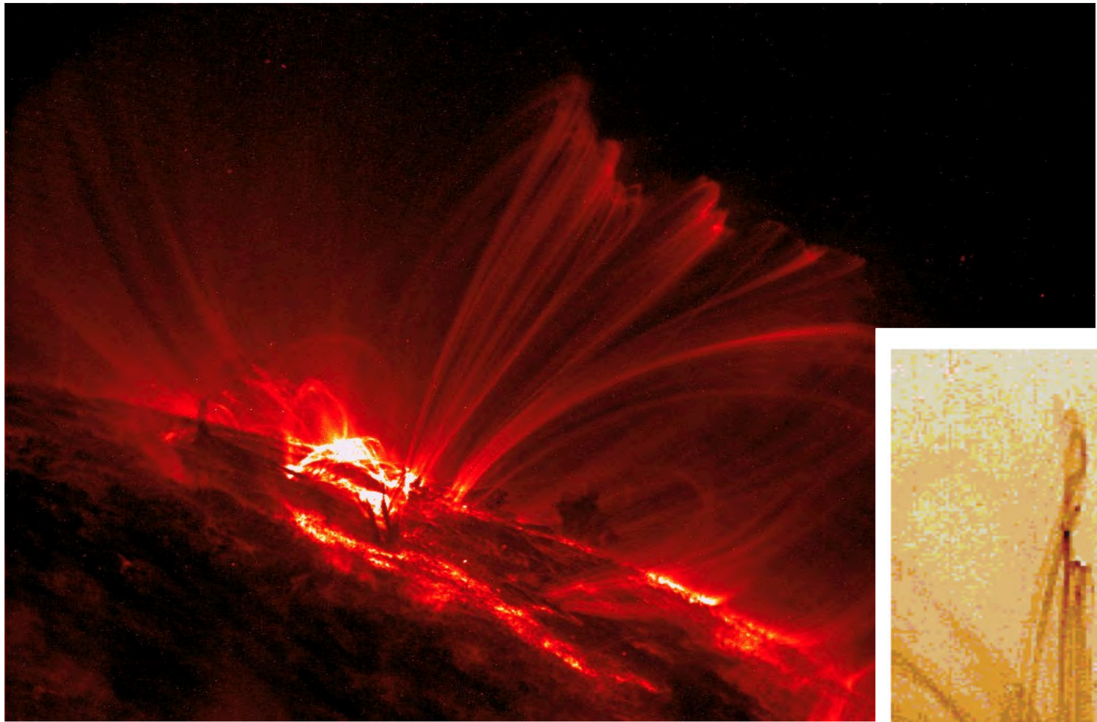
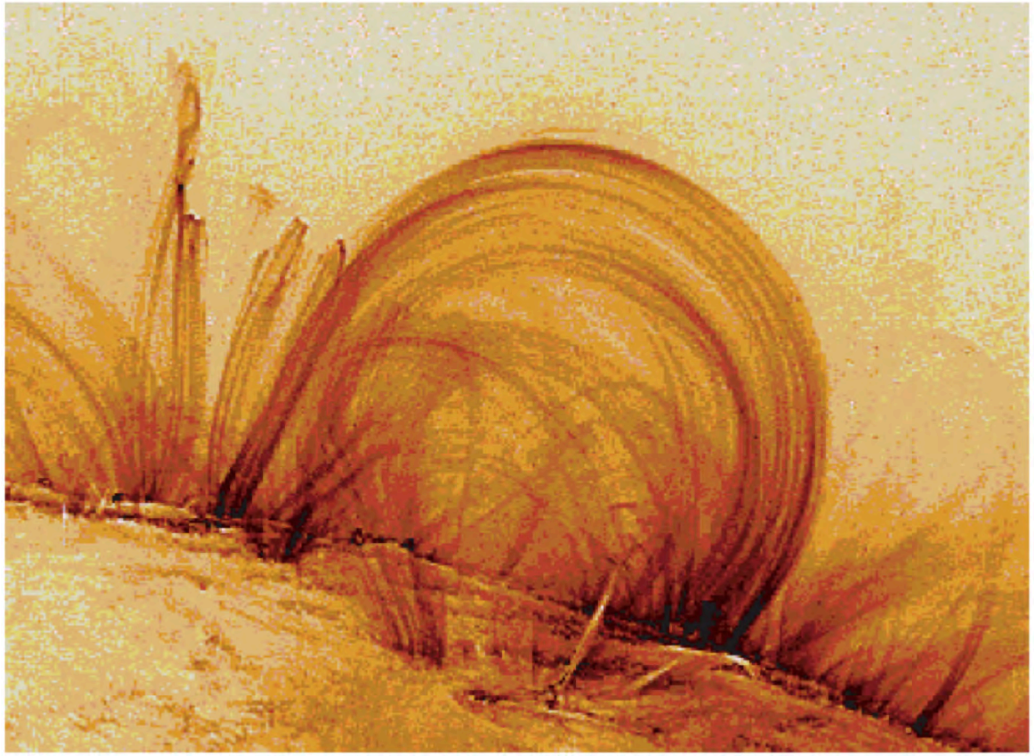
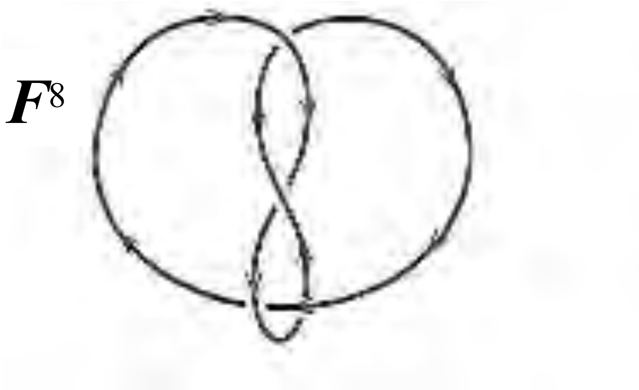


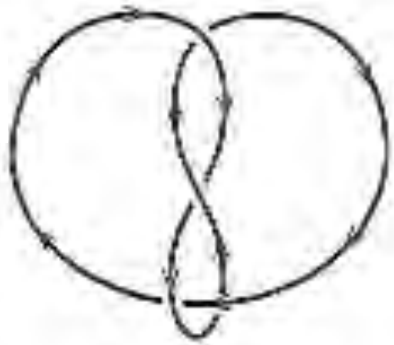


Creating a Seifert surface (orientable spanning surface)

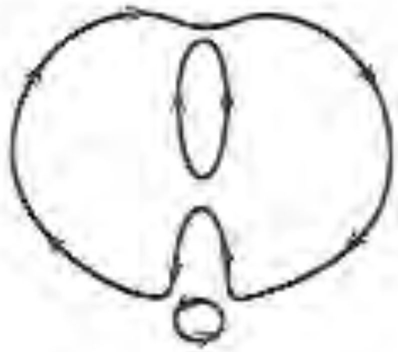


Creating a Seifert surface (orientable spanning surface)

F8



1.



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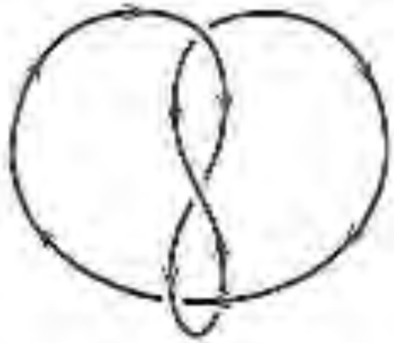
F_8



1.

2.

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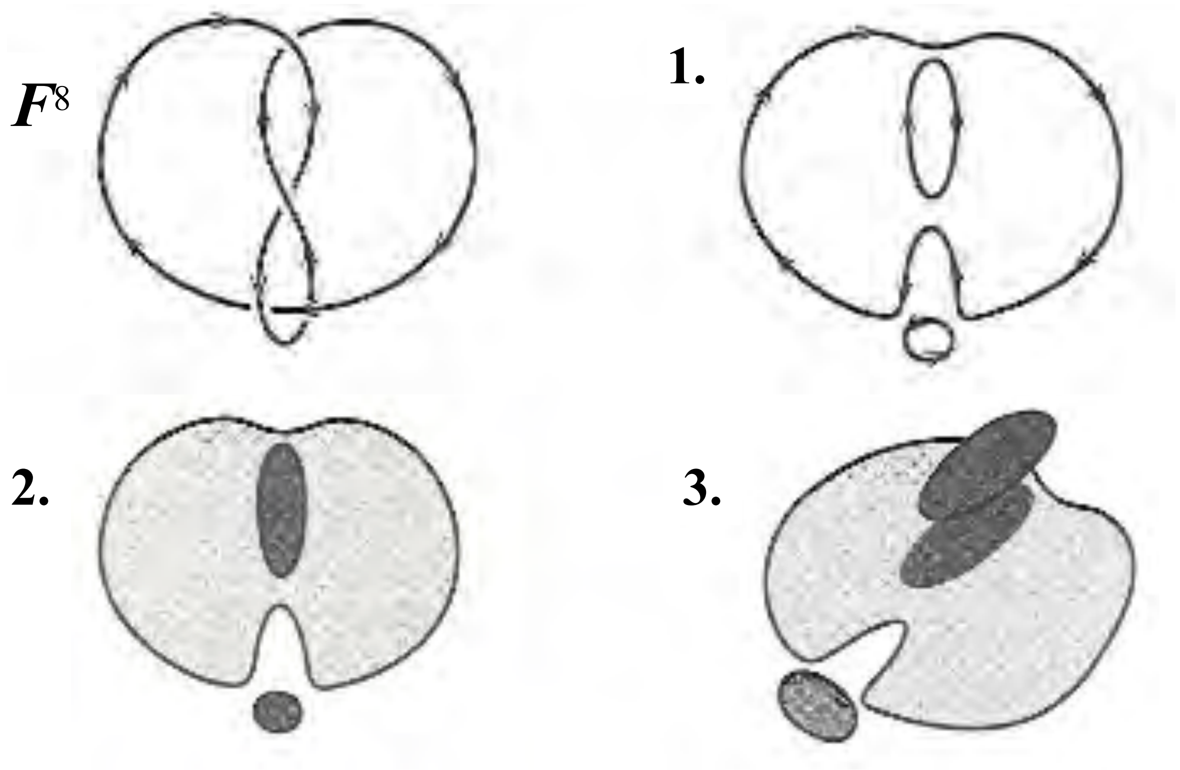
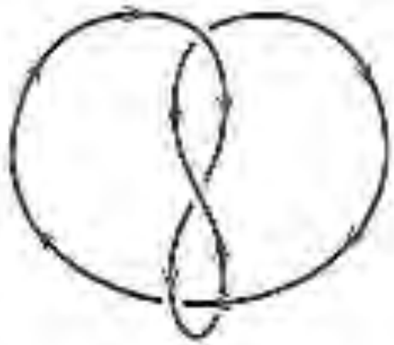


Creating a Seifert surface (orientable spanning surface)

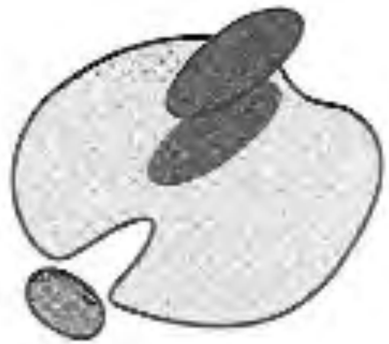
F_8



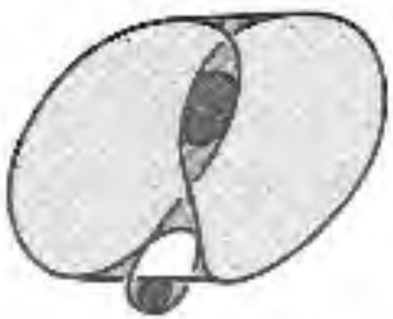
1.

2.

3.



4.



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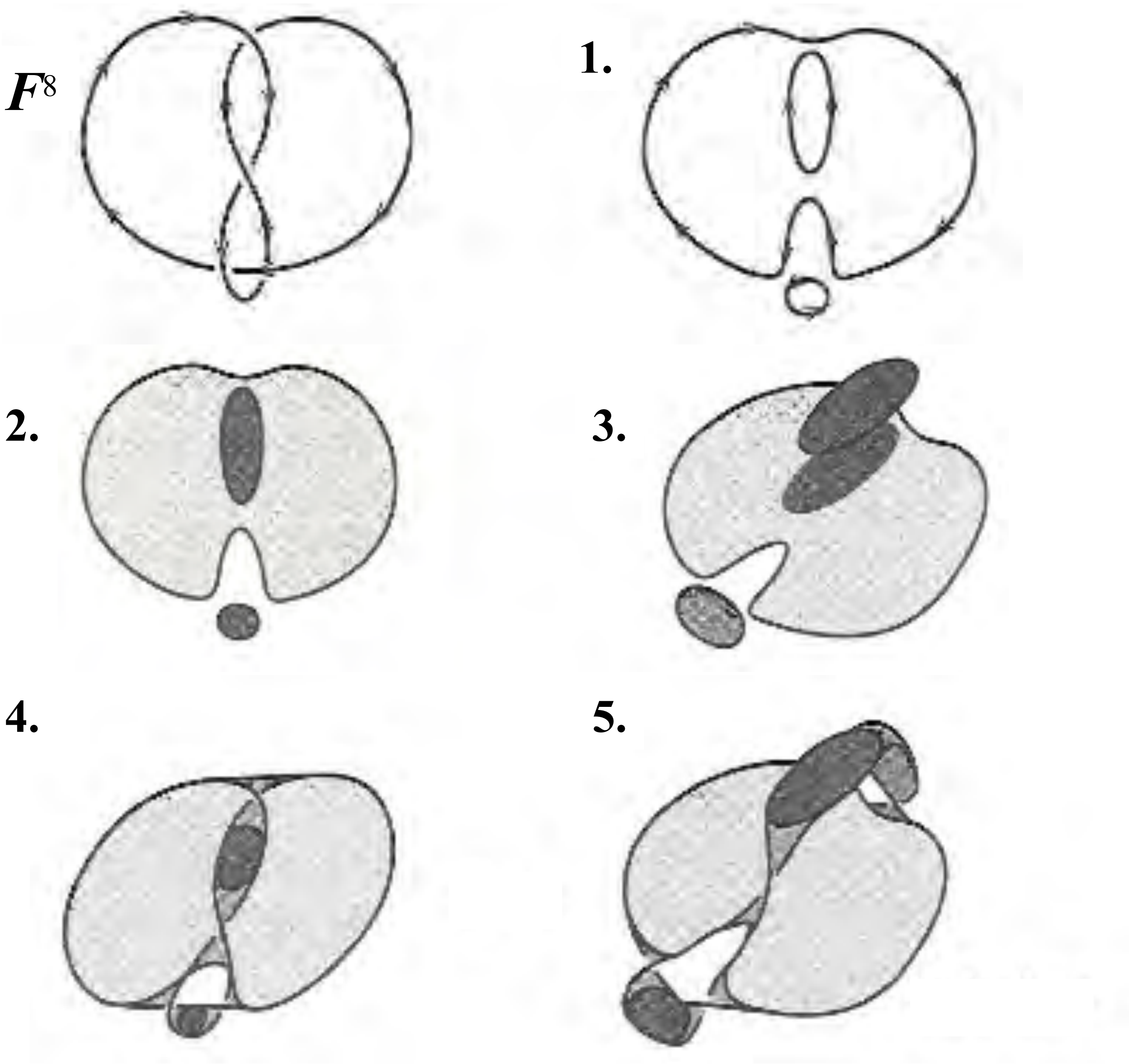

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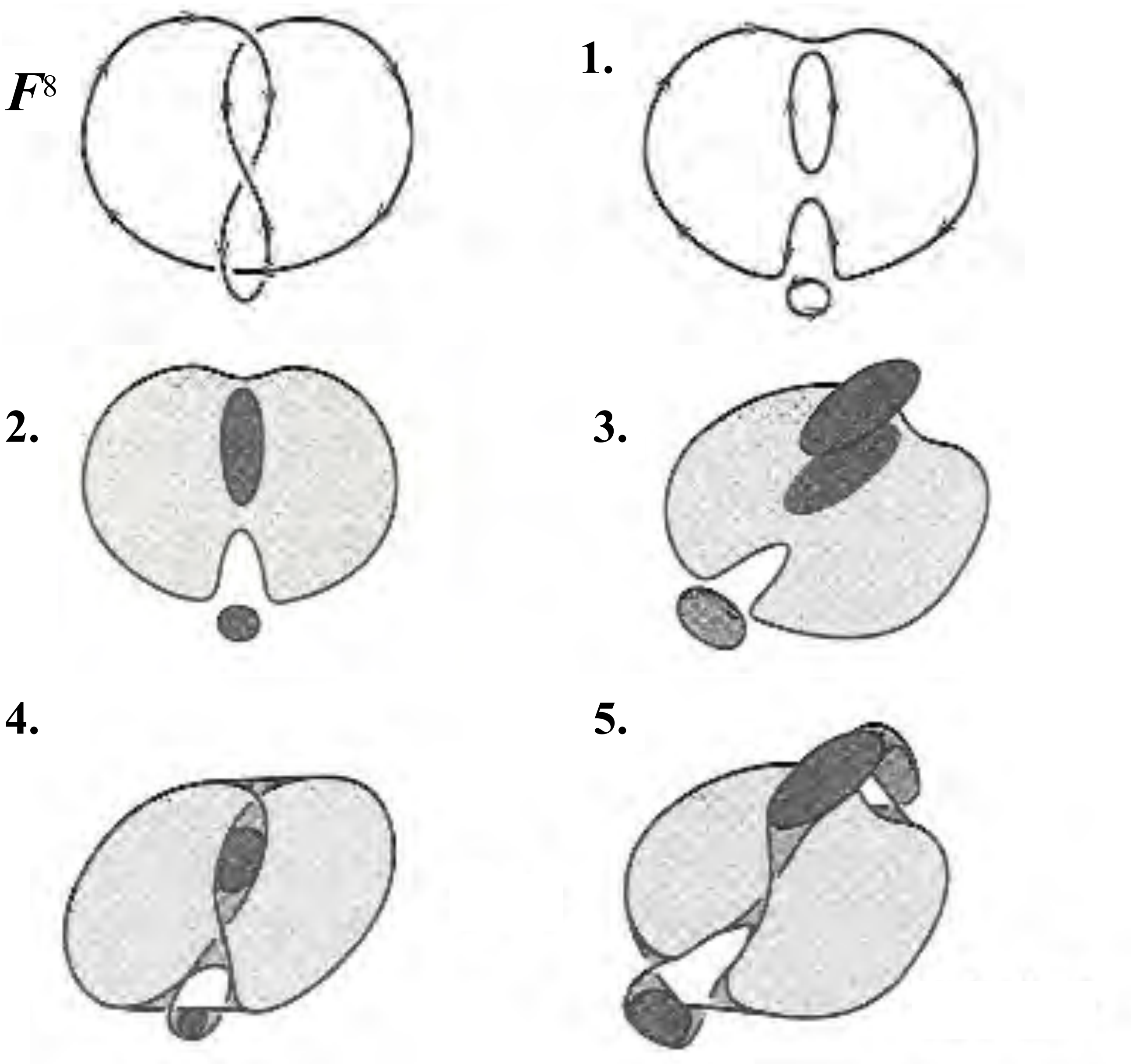
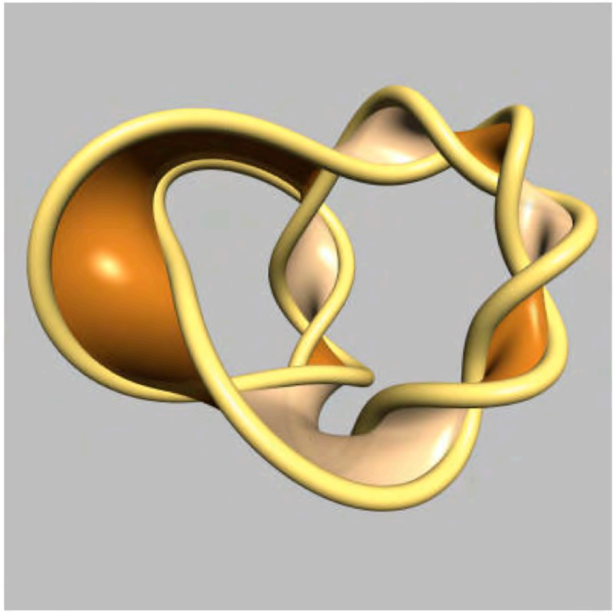


$\beta(K)$ = ***minimum number of Seifert circles in any projection of a knot.***

Examples of Seifert surfaces

a



Examples of Seifert surfaces

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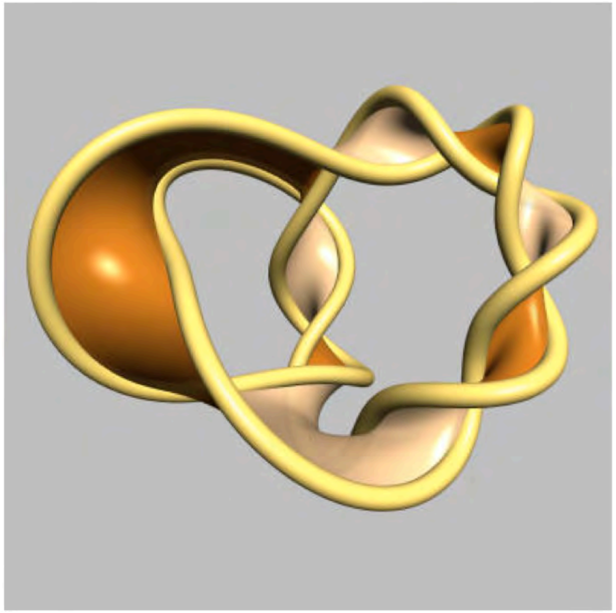
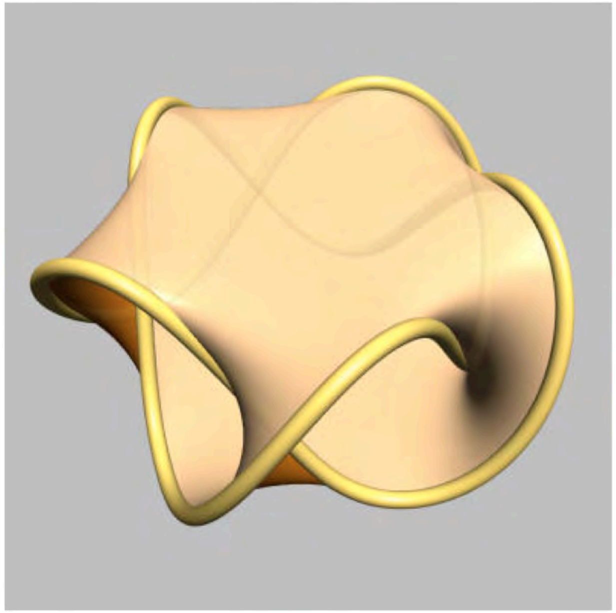
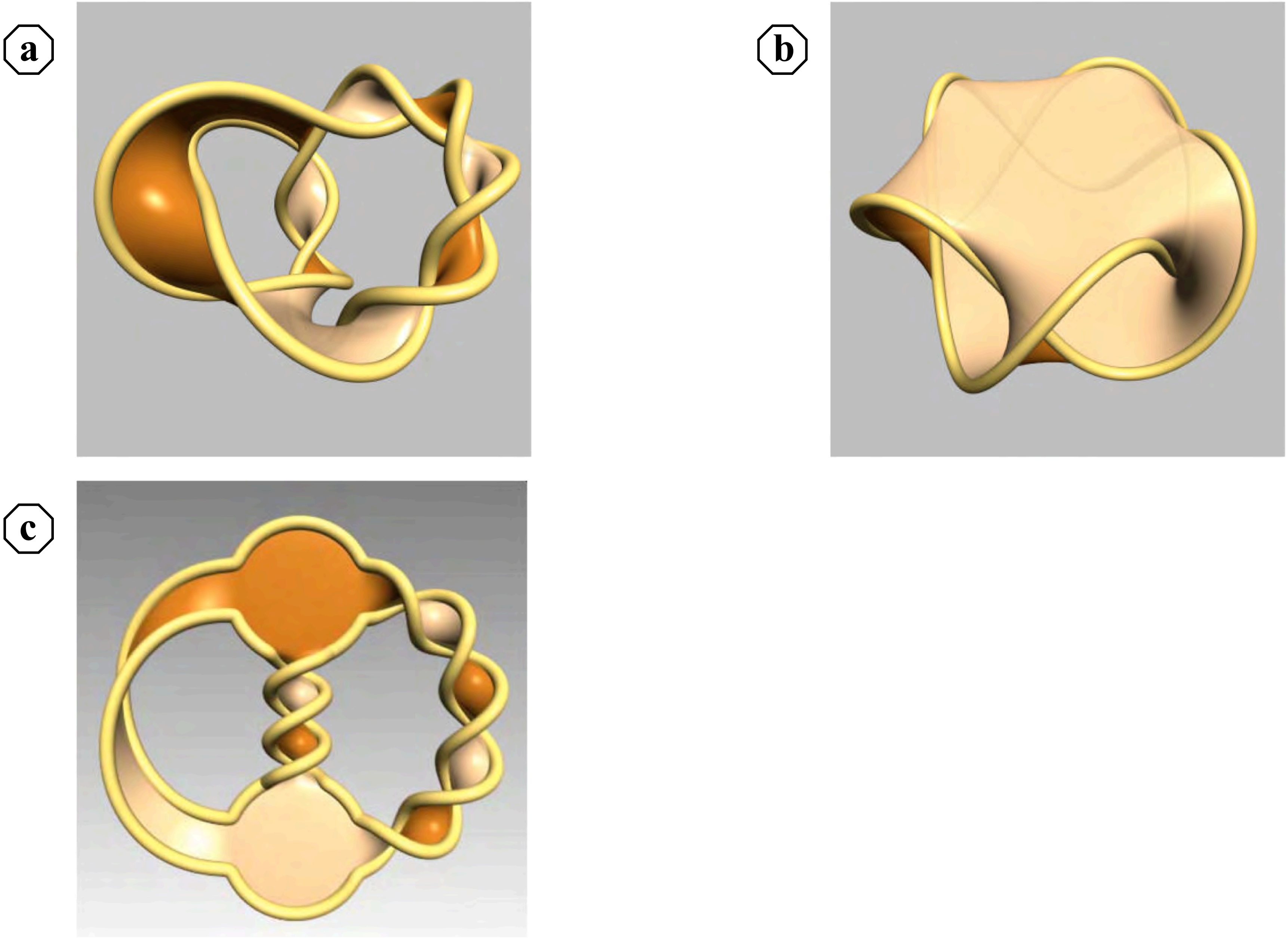


b

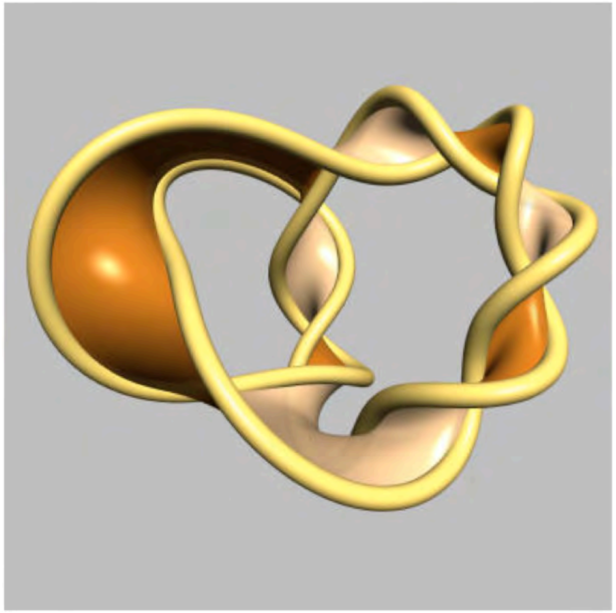


Examples of Seifert surfaces

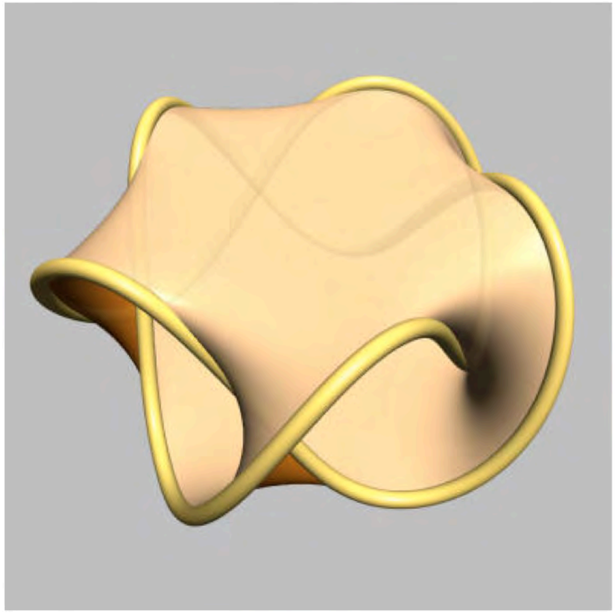


Examples of Seifert surfaces

a

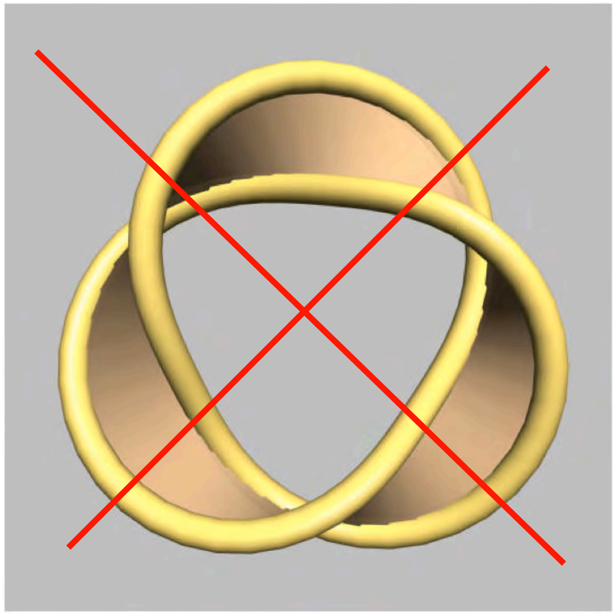


b



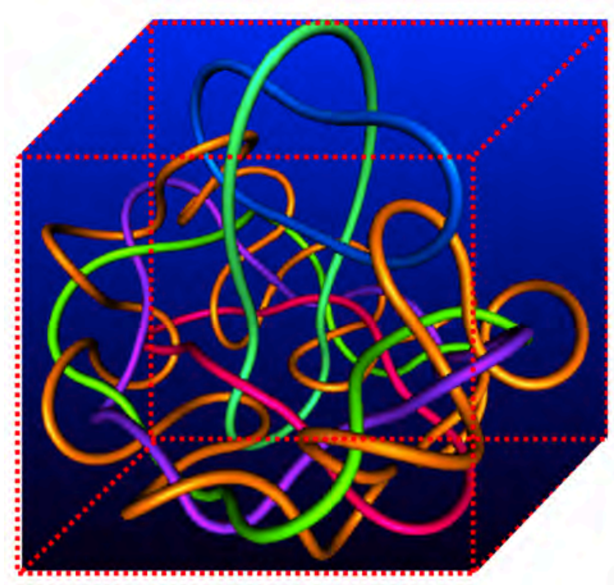
c

d



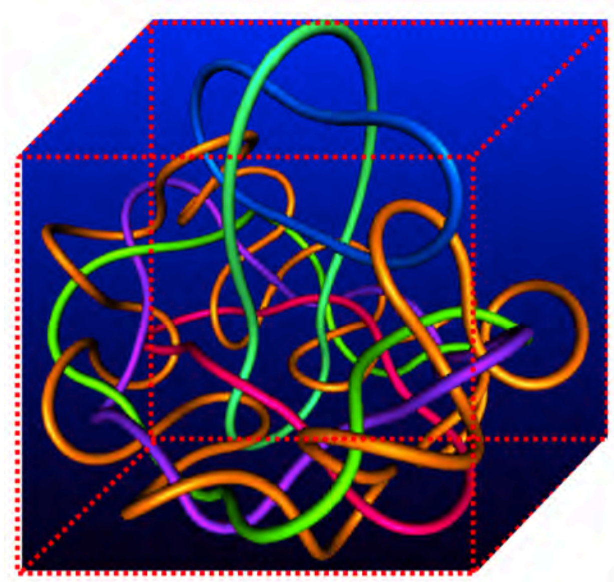
non-orientable

The polynomial era

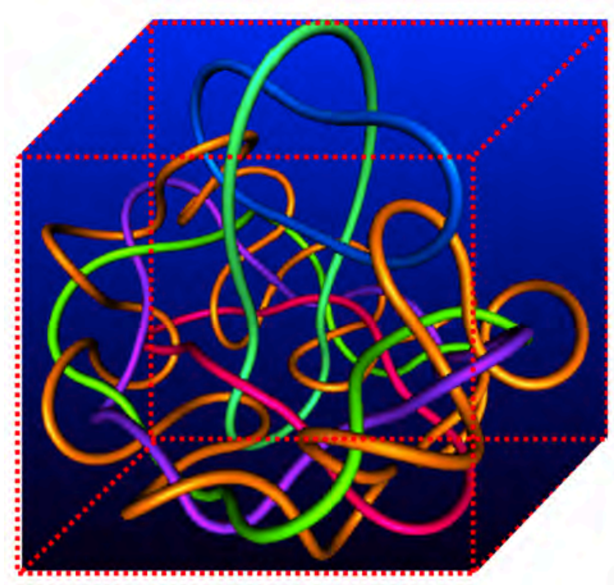


The polynomial era

1930: Alexander, ...



The polynomial era



1930: Alexander, ...

1970: Conway, ...

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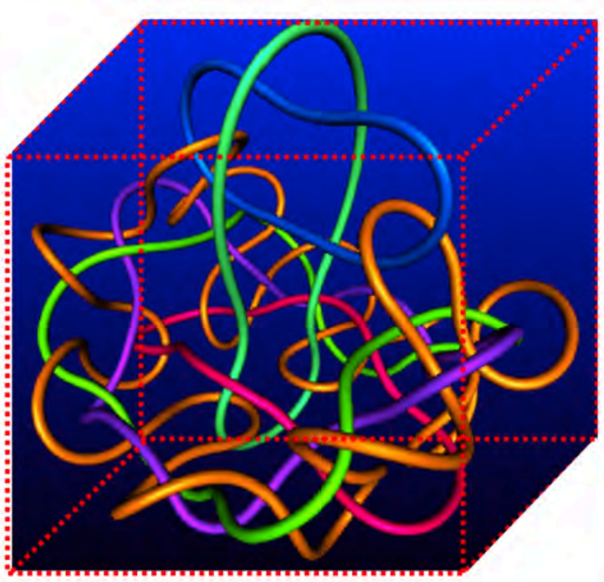
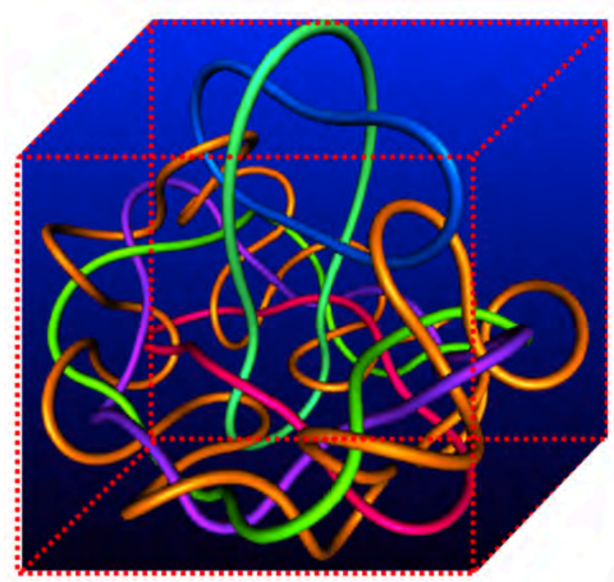


1930: Alexander, …

1970: Conway, …

1980: Kauffman, …

The polynomial era



1930: Alexander, …

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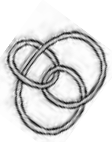
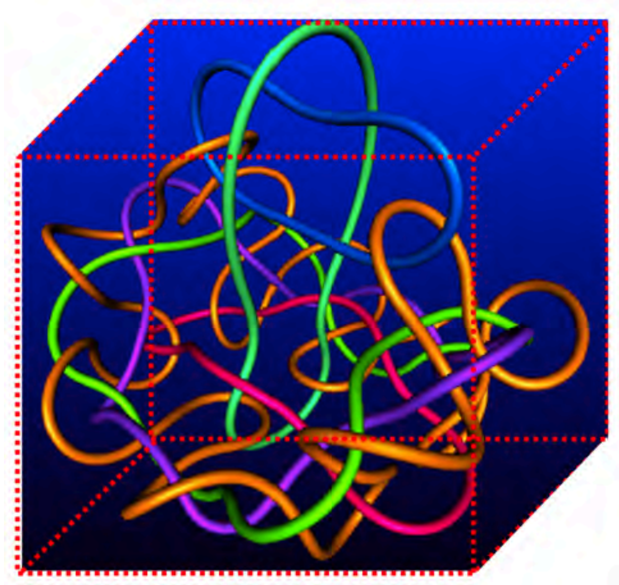
1980: Kauffman, …

1990: Jones, …

The polynomial era

$$P(\ \ ) = a^{-2} + 2a^{-1} - 5 - 2a + 3a^{2}$$





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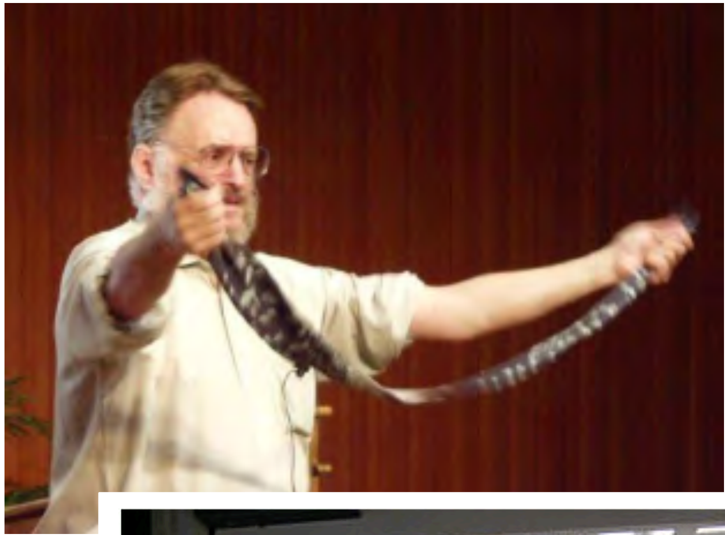

1980: Kauffman, …

1990: Jones, …

The Jones polynomial $V(\boldsymbol{K})$

$$V(\boldsymbol{K}) = f(\boldsymbol{\tau})$$

The Jones polynomial $V(\boldsymbol{K})$

- ***An example:***

$$V(\boldsymbol{K}) = f(\boldsymbol{\tau})$$

$$V\Bigg($$  $$\Bigg) =$$

The Jones polynomial $V(\boldsymbol{K})$

- ***An example:*** $$V(\boldsymbol{K}) = f(\tau)$$

$V\Big($  $\Big) = V(\boldsymbol{F}^8) = \tau^{-2} - \tau^{-1} + 1 - \tau + \tau^2$

The Jones polynomial $V(\boldsymbol{K})$

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$V\Big($  $\Big) = V(\boldsymbol{F}^8) = \boldsymbol{\tau}^{-2} - \boldsymbol{\tau}^{-1} + 1 - \boldsymbol{\tau} + \boldsymbol{\tau}^2$

- ***Skein relations:***

$V(\boldsymbol{K})$:

(V.1) $V(\mathrm{O}) = 1$

(V.2) $V\Big($  $\Big) = \tau^2 V\Big($  $\Big) + (\tau^{\frac{3}{2}} - \tau^{\frac{1}{2}}) V\Big($  $\Big)$

\+   −   =

The Jones polynomial $V(\boldsymbol{K})$

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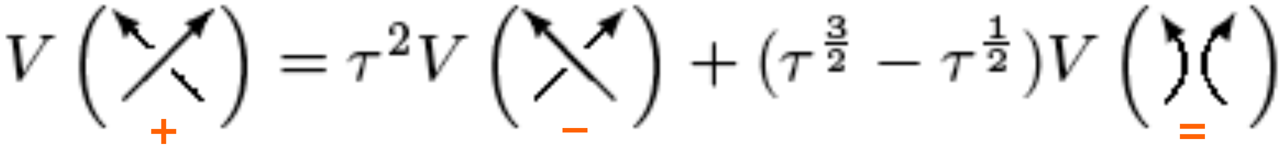
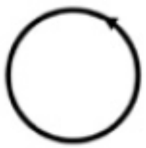
(V.2) 

V.1:



unknot

The Jones polynomial $V(\boldsymbol{K})$

- ***An example:***

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$$V(\;\cdot\;) = V(\boldsymbol{F}^8) = \boldsymbol{\tau}^{-2} - \boldsymbol{\tau}^{-1} + 1 - \boldsymbol{\tau} + \boldsymbol{\tau}^2$$

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+

−

=

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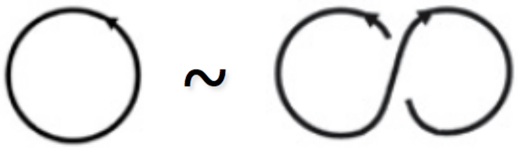

unknot γ_+

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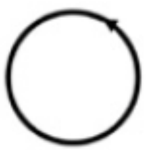
 $\sim$  $\sim$ 

unknot γ_+ γ_-

The Jones polynomial $V(K)$

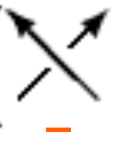
- ***An example:*** $V(K) = f(\tau)$

$V\Big($  $\Big) = V(F^8) = \tau^{-2} - \tau^{-1} + 1 - \tau + \tau^2$

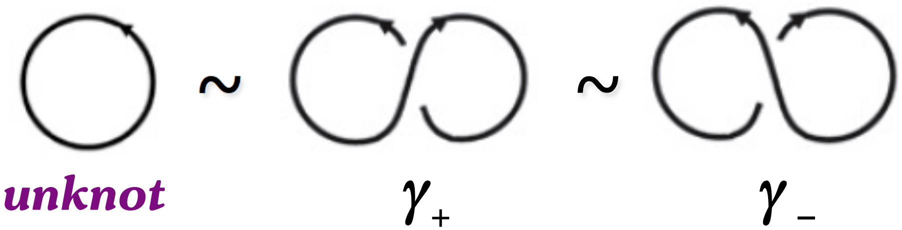
- ***Skein relations:***

$V(K)$:

(V.1) $V(\bigcirc) = 1$

(V.2) $V\Big($  $\Big) = \tau^2 V\Big($  $\Big) + (\tau^{\frac{3}{2}} - \tau^{\frac{1}{2}}) V\Big($  $\Big)$

V.1:



$V(\bigcirc) = V(\gamma_+) = V(\gamma_-) = 1$

The Jones polynomial $V(\boldsymbol{K})$

- **An example:** $V(\boldsymbol{K}) = f(\tau)$

$V\Big($  $\Big) = V(\boldsymbol{F}^8) = \tau^{-2} - \tau^{-1} + 1 - \tau + \tau^2$

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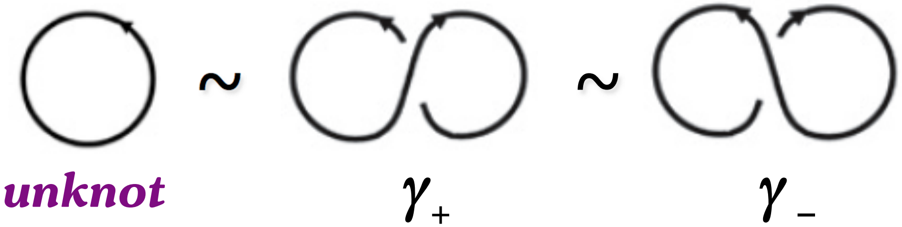
+   −   =

V.1:



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V.2:

γ_+

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V.1:

unknot $\quad \gamma_+ \quad \gamma_-$

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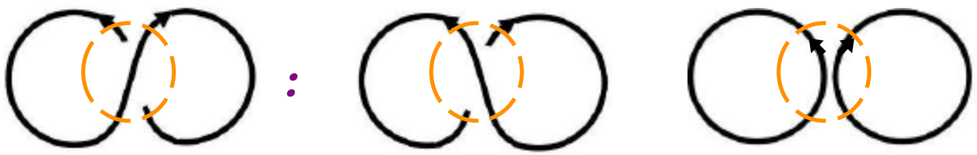
V.2:



$\gamma_+ \quad \gamma_- \quad \boldsymbol{U}^2$

The Jones polynomial $V(\boldsymbol{K})$

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\+   −   =

V.1:

unknot γ_+ γ_-

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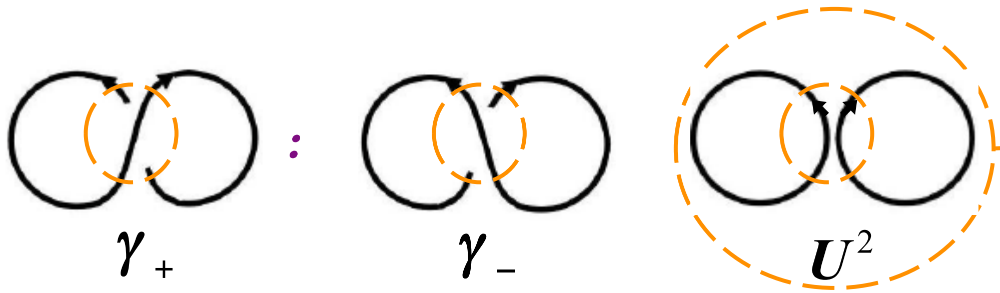
V.2:



$$V(U^2) = -\tau^{-\frac{1}{2}} - \tau^{\frac{1}{2}}$$

Worked-out examples

- Hopf link H_- :

H_-

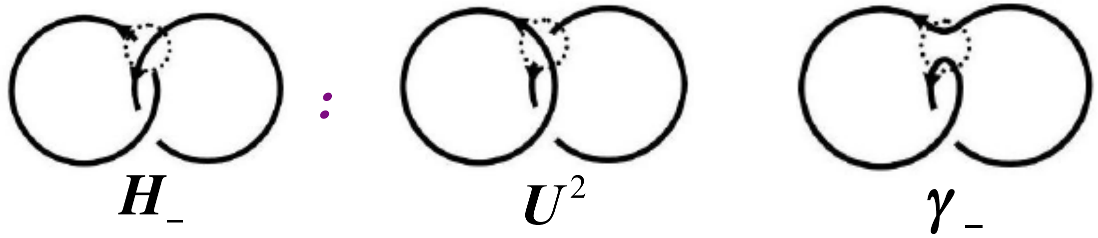
Worked-out examples

- *Hopf link* H_{-} *:*

:

Worked-out examples

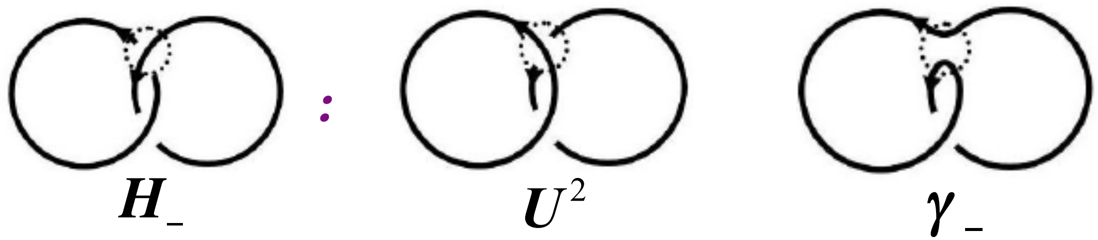
- *Hopf link* H_- *:*



Worked-out examples

- ***Hopf link*** H_- **:**

$$V(H_-) = -\tau^{-\frac{1}{2}} - \tau^{-\frac{5}{2}}$$



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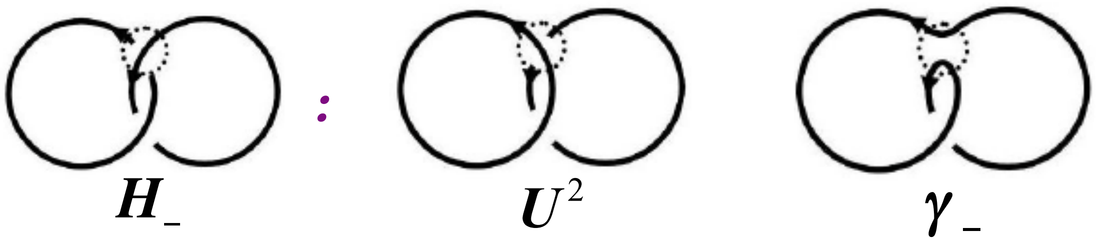
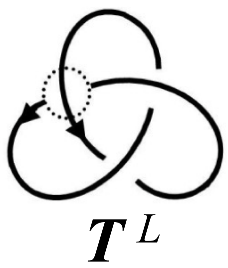


- ***Left-handed trefoil knot*** T^L **:**



Worked-out examples

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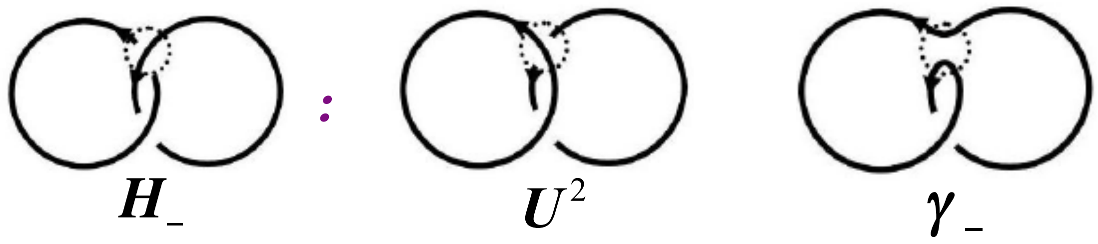

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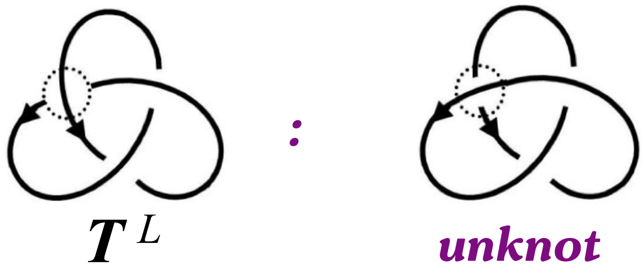

Worked-out examples

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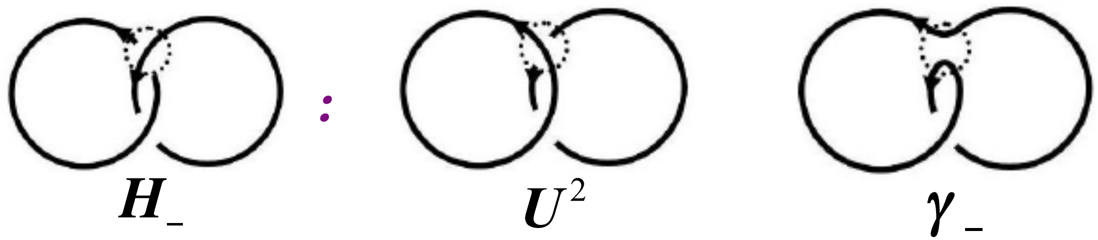

- ***Left-handed trefoil knot*** T^L **:**

Worked-out examples

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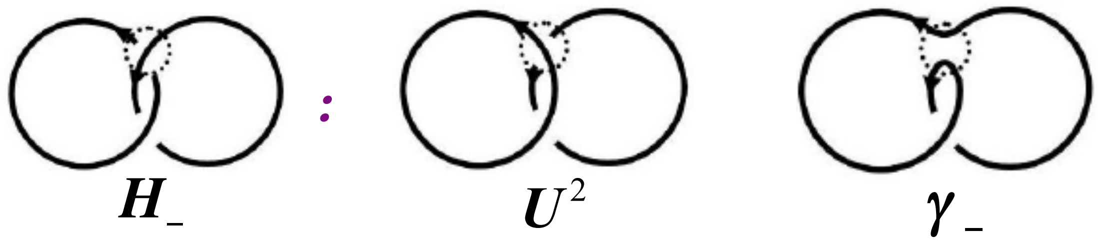


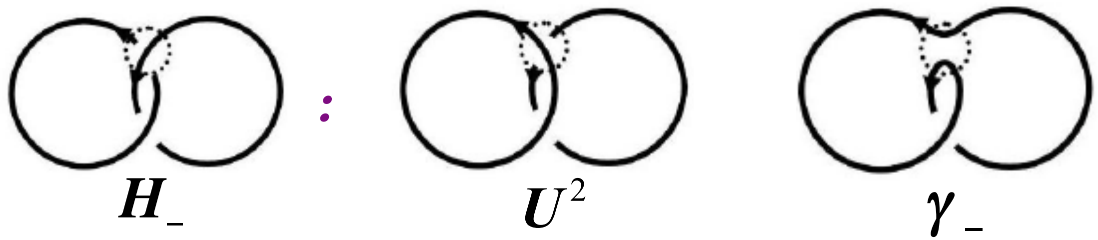
- ***Left-handed trefoil knot T^L :***

$$V(T^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$

Worked-out examples

- **Hopf link H_- :**

$$V(H_-) = -\tau^{-\frac{1}{2}} - \tau^{-\frac{5}{2}}$$



- **Left-handed trefoil knot T^L :**

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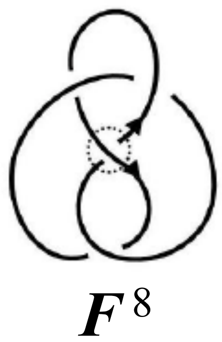
- **Figure-of-eight knot F^8 :**



Worked-out examples

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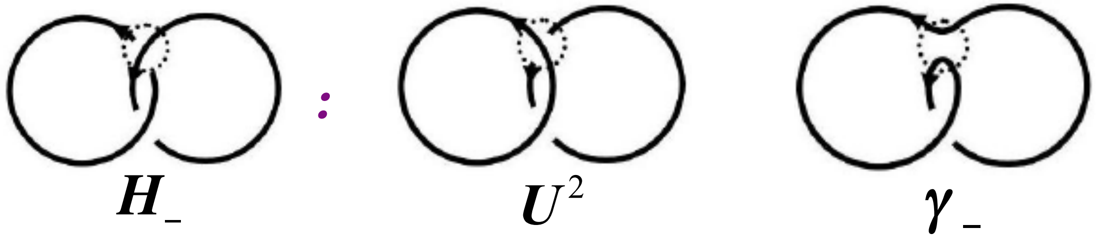

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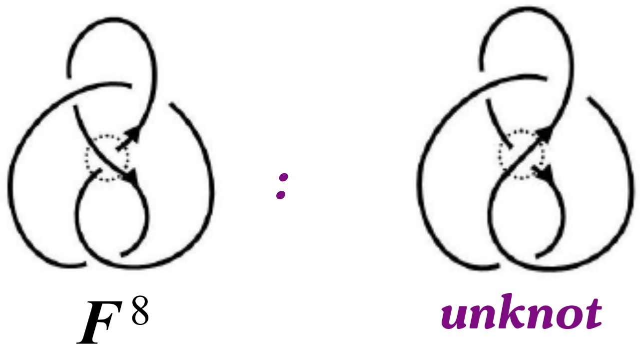

Worked-out examples

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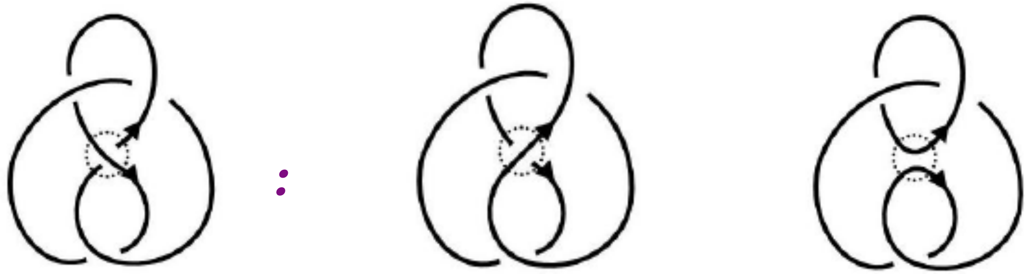
H_- : U^2 γ_-

- ***Left-handed trefoil knot T^L :***

$$V(T^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$

T^L : **unknot** H_-

- ***Figure-of-eight knot F^8 :***



F^8 : **unknot** H_-

Worked-out examples

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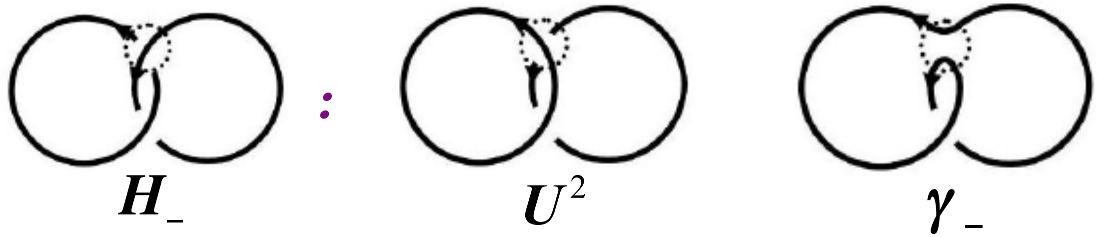

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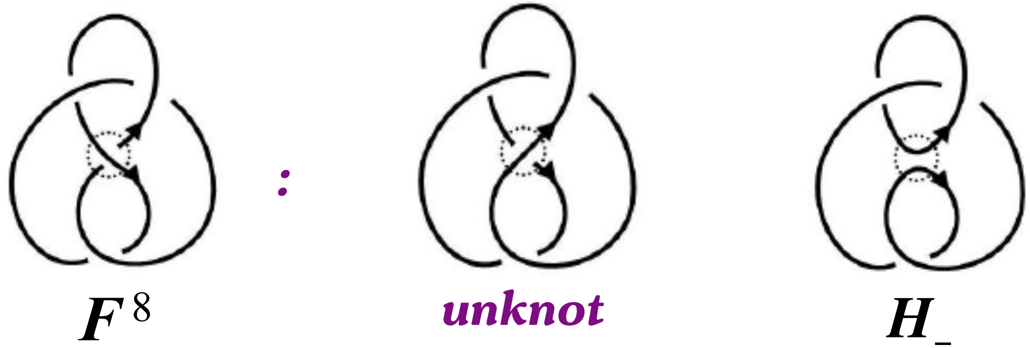
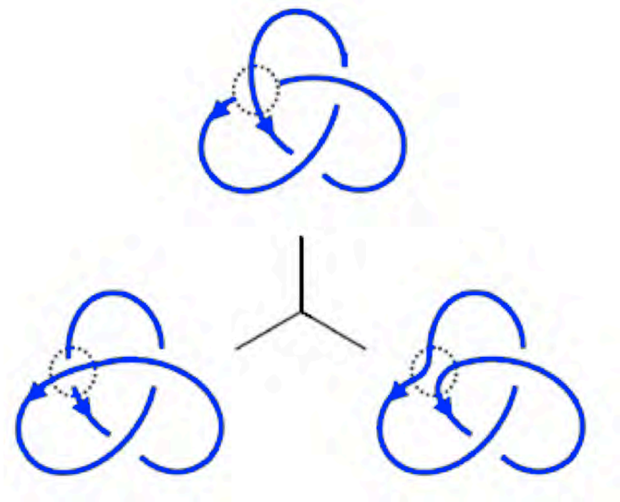
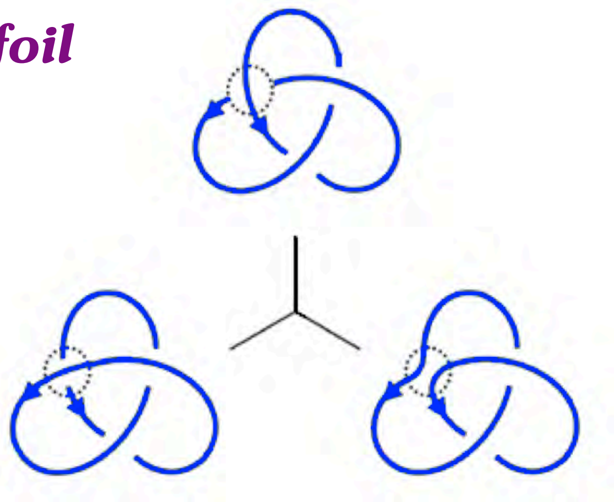


Jones distinguishes mirror knots



Jones distinguishes mirror knots

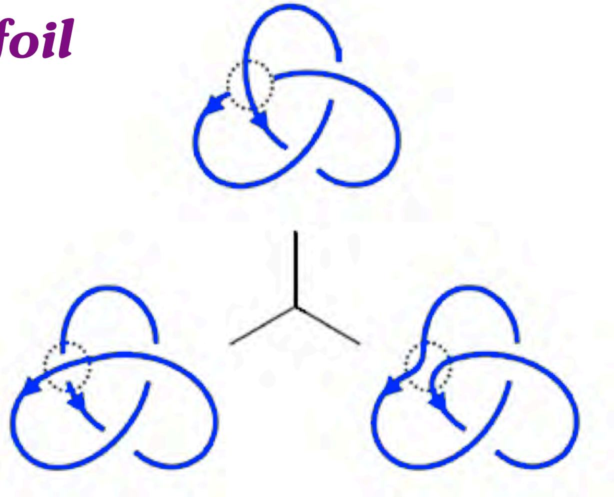
Left trefoil



$$V(\boldsymbol{T}^{L}) = \boldsymbol{\tau}^{-1} + \boldsymbol{\tau}^{-3} - \boldsymbol{\tau}^{-4}$$

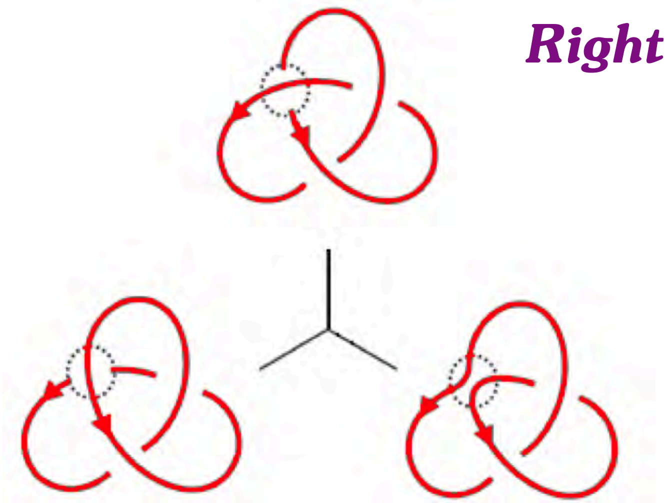
Jones distinguishes mirror knots

Left trefoil



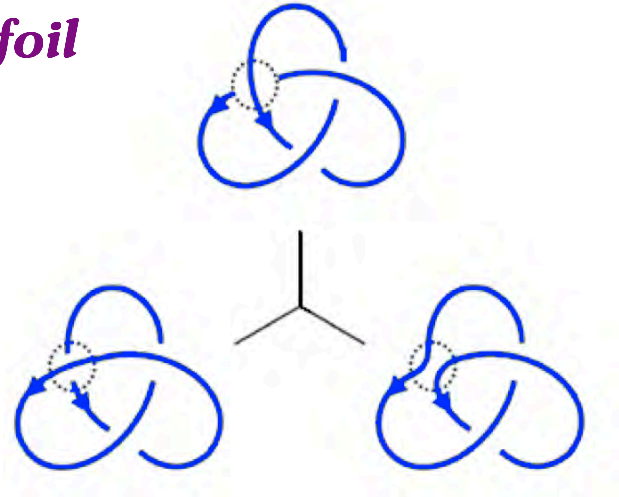
$$V(\boldsymbol{T}^{L}) = \boldsymbol{\tau}^{-1} + \boldsymbol{\tau}^{-3} - \boldsymbol{\tau}^{-4}$$

Right trefoil



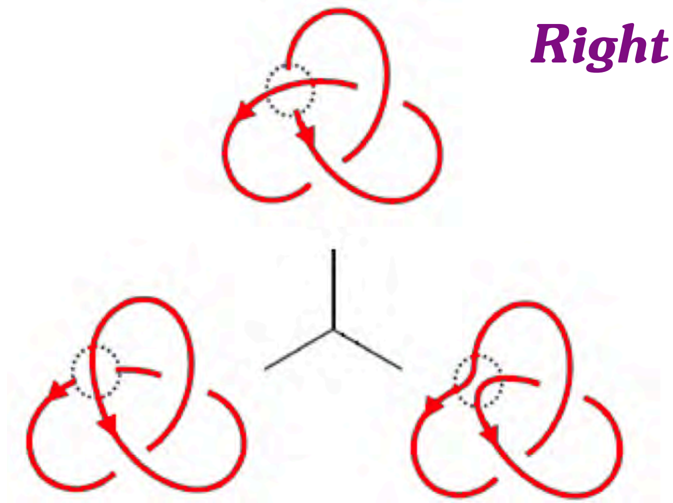
Jones distinguishes mirror knots

Left trefoil



$$V(\boldsymbol{T}^{L}) = \boldsymbol{\tau}^{-1} + \boldsymbol{\tau}^{-3} - \boldsymbol{\tau}^{-4}$$

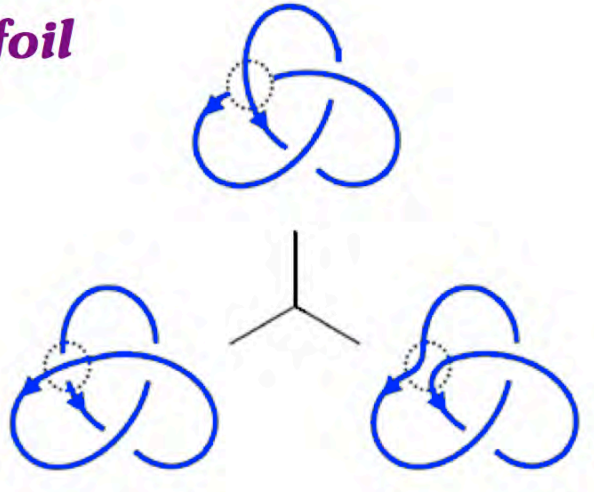
Right trefoil



$$V(\boldsymbol{T}^{R}) = \boldsymbol{\tau} + \boldsymbol{\tau}^{3} - \boldsymbol{\tau}^{4}$$

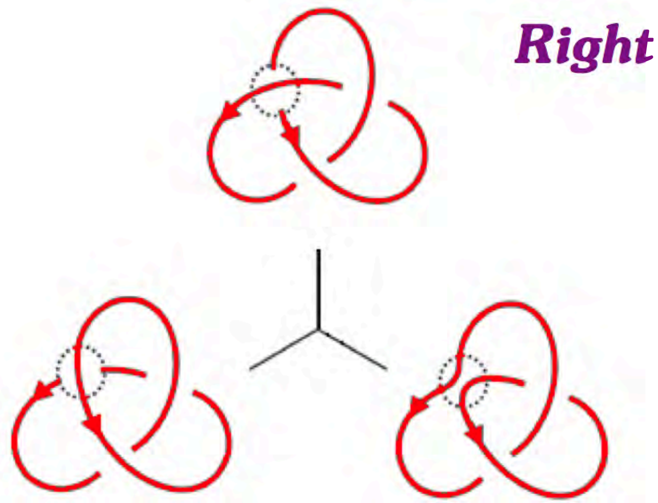
Jones distinguishes mirror knots

Left trefoil



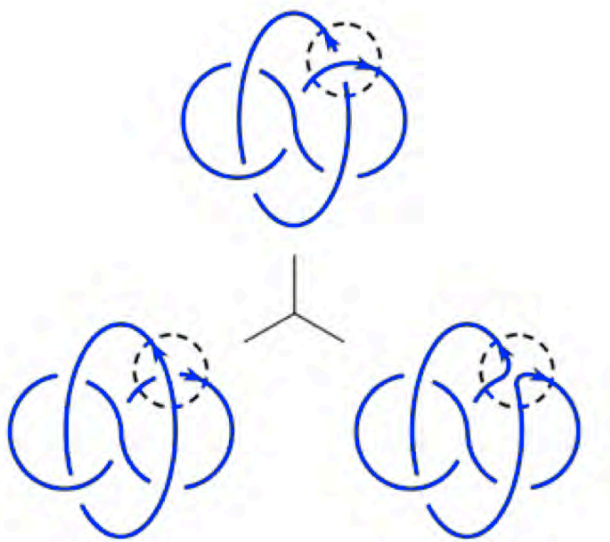
$$V(\boldsymbol{T}^{L}) = \boldsymbol{\tau}^{-1} + \boldsymbol{\tau}^{-3} - \boldsymbol{\tau}^{-4}$$

Right trefoil



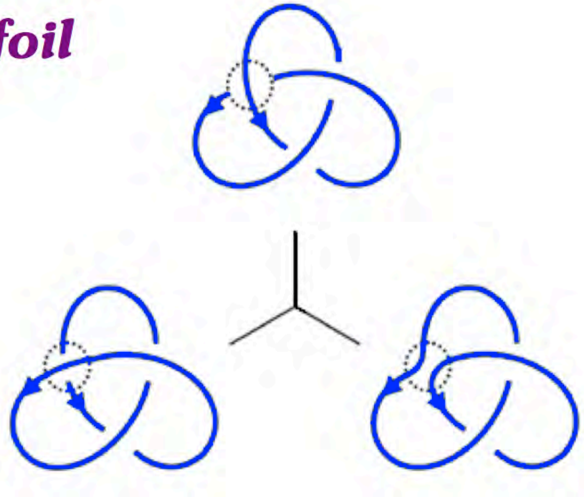
$$V(\boldsymbol{T}^{R}) = \boldsymbol{\tau} + \boldsymbol{\tau}^{3} - \boldsymbol{\tau}^{4}$$

Whitead link



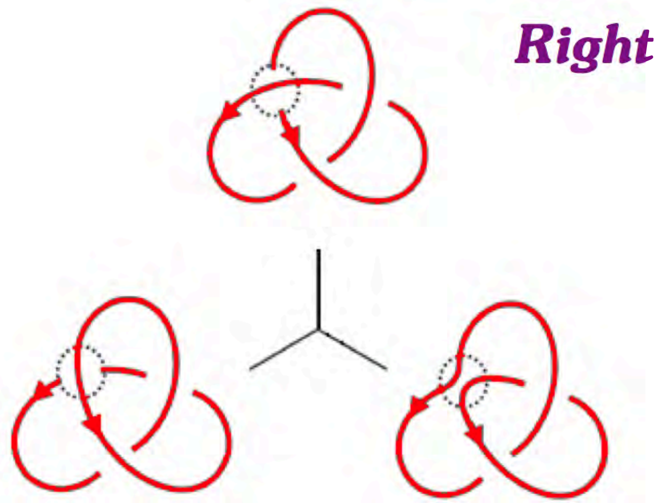
Jones distinguishes mirror knots

Left trefoil



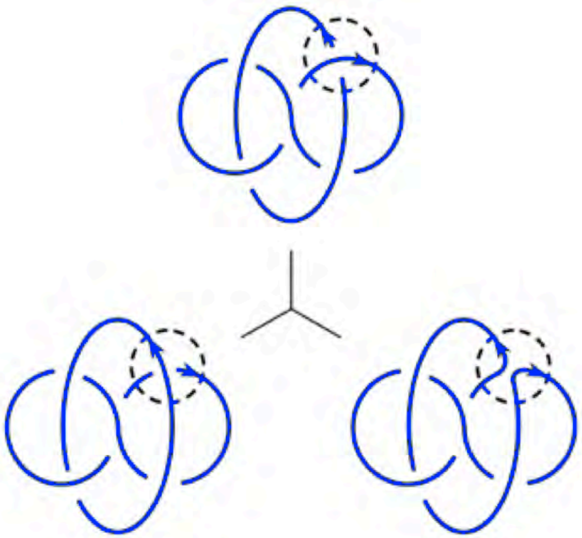
$$V(T^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$

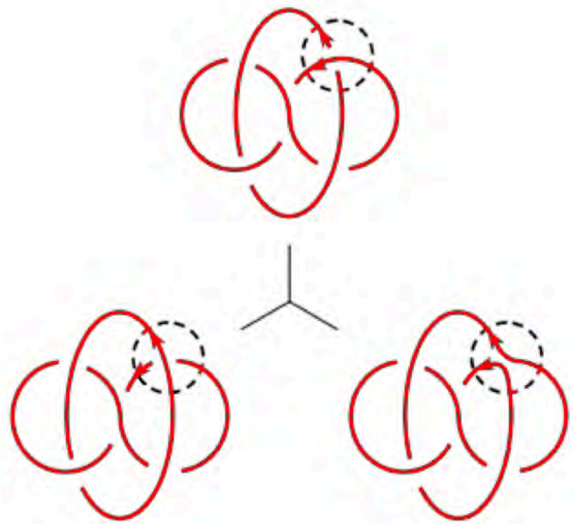
Right trefoil



$$V(T^R) = \tau + \tau^{3} - \tau^{4}$$

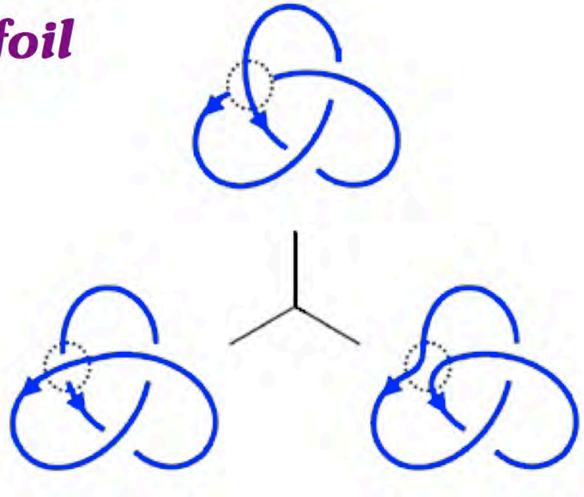
Whitead link



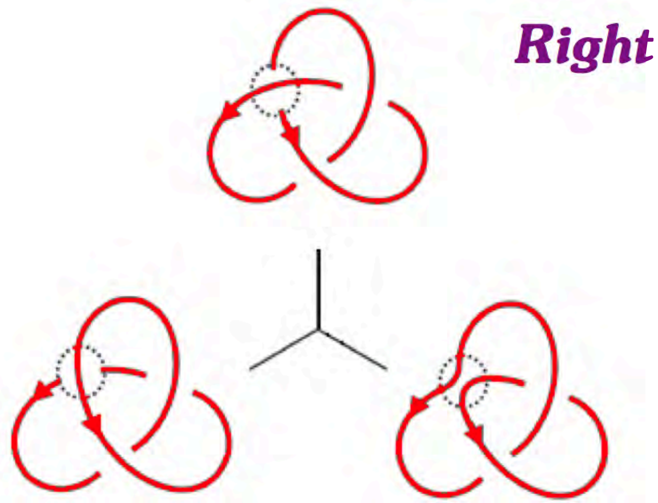


Jones distinguishes mirror knots

Left trefoil



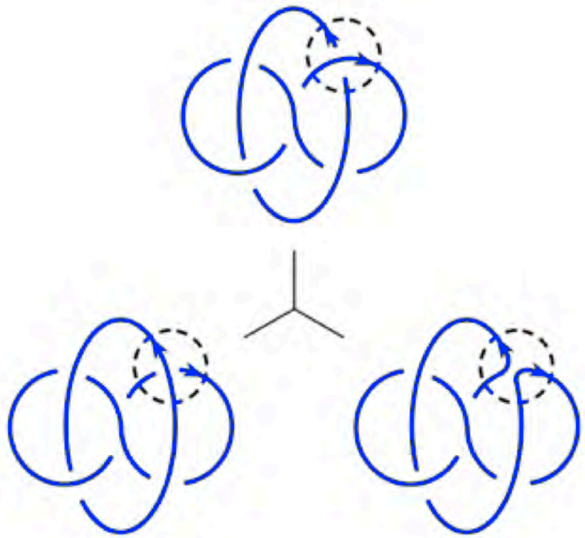
Right trefoil

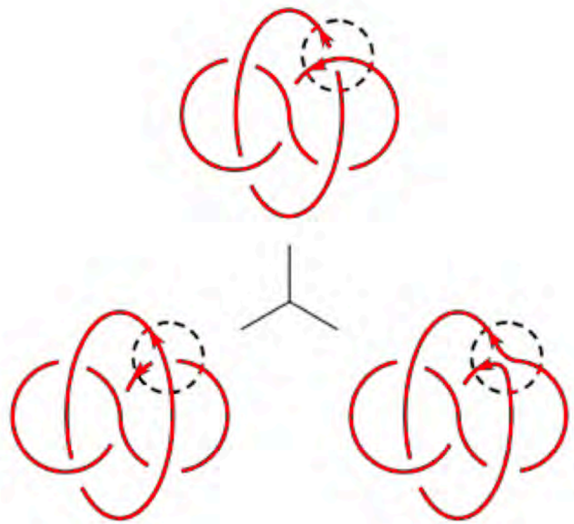


$$V(\boldsymbol{T}^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$

$$V(\boldsymbol{T}^R) = \tau + \tau^3 - \tau^4$$

Whitead link





$$V(\boldsymbol{W}) = \tau^{-\frac{7}{2}} - 2\tau^{-\frac{5}{2}} + \tau^{-\frac{3}{2}} - 2\tau^{-\frac{1}{2}} + \tau^{\frac{1}{2}} - \tau^{\frac{3}{2}}$$

From Leonardo to visiometrics

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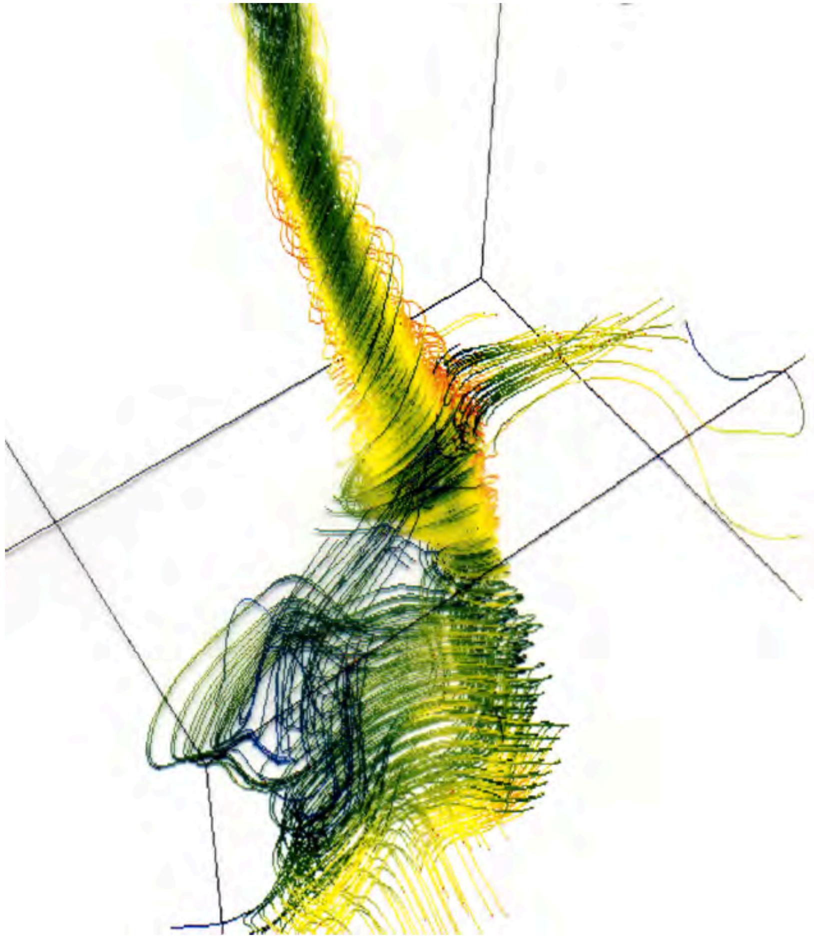
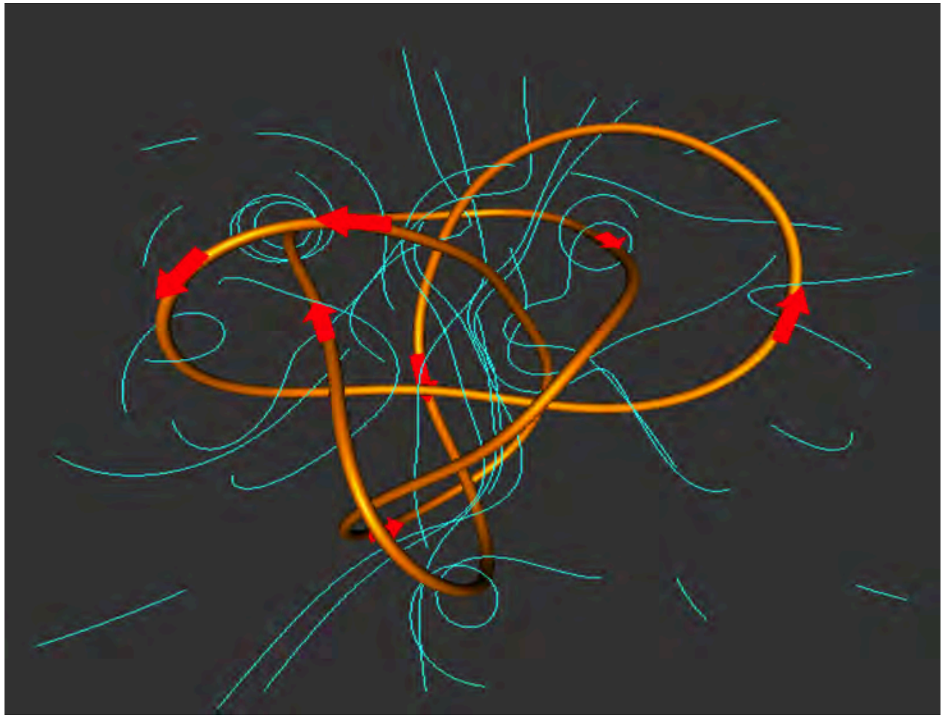
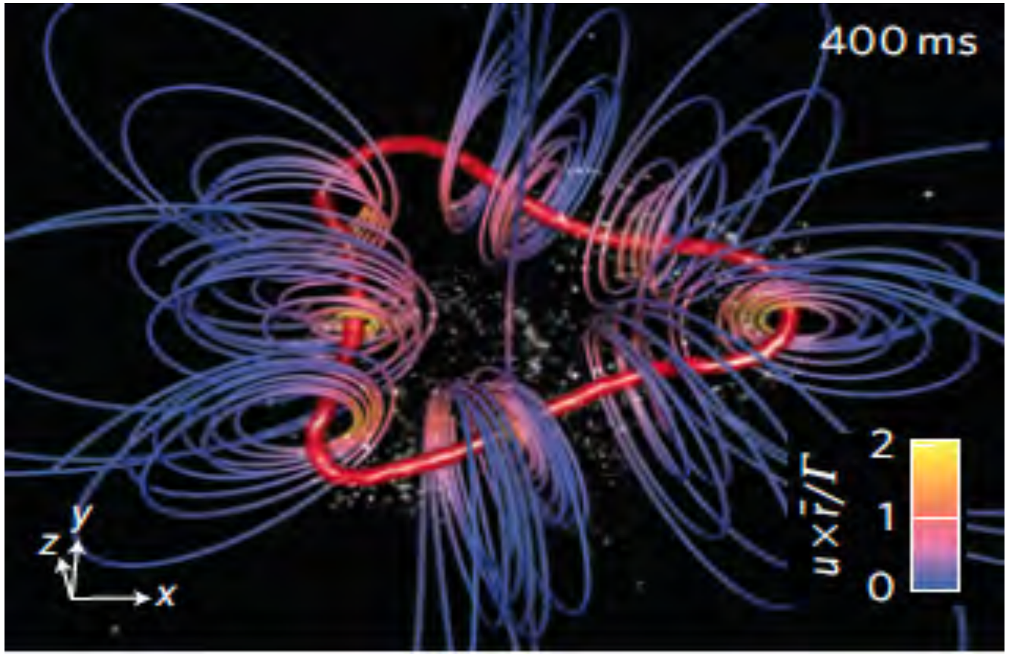


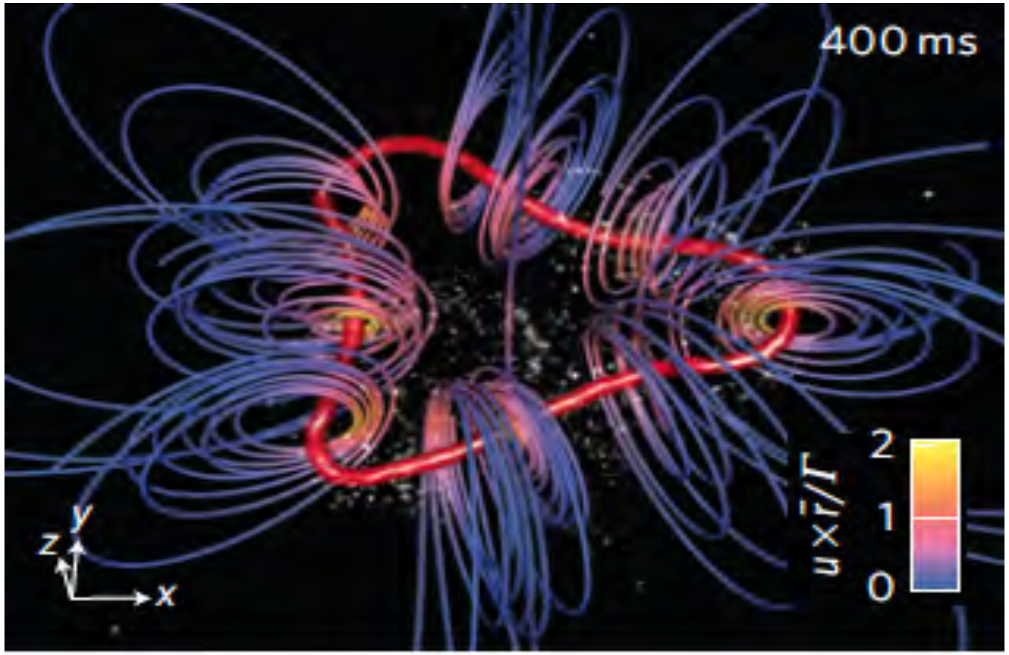
From Leonardo to visiometrics

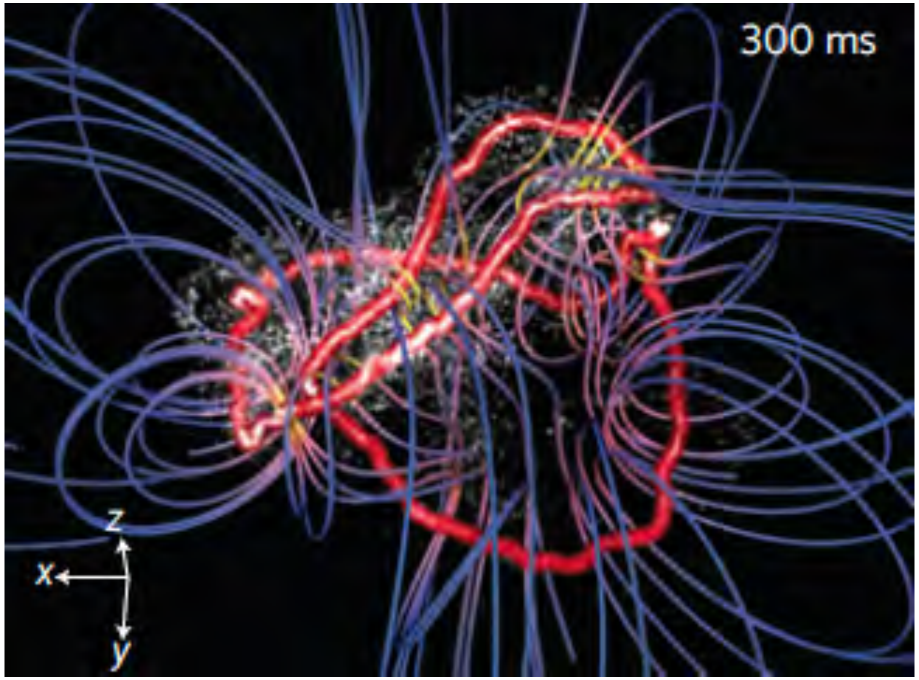


Examples of topological visiometrics

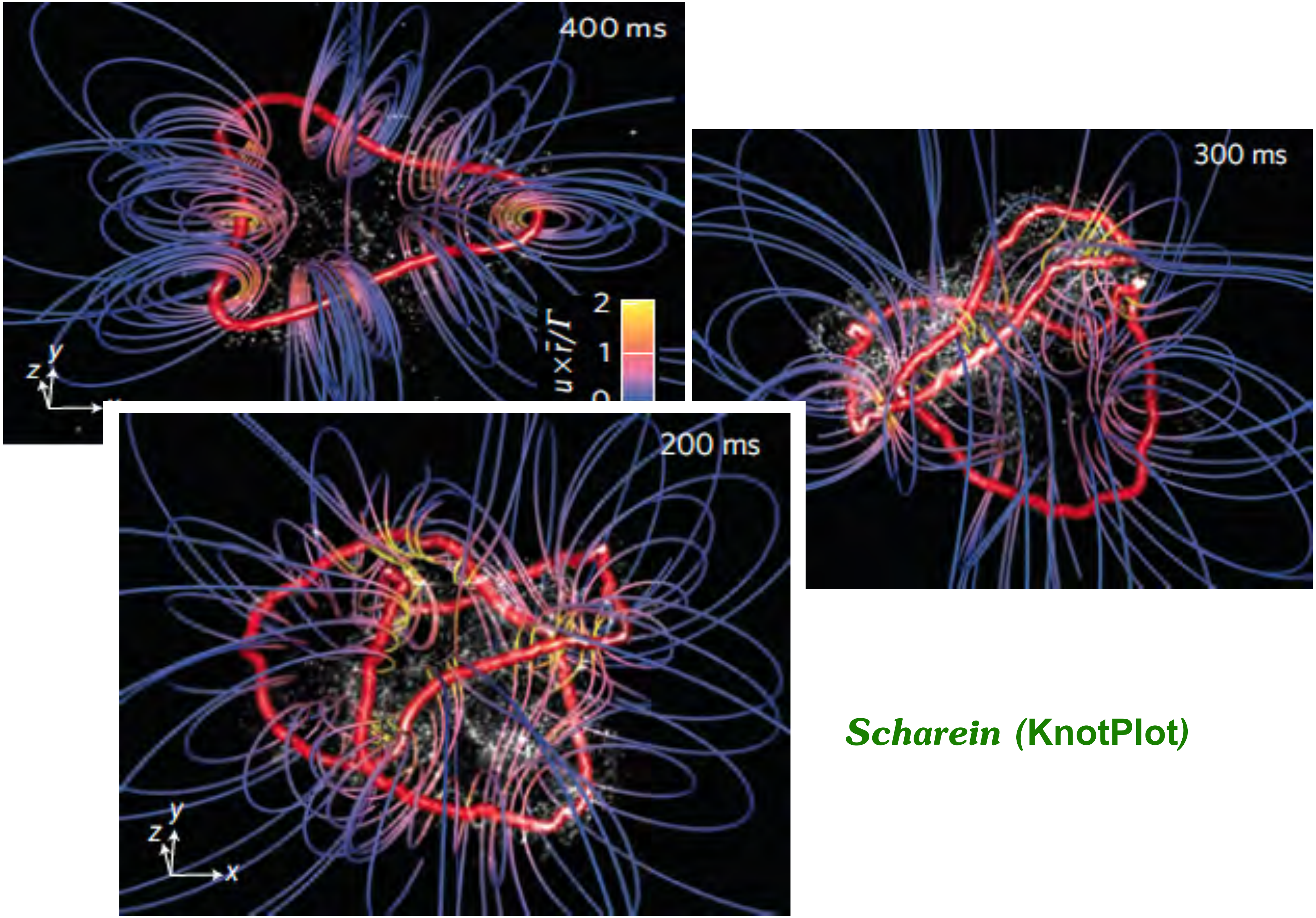


Examples of topological visiometrics



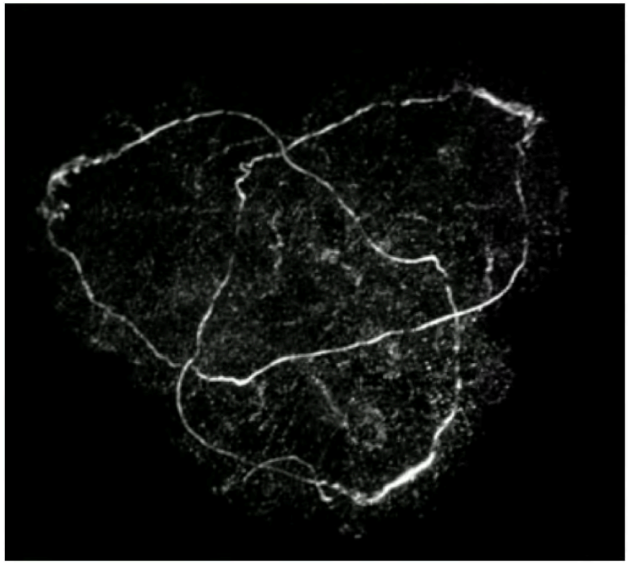


Examples of topological visiometrics



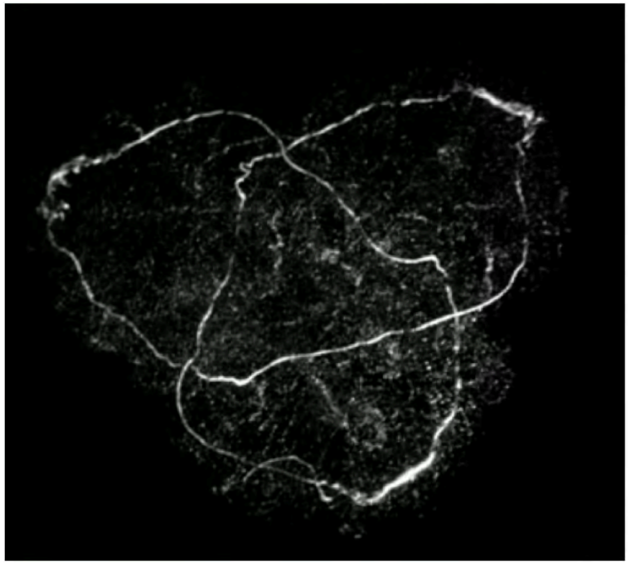
Scharein **(KnotPlot)**

Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)



$t = 1$

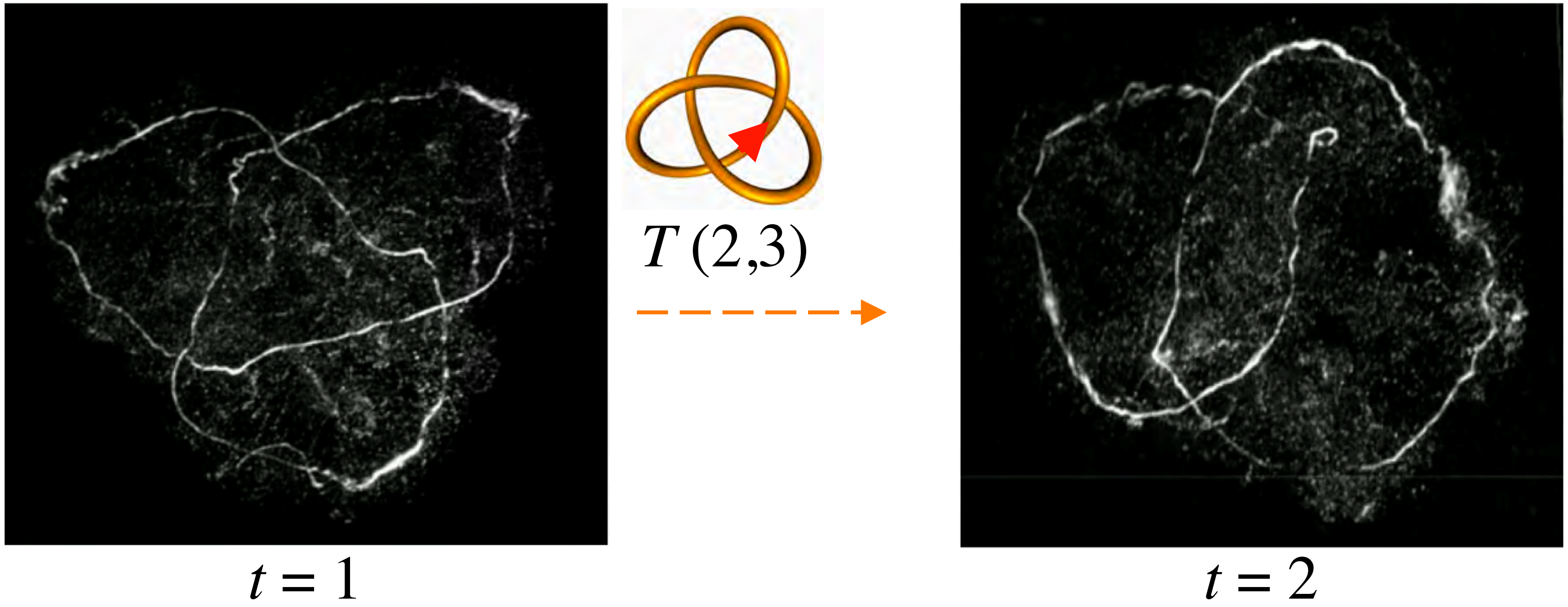
Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)



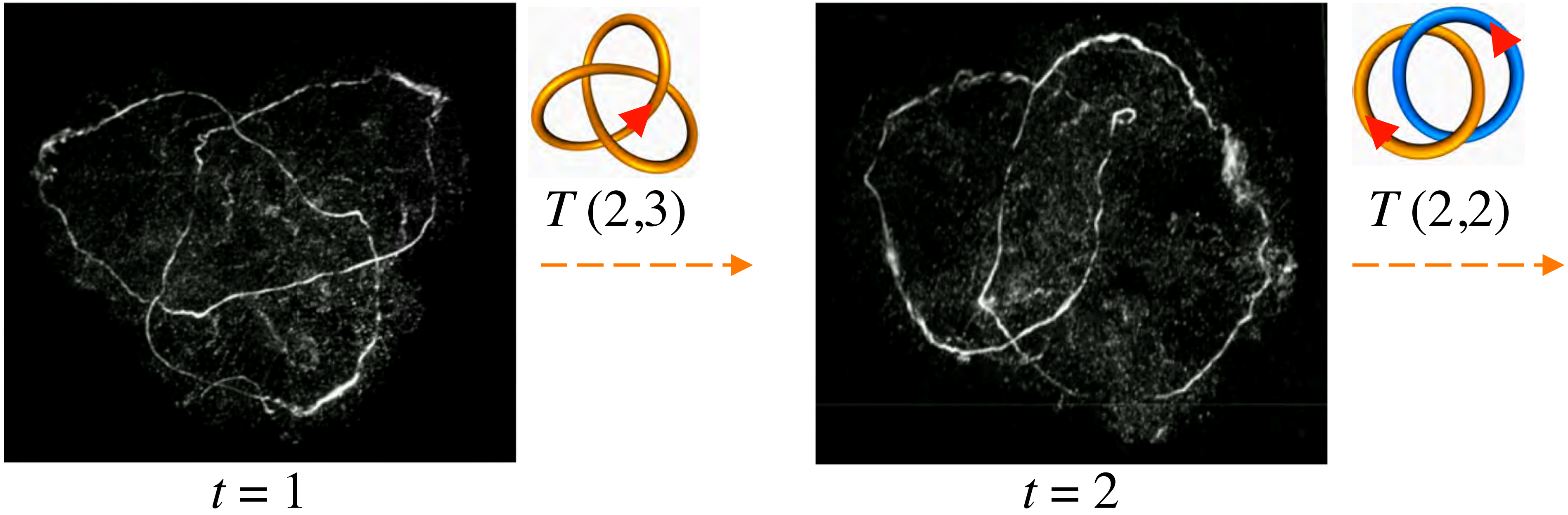
$t = 1$

$T\,(2,3)$

Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)



Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)



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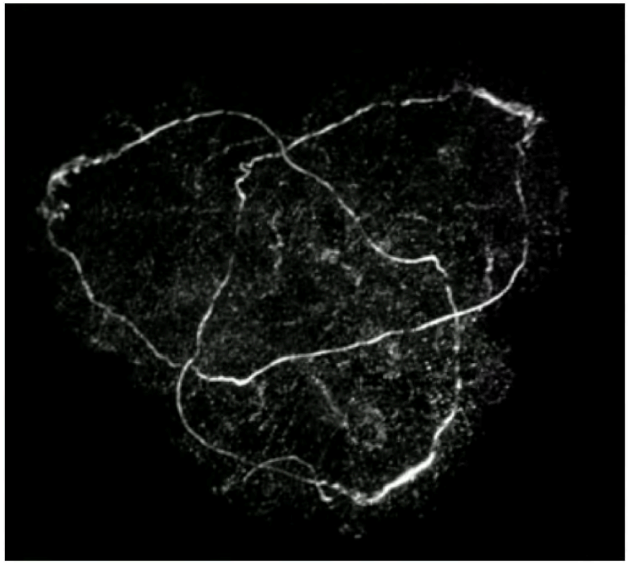
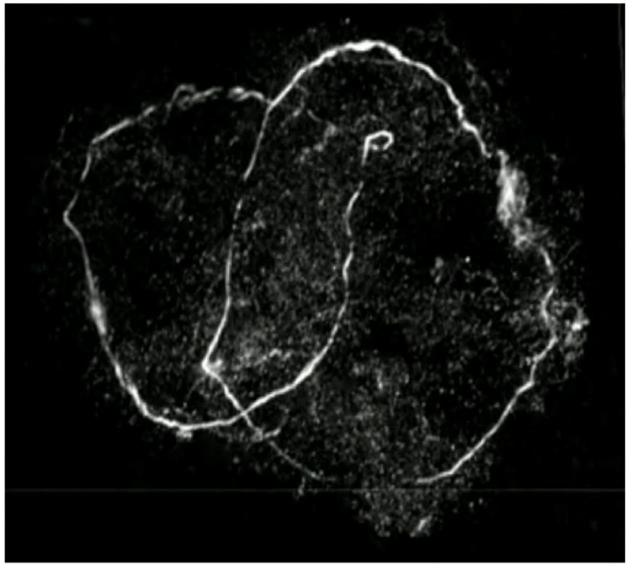
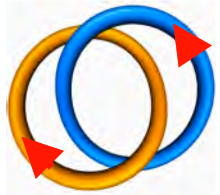


$t = 1$

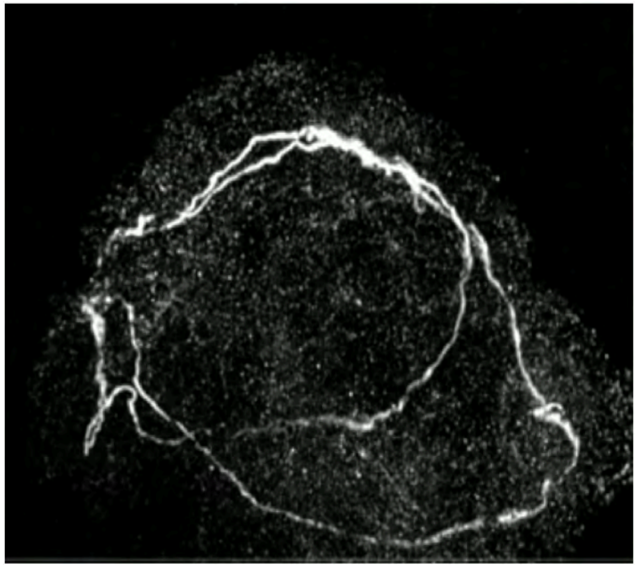
$T\,(2,3)$



$t = 2$

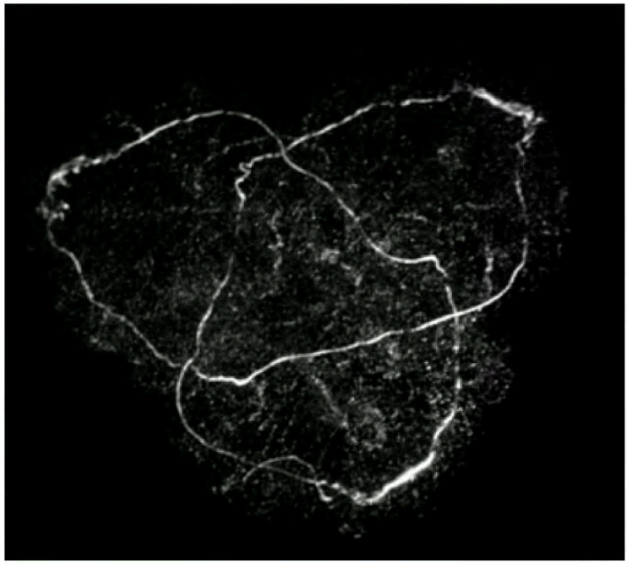


$T\,(2,2)$



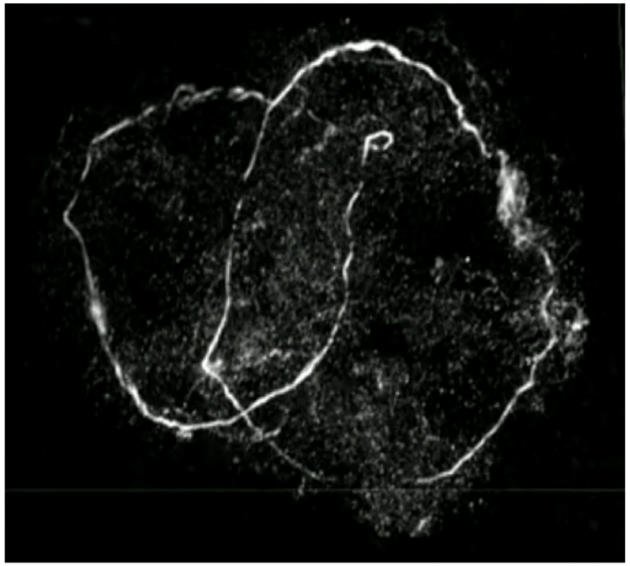
$t = 3$

Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)

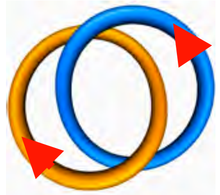


$t = 1$

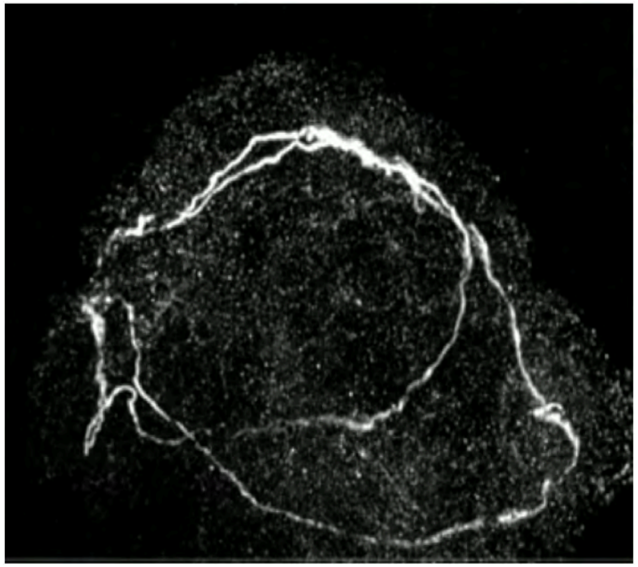
$T\,(2,3)$



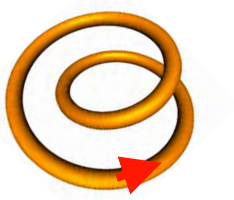
$t = 2$



$T\,(2,2)$

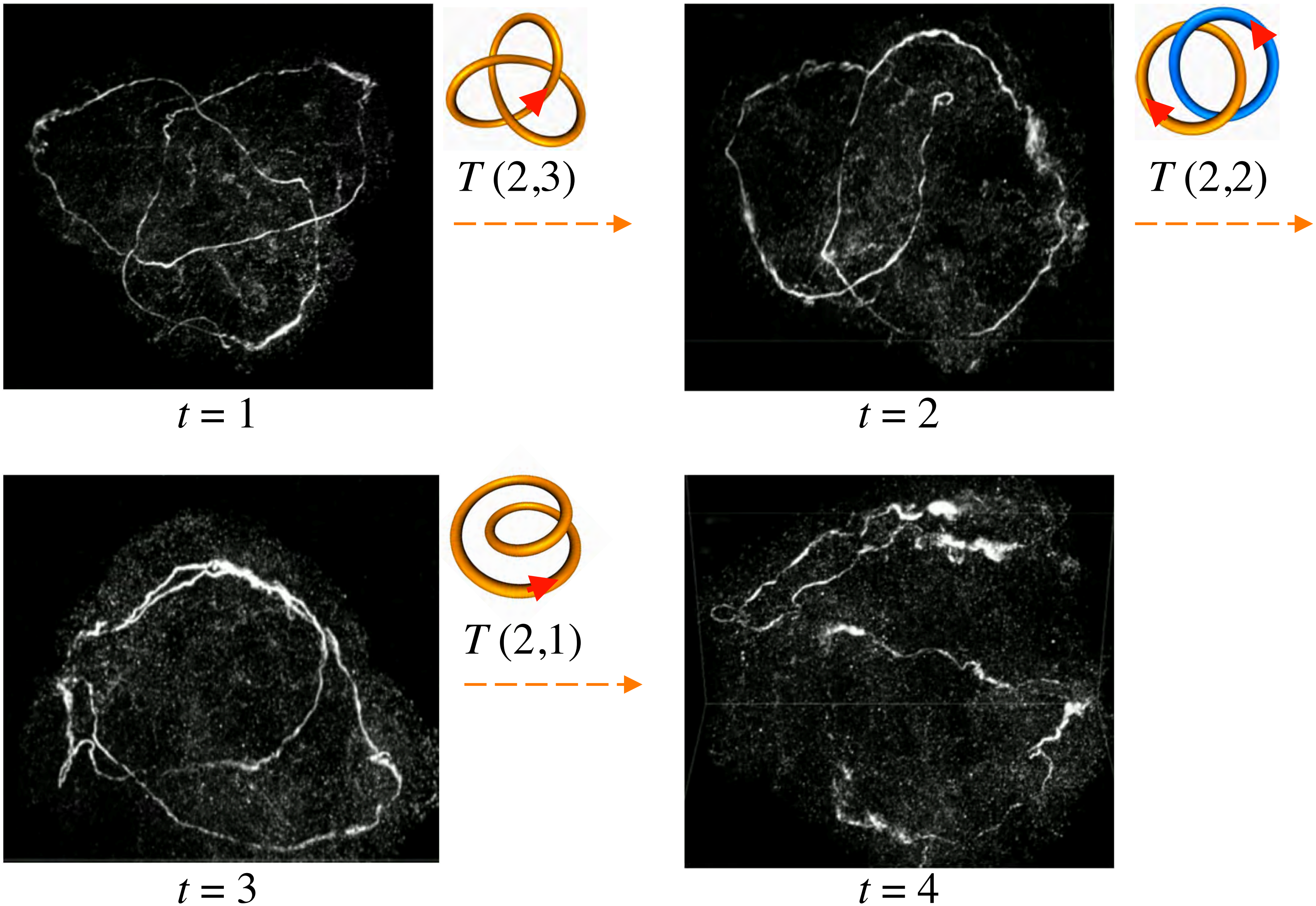


$t = 3$

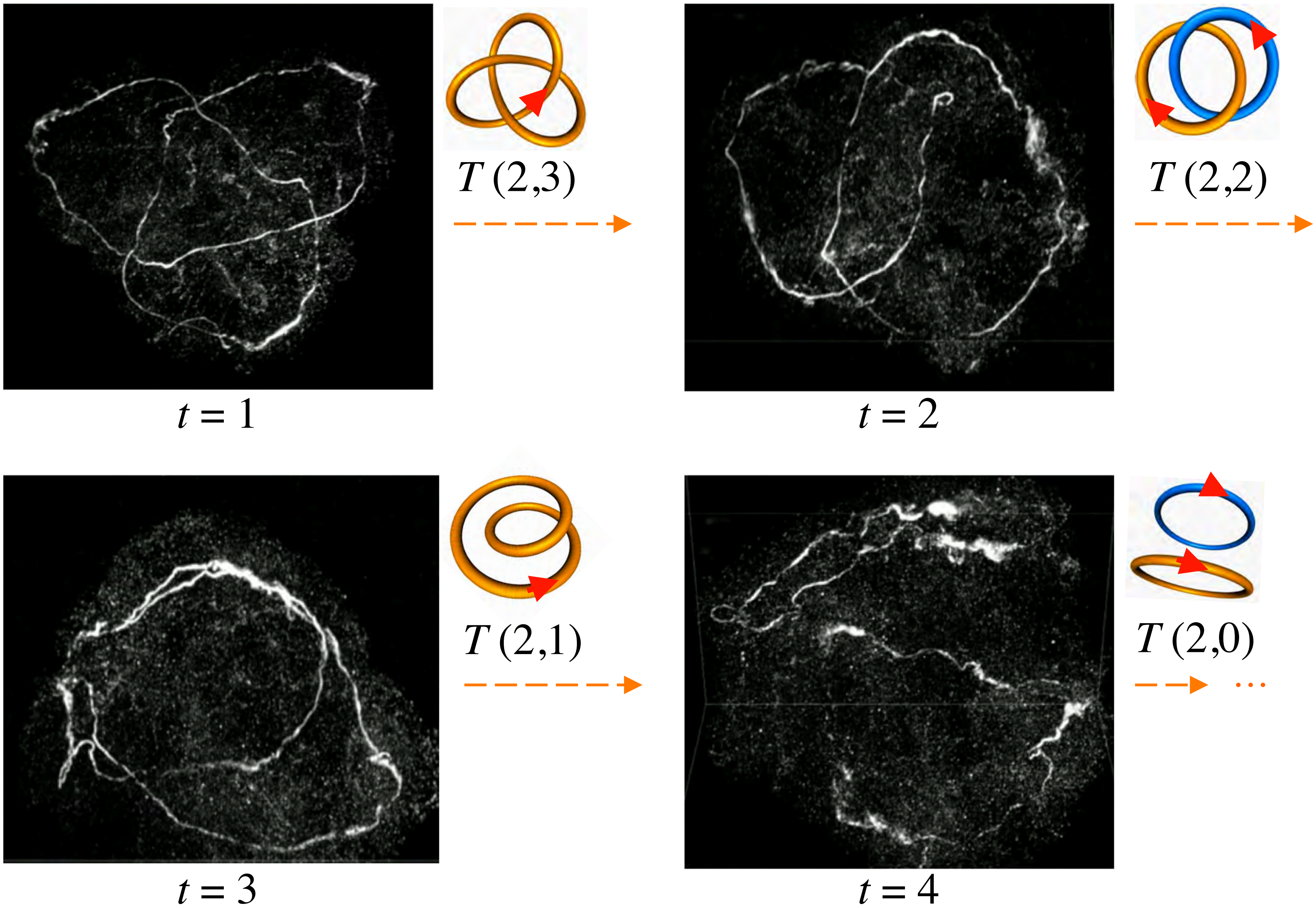


$T\,(2,1)$

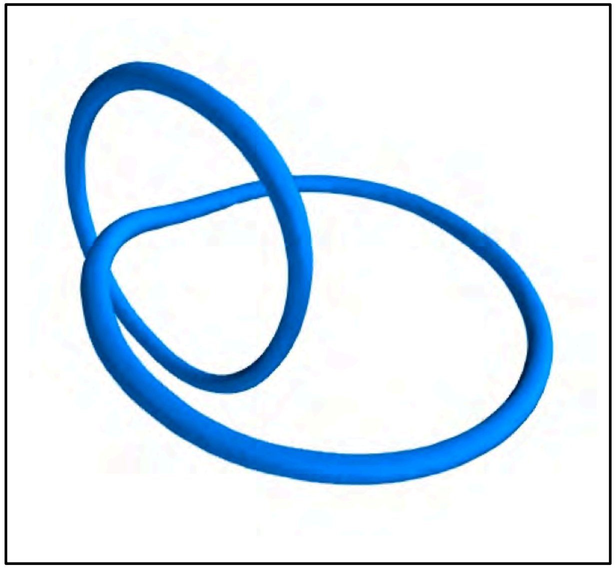
Production of a vortex trefoil knot in water *(Kleckner & Irvine, 2013)*



Production of a vortex trefoil knot in water (Kleckner & Irvine, 2013)

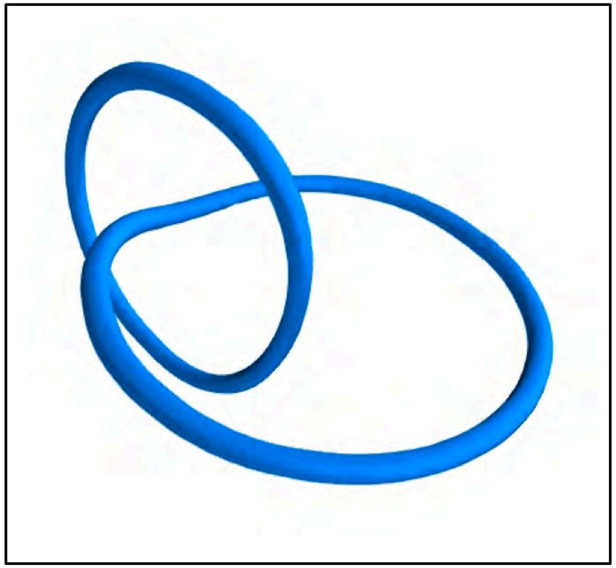


Production of a vortex link in BECs (Zuccher & Ricca, submitted)



$t = 1$

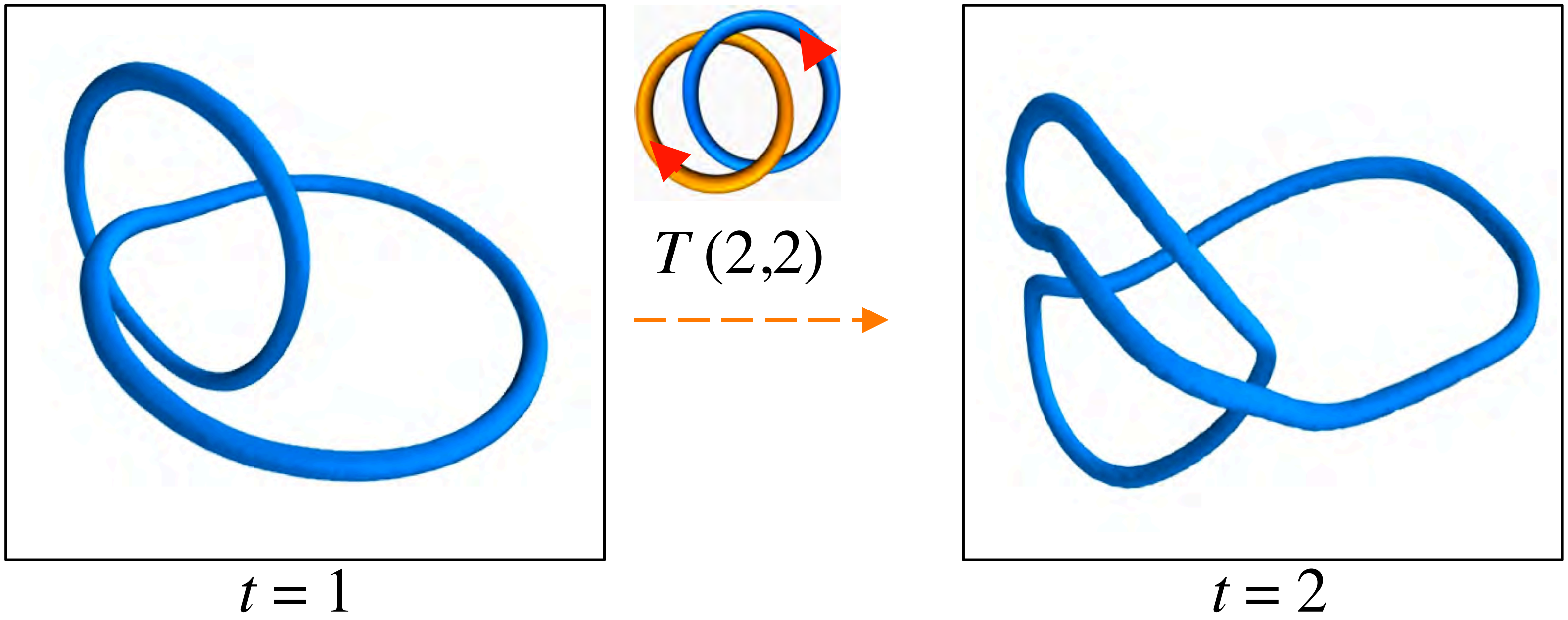
Production of a vortex link in BECs (Zuccher & Ricca, submitted)



$t = 1$

$T\,(2,2)$

Production of a vortex link in BECs (Zuccher & Ricca, submitted)



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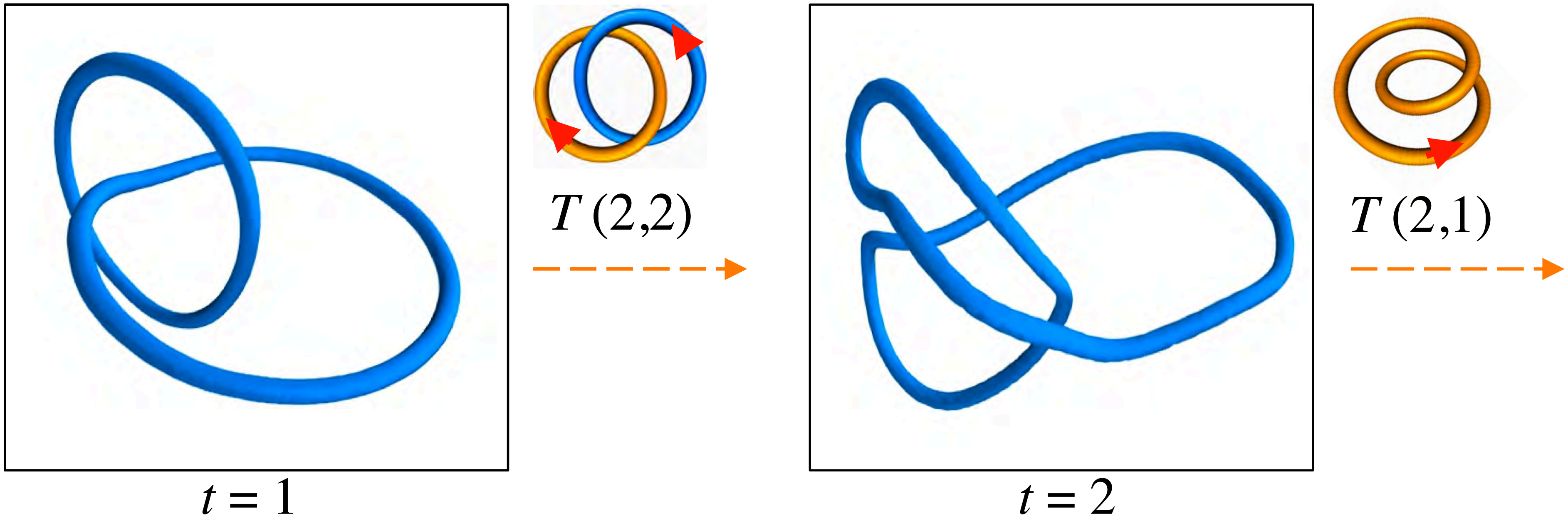

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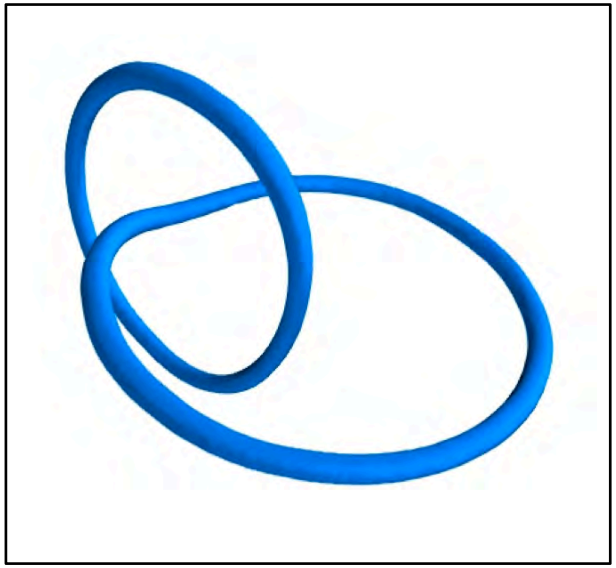
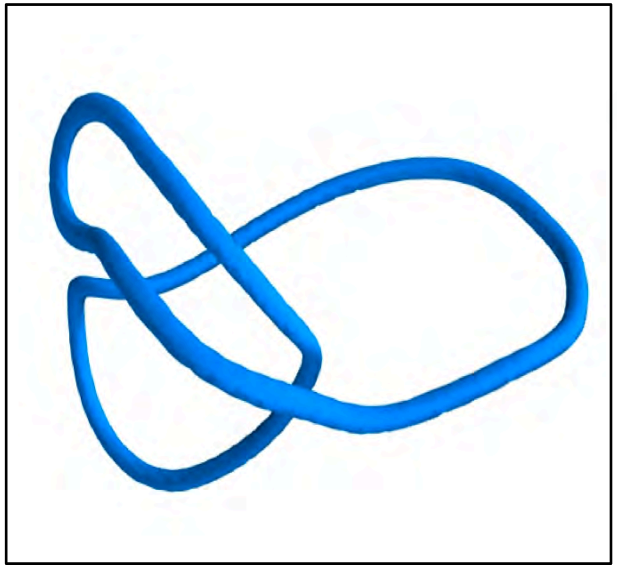


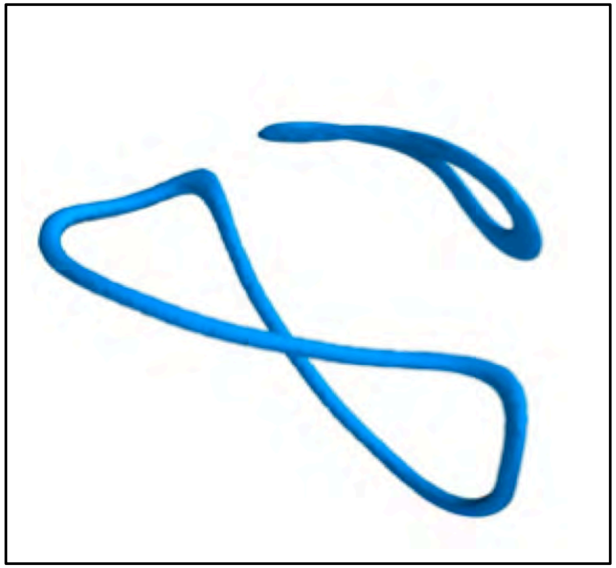
$t = 1$

$T\,(2,2)$



$t = 2$

$T\,(2,1)$



$t = 3$

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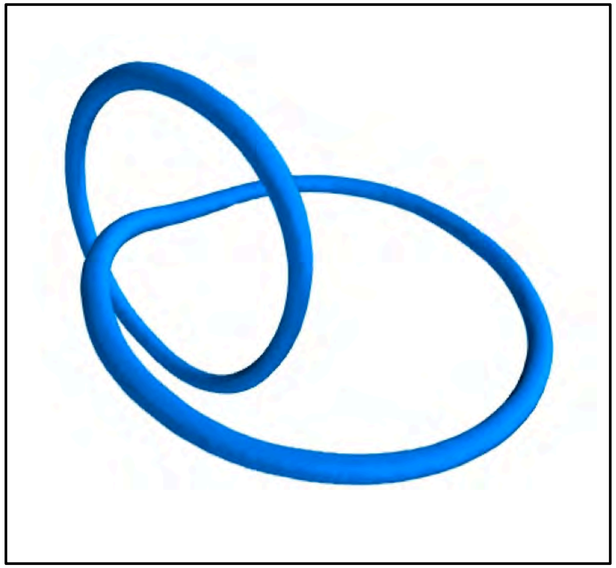
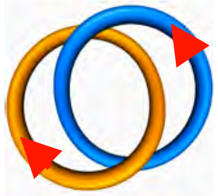
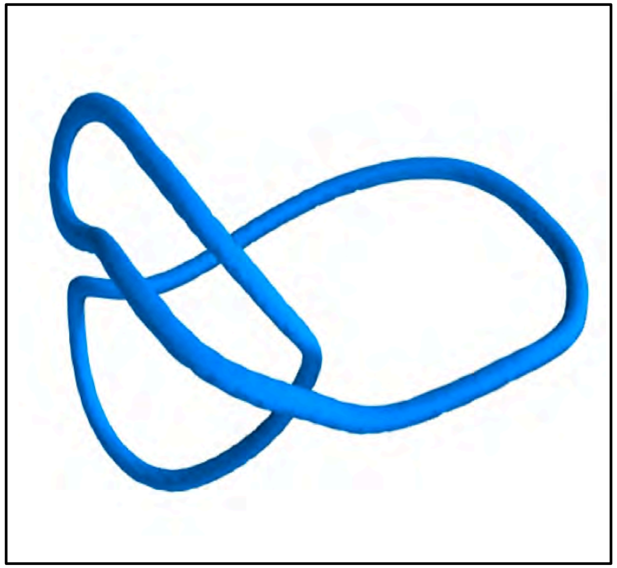


$t = 1$

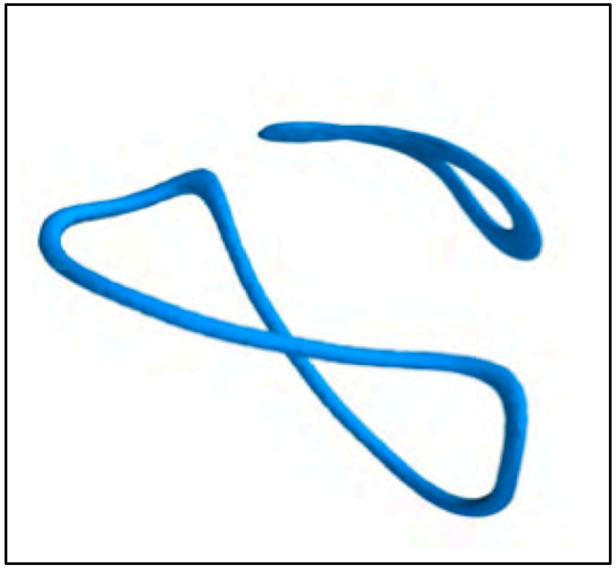


$T\,(2,2)$



$t = 2$

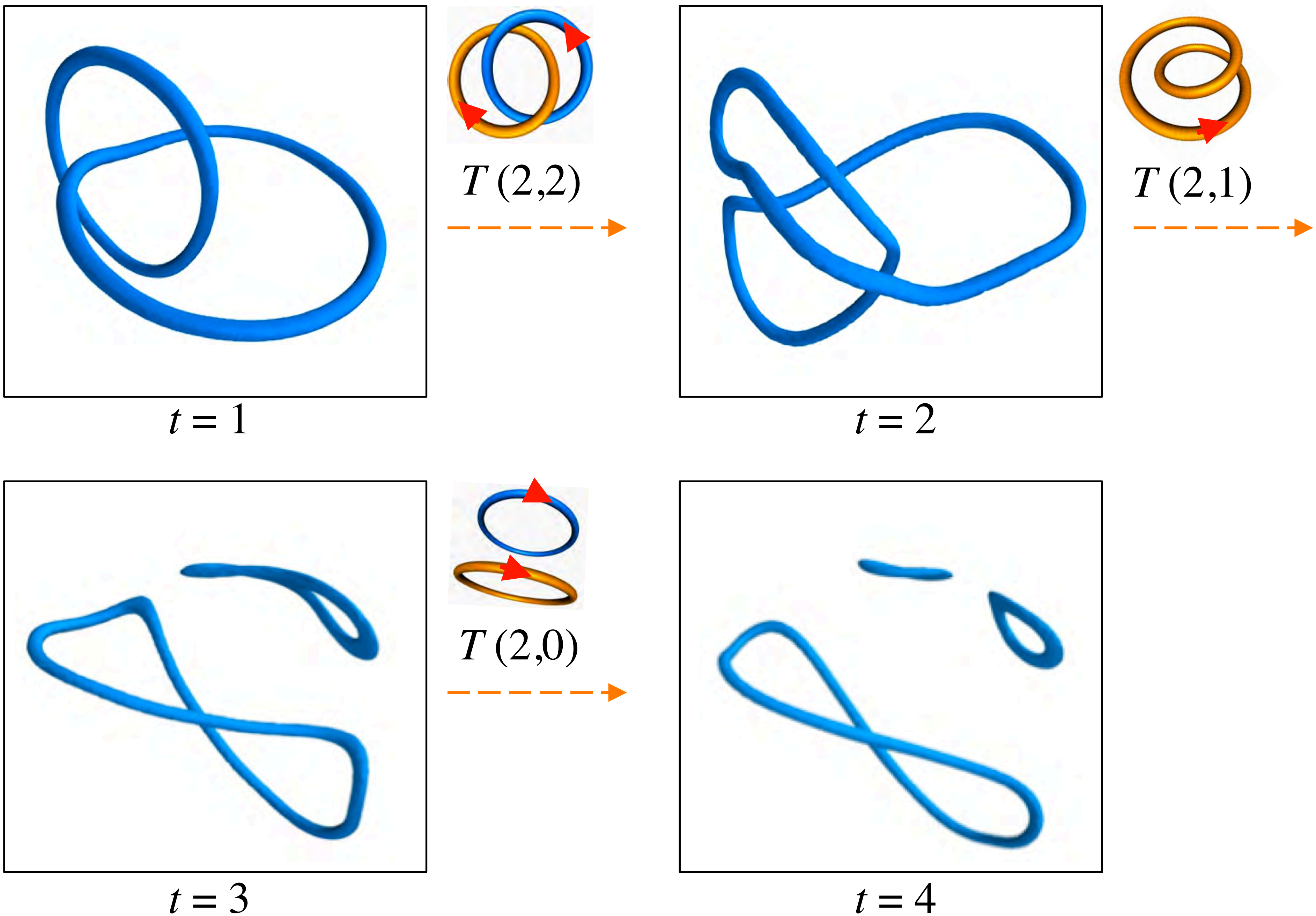
$T\,(2,1)$



$t = 3$

$T\,(2,0)$

Production of a vortex link in BECs (Zuccher & Ricca, submitted)



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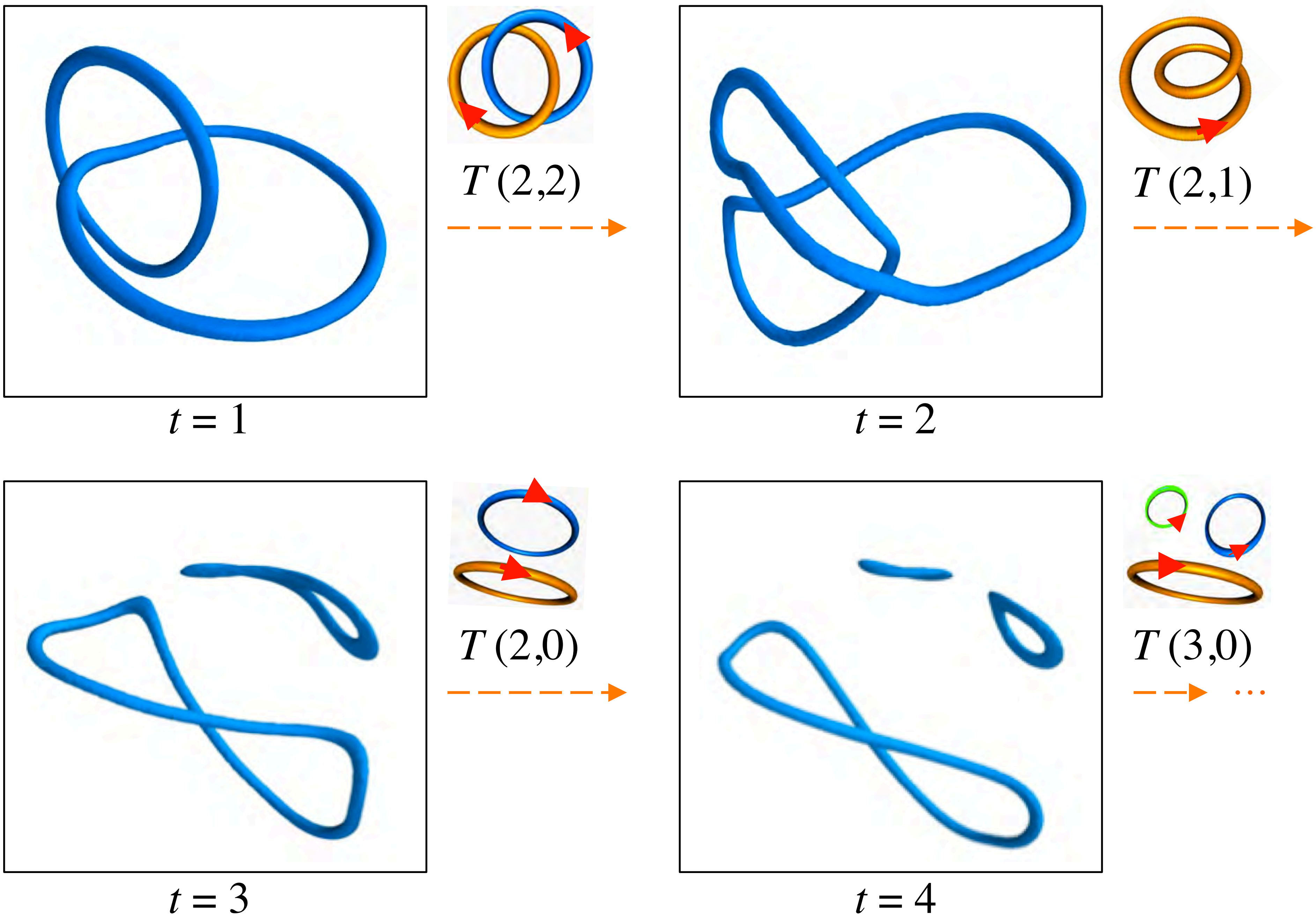


Ideal torus knots & links cascade

Consider the cascade process:

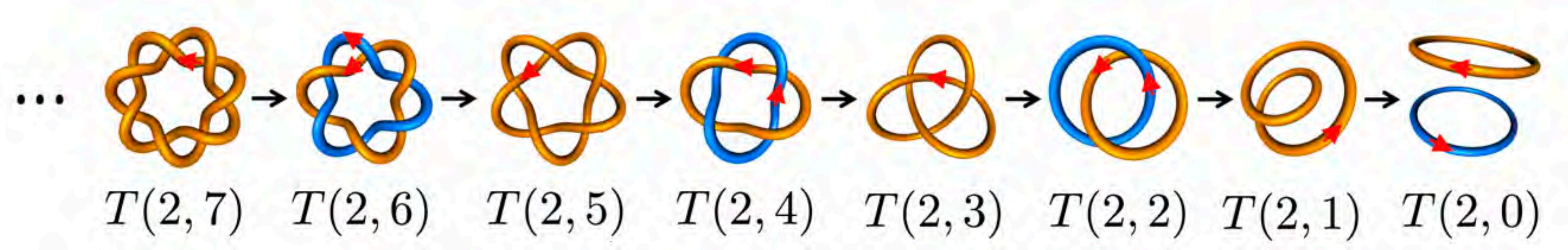
(i)

Ideal torus knots & links cascade

Consider the cascade process:

(i)

(ii)



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Consider the cascade process:

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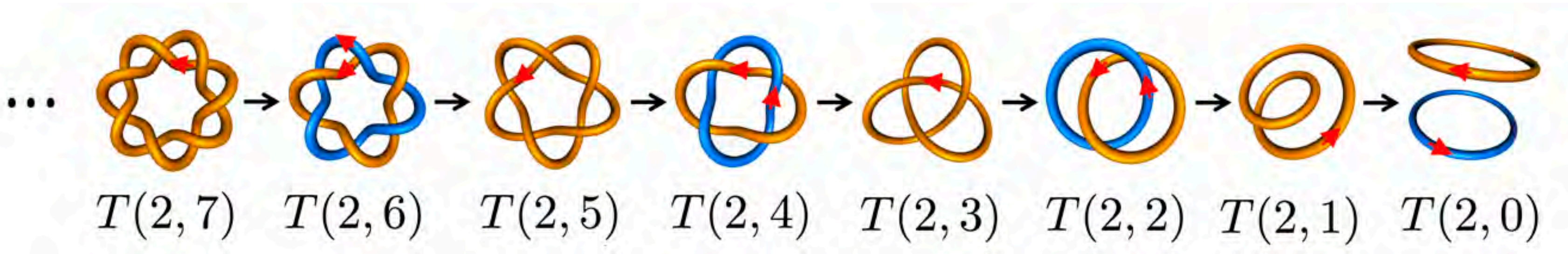

$$\{T(2,n)\}: \quad \ldots \to T(2,2n+1) \to T(2,n) \to \ldots \to T(2,0) \; .$$

Ideal torus knots & links cascade

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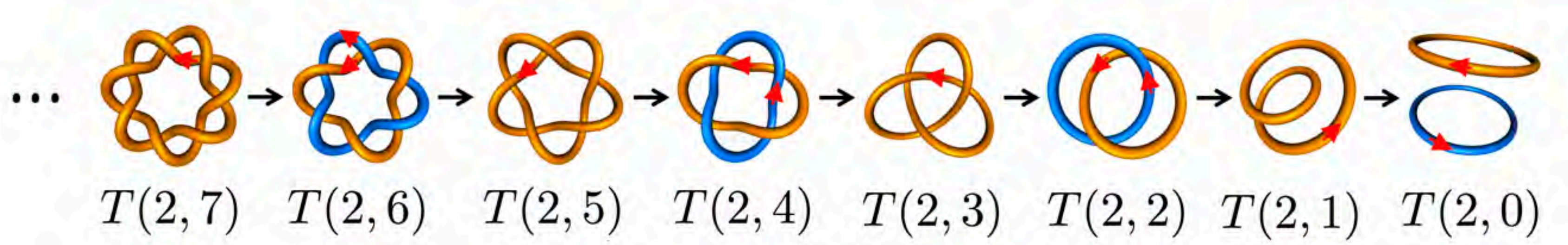

$$\{T(2,n)\}: \quad \ldots \to T(2,2n+1) \to T(2,n) \to \ldots \to T(2,0)\ .$$

Assumptions:

- ***all torus knots*** $T(2,2n+1)$ ***and links*** $T(2,2n)$ ***are standardly embedded on a mathematical torus in closed braid form;***

Ideal torus knots & links cascade

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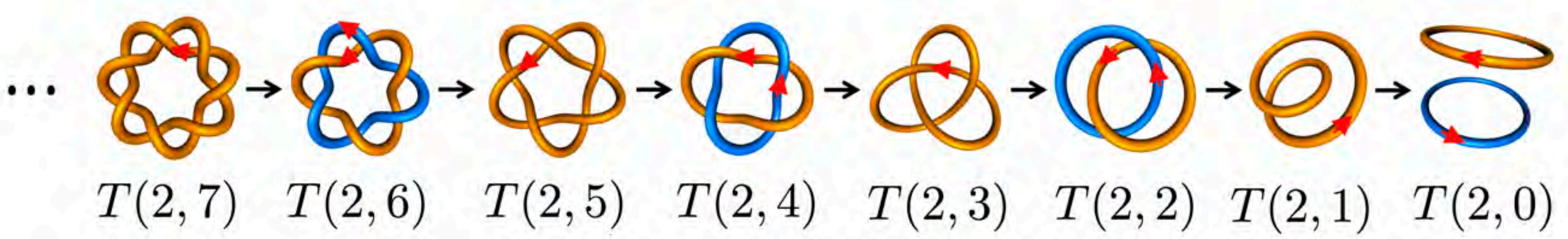

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Assumptions:

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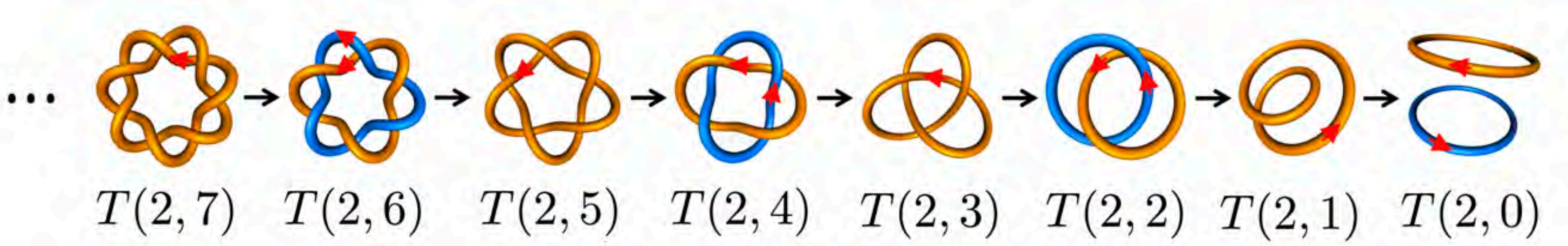
- ***all torus knots and links form an ordered set*** $\{T(2,n)\}$ ***of elements listed according to their decreasing value of topological complexity given by*** $c_{\min} = n$ ***;***

Ideal torus knots & links cascade

Consider the cascade process:

(i)

(ii)



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Assumptions:

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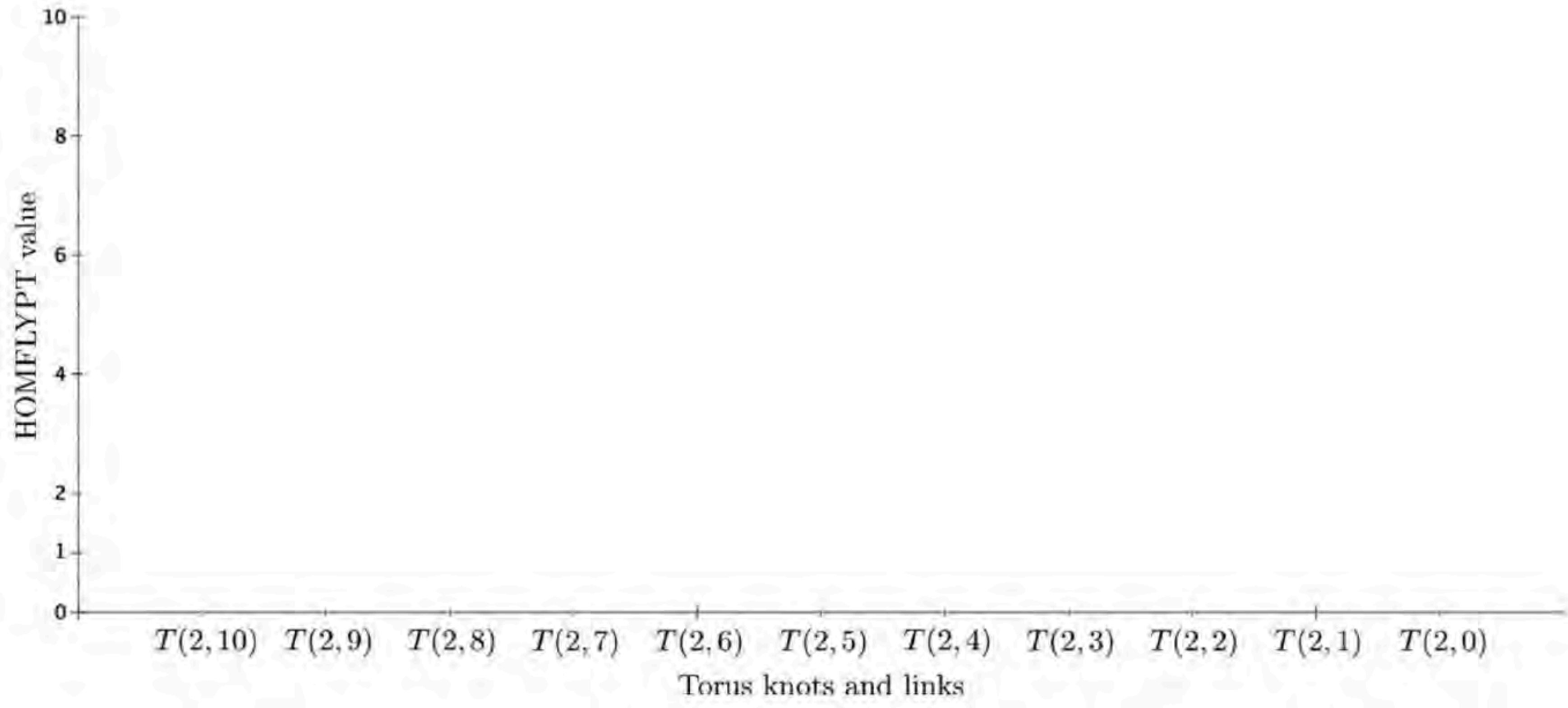
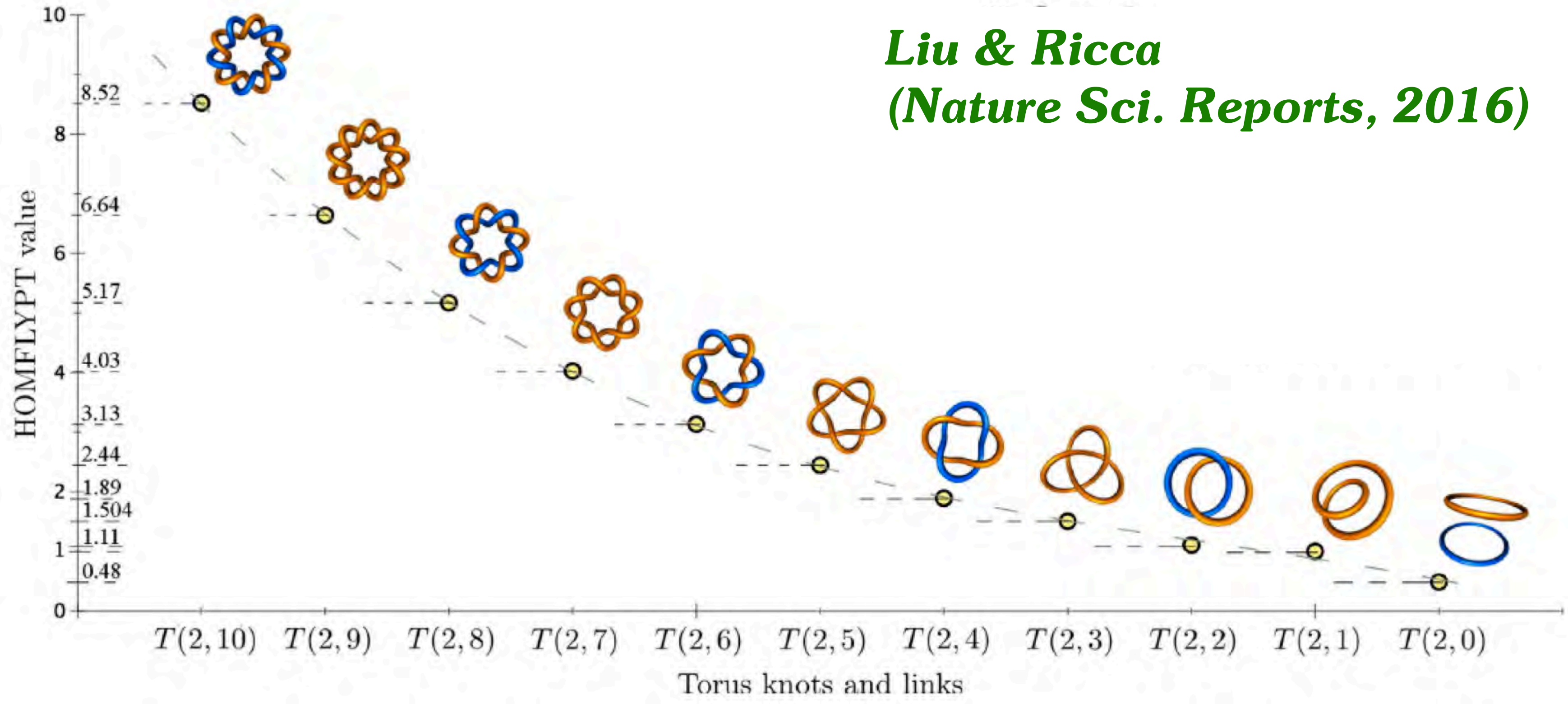
- ***all torus knots and links form an ordered set*** $\{T(2,n)\}$ ***of elements listed according to their decreasing value of topological complexity given by*** $c_{\min} = n$ ***;***
- ***any topological transition between two contiguous elements of*** $\{T(2,n)\}$ ***is determined by a single, orientation-preserving reconnection event.***

HOMFLYPT is the best quantifier of topological complexity



HOMFLYPT is the best quantifier of topological complexity

Liu & Ricca
(Nature Sci. Reports, 2016)



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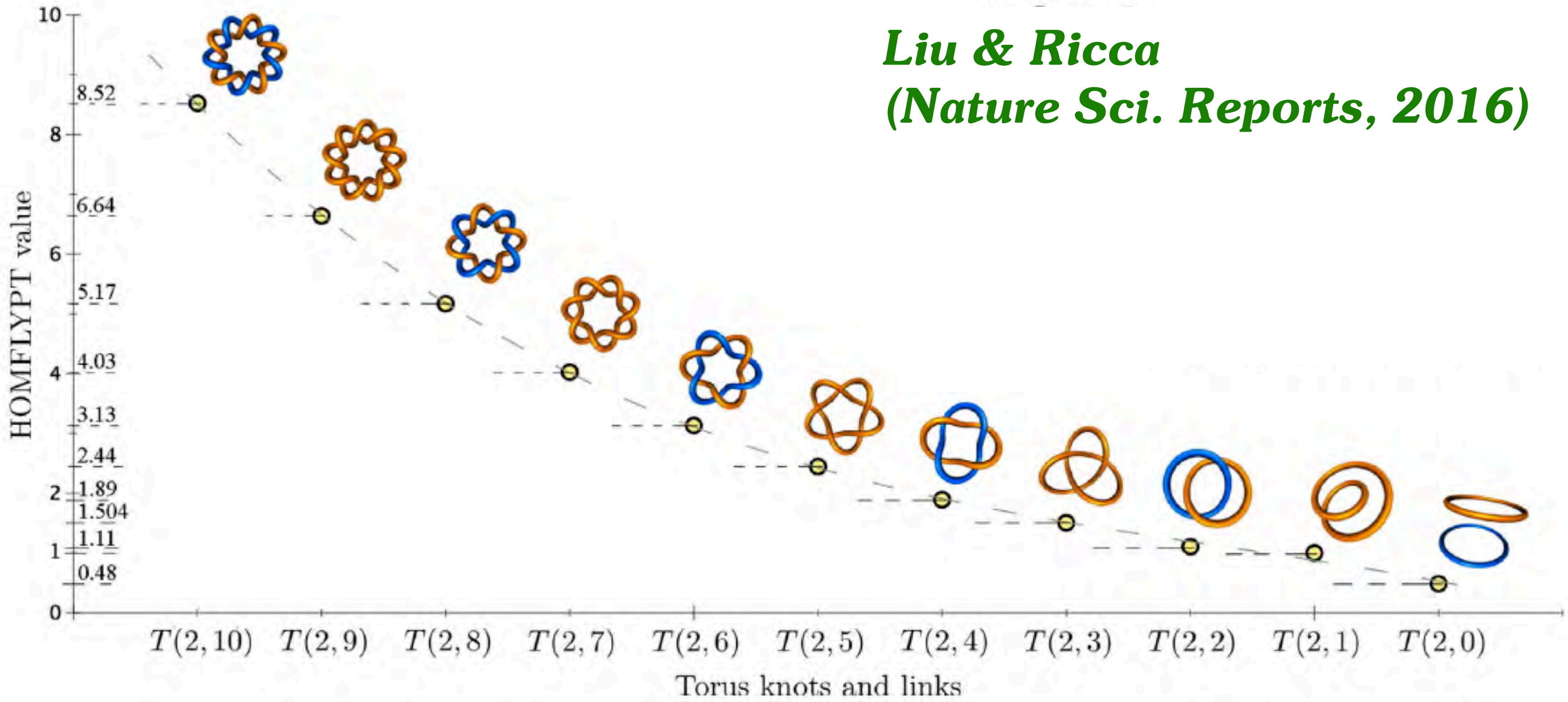
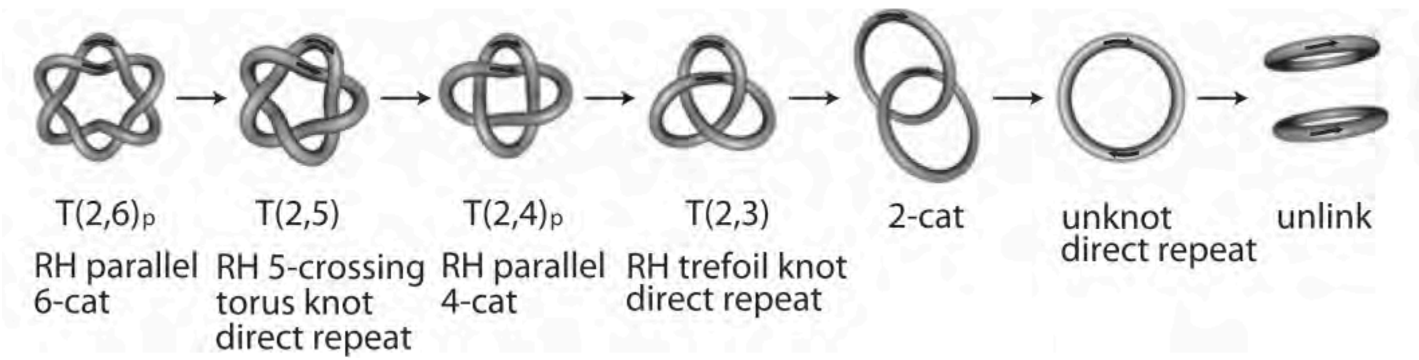


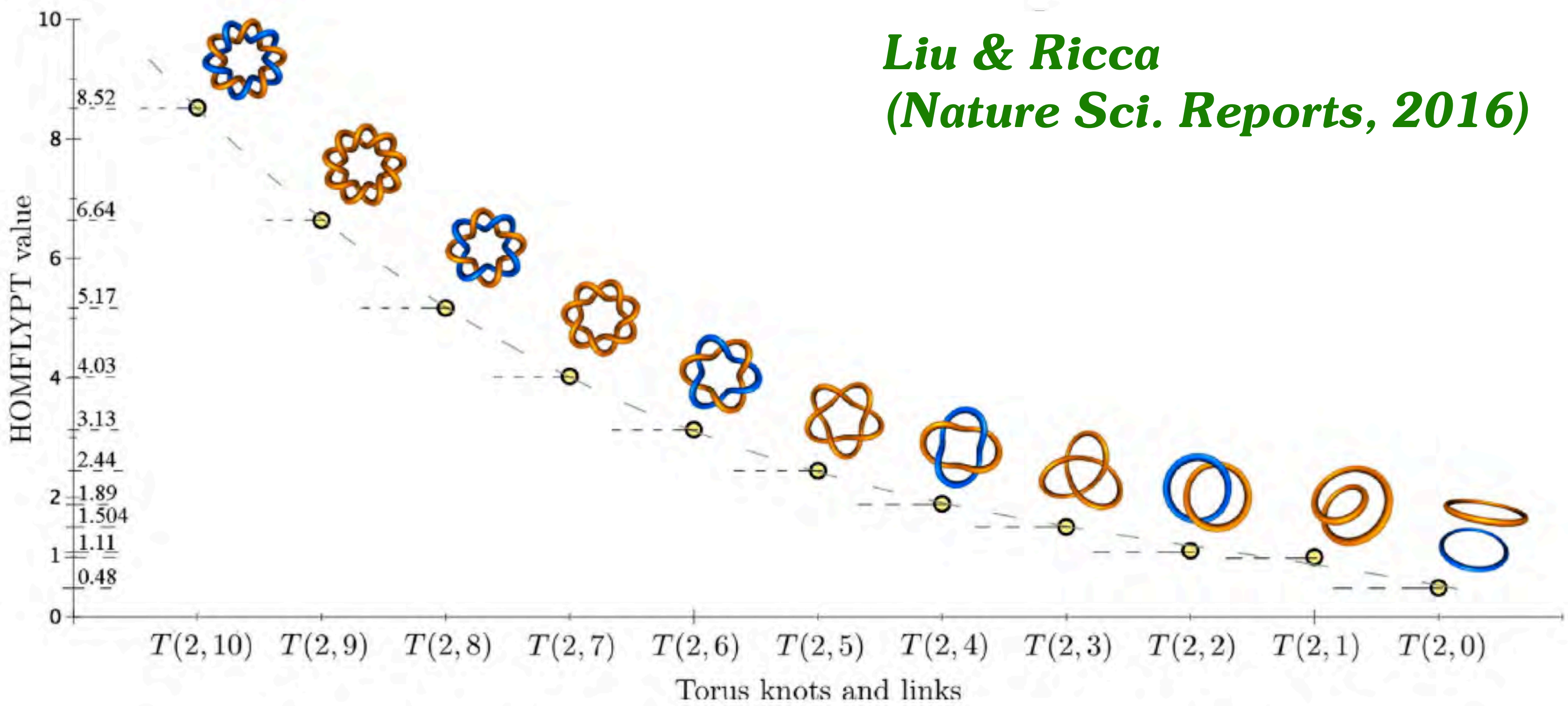
Liu & Ricca
(Nature Sci. Reports, 2016)

- ***Similar cascade process in E-coli DNA replication:***



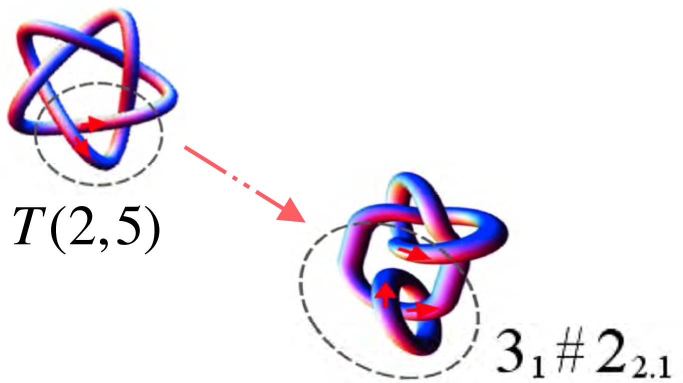
(Shimokawa et al., 2013)

HOMFLYPT is the best quantifier of topological complexity



Liu & Ricca
(Nature Sci. Reports, 2016)

- ***Optimal path to cascade?***



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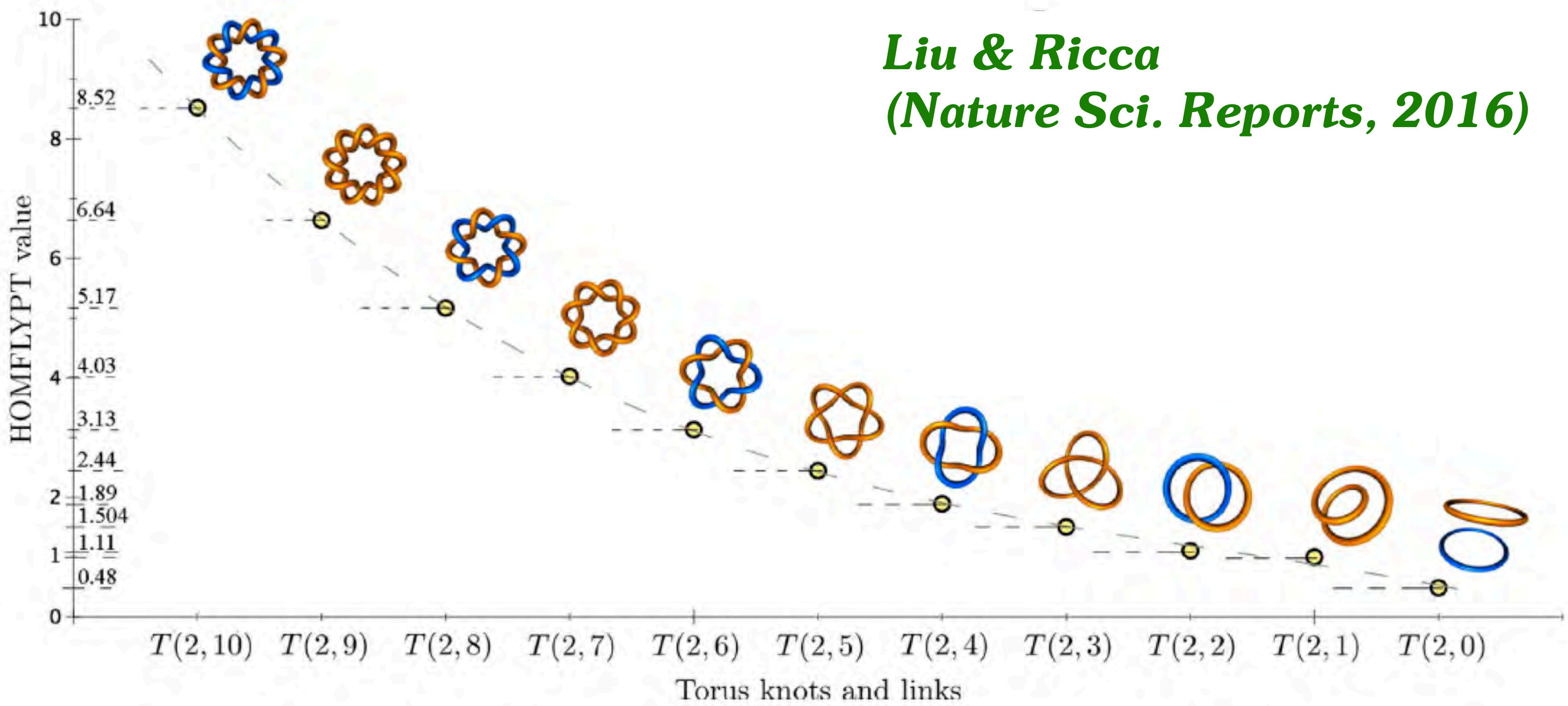
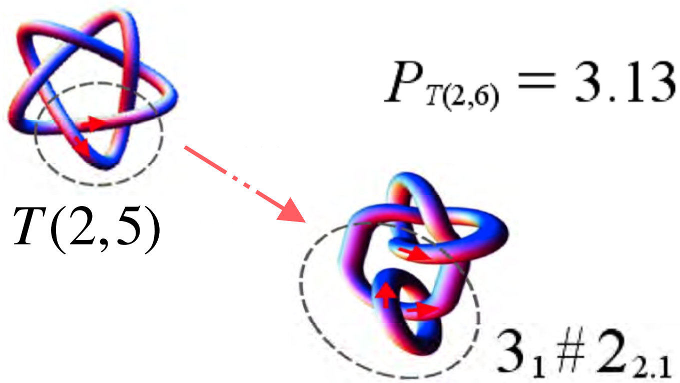


- *Optimal path to cascade?*

$P_{T(2,6)} = 3.13 \rightarrow P_{T(2,5)} = 2.44 \rightarrow P_{T(2,4)} = 1.89 \rightarrow P_{T(2,3)} = 1.50$



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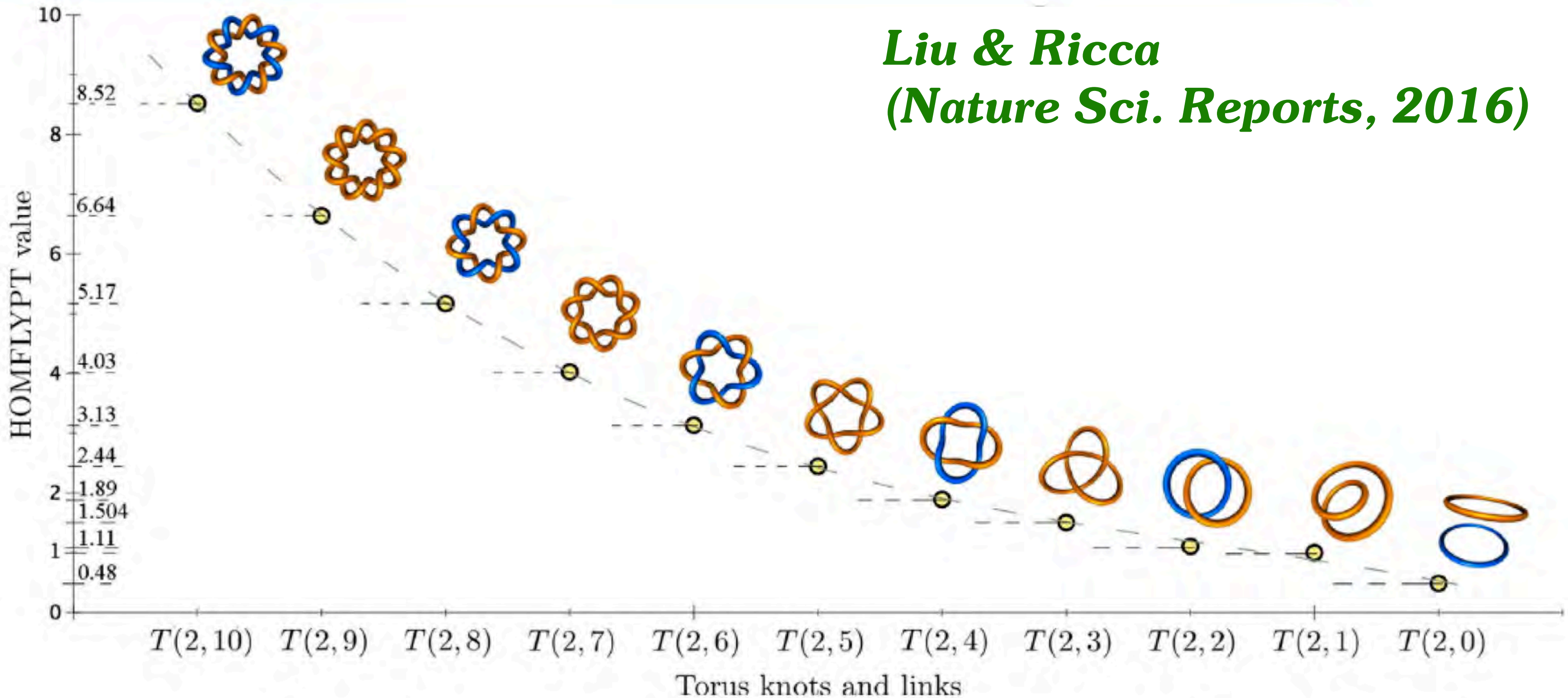
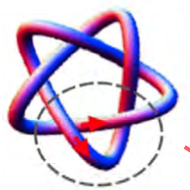
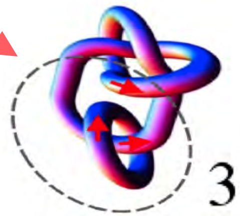


Liu & Ricca
(Nature Sci. Reports, 2016)

- ***Optimal path to cascade?***



$T(2,5)$



$3_1\#2_{2.1}$

$$P_{T(2,5)} = 2.44 \rightarrow P_{T(2,4)} = 1.89$$

$$P_{T(2,6)} = 3.13 \qquad\qquad P_{T(2,3)} = 1.50$$

$$P_{3_1\#3_1} = 2.26 \rightarrow P_{3_1\#2_{2.1}} = 1.66$$

End of Lecture 2

Laboratory

- ***From Gauss coding to modern tabulation:***

  - Alexander-Briggs notation

  - Braid words, Gauss and Dowker-Thistlethwaite code

  - Jones polynomial

  - Ropelength, tight knots and ideal shapes

- **KnotAtlas** ***(Bar Natan, 2000, 2004):***

  - online database of knots and invariants

- **LinKnot** ***(Jablan-Razdanovic, 2006):***

  - online knot theory software

- **KnotPlot** ***(Scharein, 2011):***

  - visualization and mathematical exploration software

  - tangle calculator

  - dynamical systems interface

  - mathematical experimentation

Topological crossing number and knot types

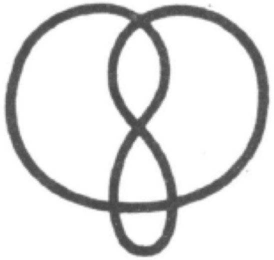
c_{min}	# *of knot types*
0	1
1	0
2	0
3	1
4	1
5	2
6	3
7	7
8	21
9	49
10	165
11	552
12	2176
13	9988
14	46,972
15	253,293
16	1,388,705

The standard table of knots (Reidemeister, Knotetheorie, 1932)

Knotentabelle.

Die Tabelle der folgenden Knotenprojektionen bis zu neun Überkreuzungen wurde der Arbeit von ALEXANDER und BRIGGS (5) entnommen. Verbessert wurden die Kurven 8_4 und 9_7, bei denen die Anzahl der Überkreuzungen nicht stimmte.

3_1



4_1

5_1

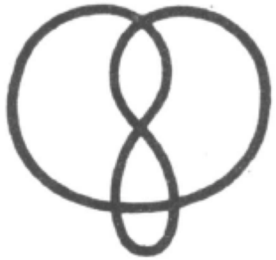
5_2

The standard table of knots (Reidemeister, Knotetheorie, 1932)

Knotentabelle.

Die Tabelle der folgenden Knotenprojektionen bis zu neun Überkreuzungen wurde der Arbeit von ALEXANDER und BRIGGS (5) entnommen. Verbessert wurden die Kurven 8_4 und 9_7, bei denen die Anzahl der Überkreuzungen nicht stimmte.

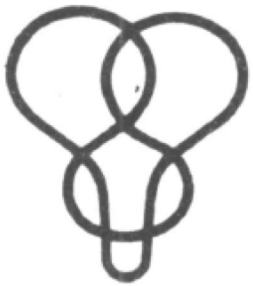
3_1



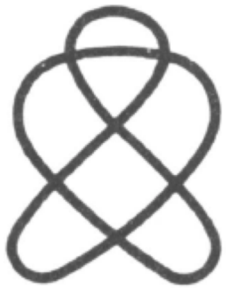
4_1

5_1

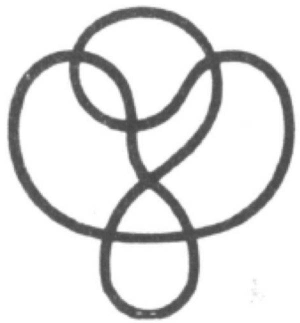
5_2



6_1



6_2

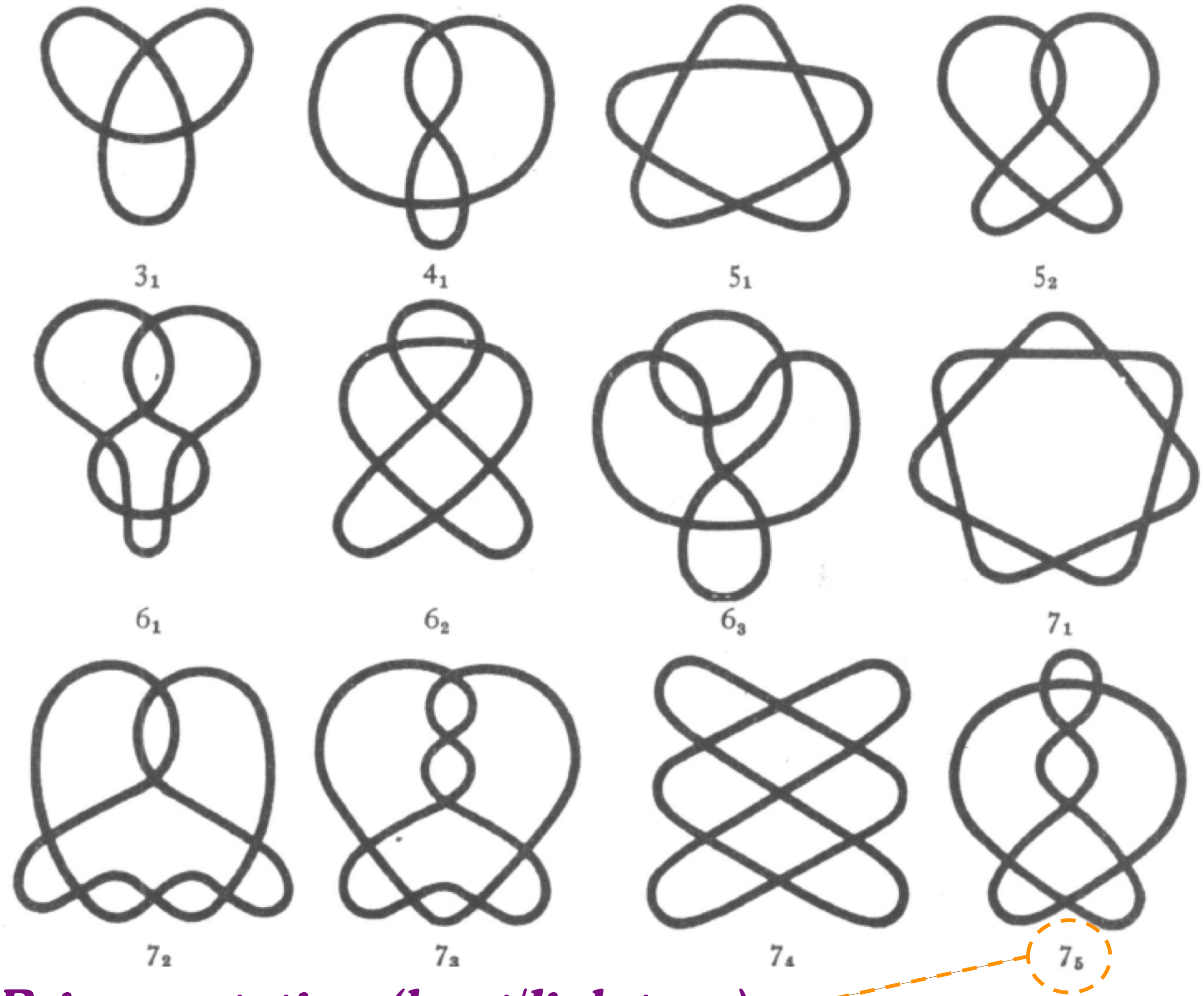


6_3

The standard table of knots (Reidemeister, Knotetheorie, 1932)

Knotentabelle.

Die Tabelle der folgenden Knotenprojektionen bis zu neun Überkreuzungen wurde der Arbeit von Alexander und Briggs (5) entnommen. Verbessert wurden die Kurven 8_4 und 9_7, bei denen die Anzahl der Überkreuzungen nicht stimmte.



… …

Alexander-Briggs notation (knot/link type)

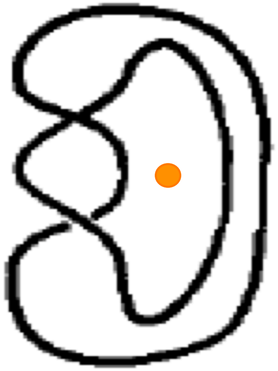
Braid words

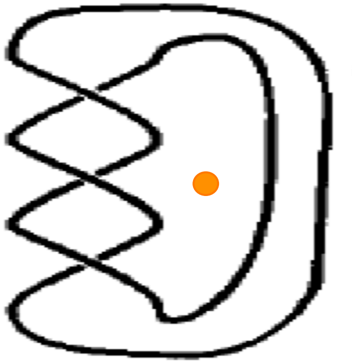
Hopf link:

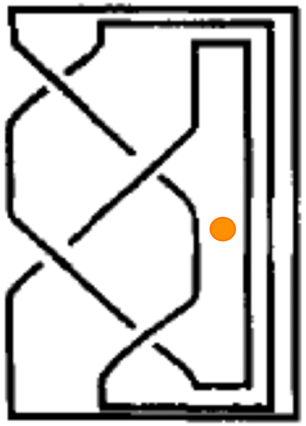
Trefoil knot:

F^8 knot:

Braid words

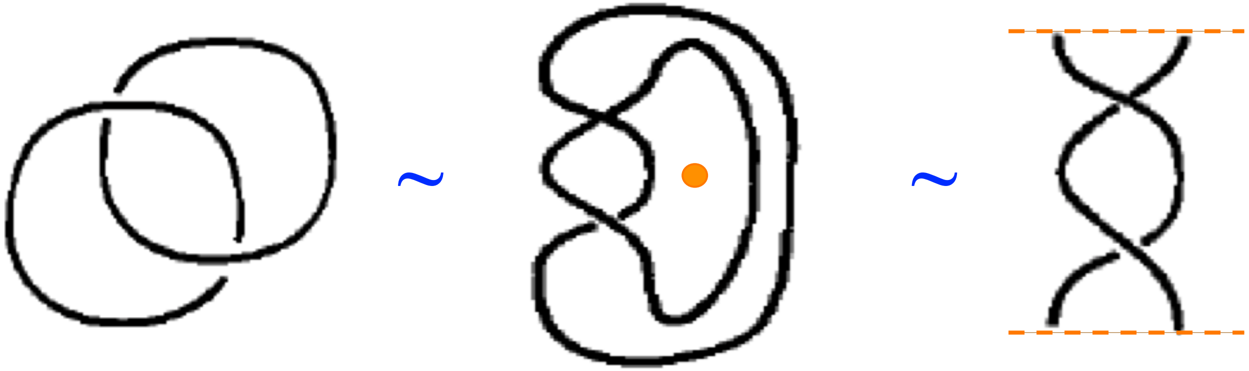
Hopf link:  ~ 

Trefoil knot:  ~ 

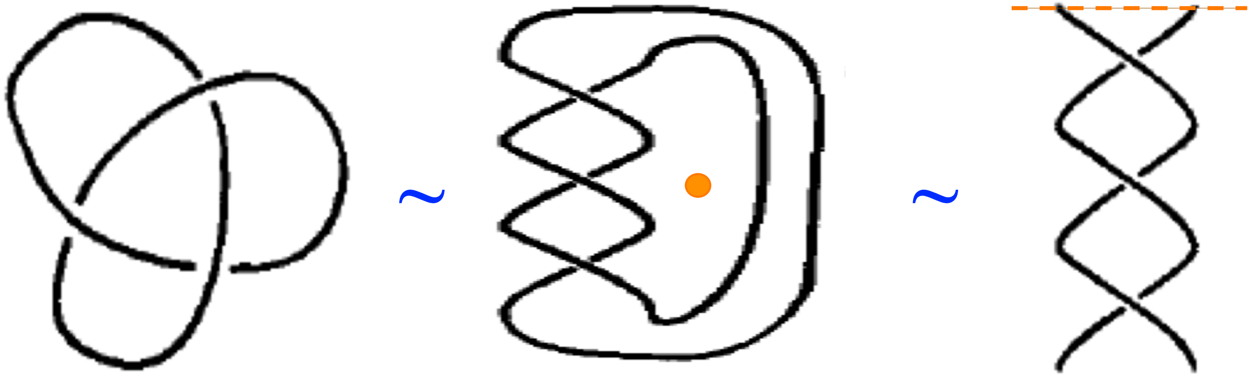
F^8 *knot:*  ~ 

Braid words

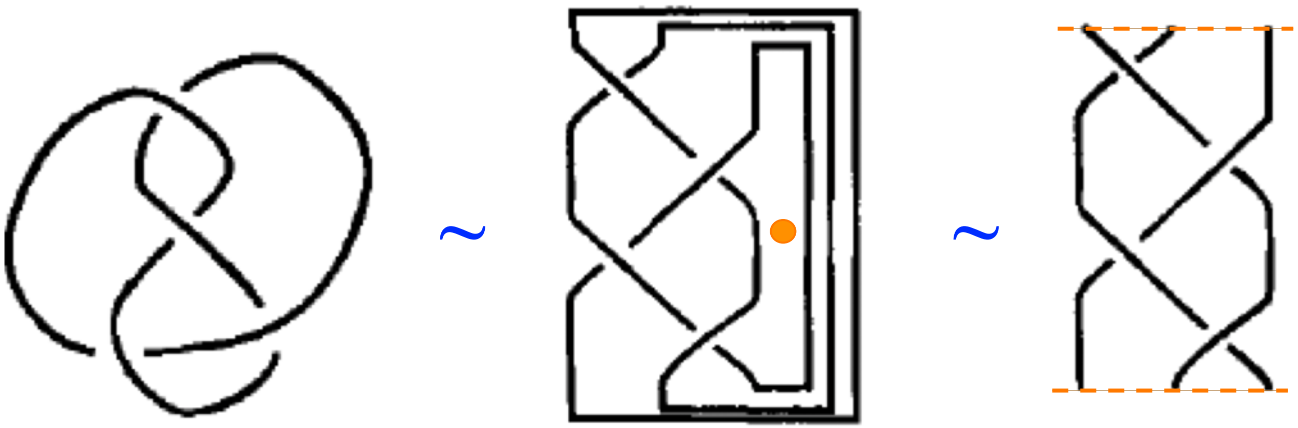
Hopf link:



Trefoil knot:

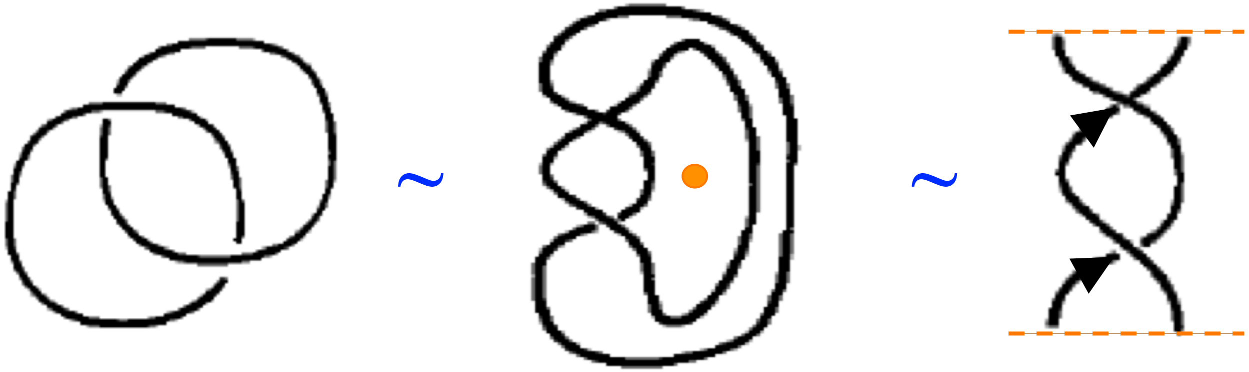


F^8 knot:

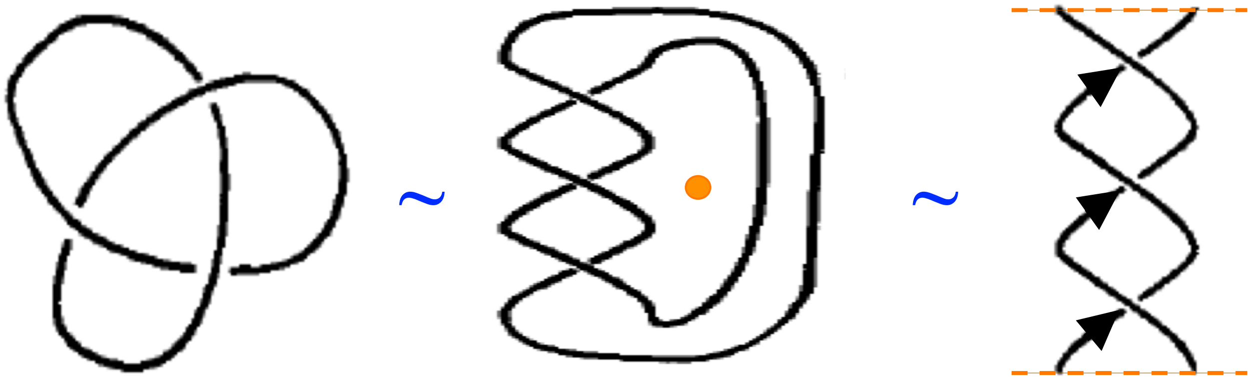


Braid words

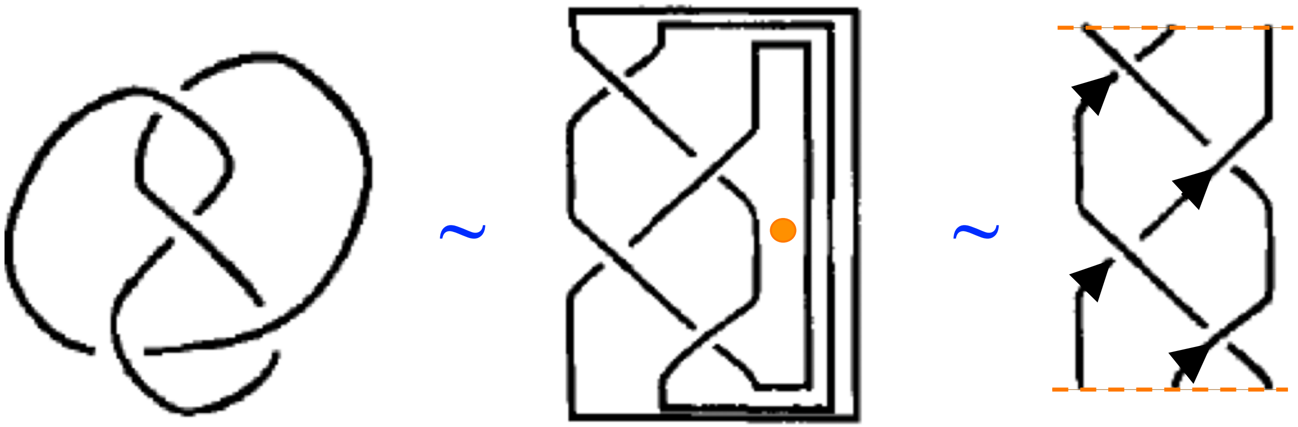
Hopf link:



Trefoil knot:

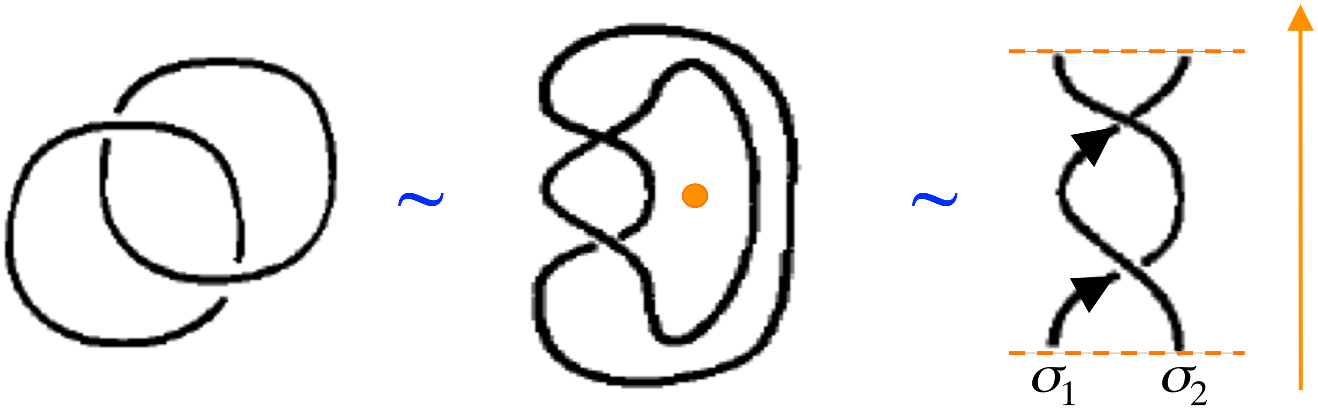


***F^8* knot:**

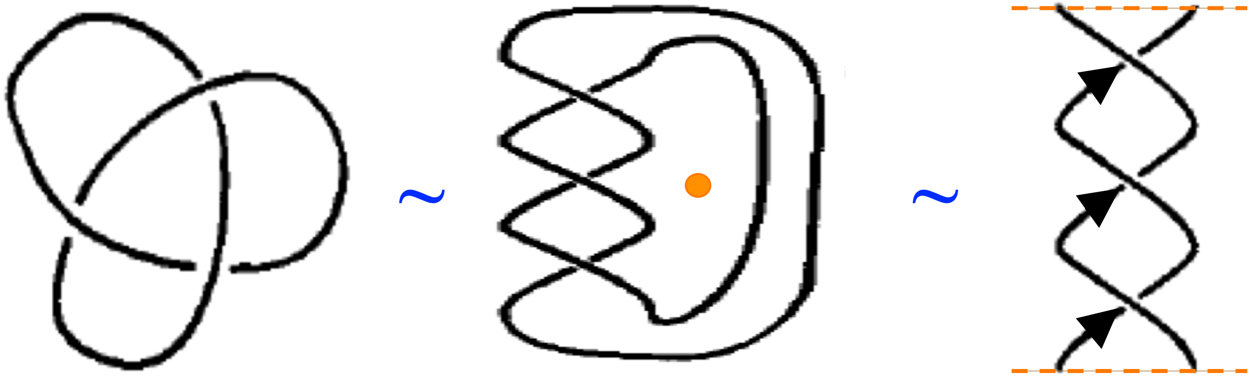


Braid words

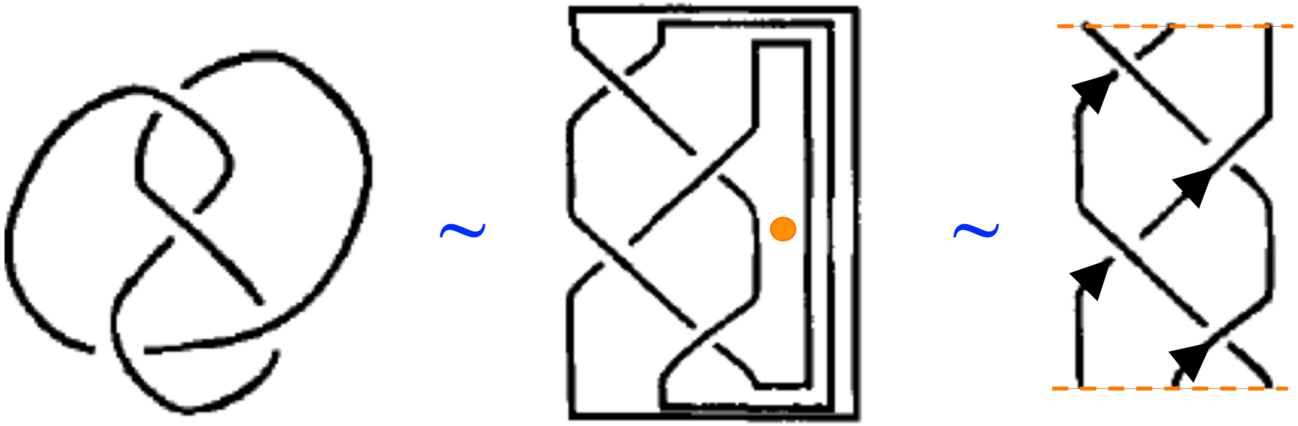
Hopf link:



Trefoil knot:

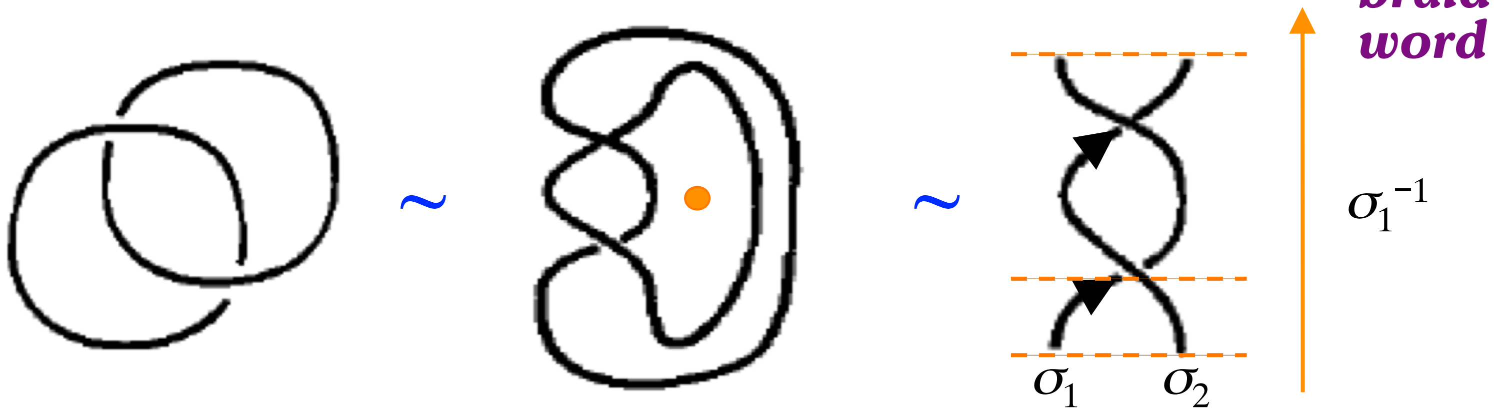


F^8 *knot:*

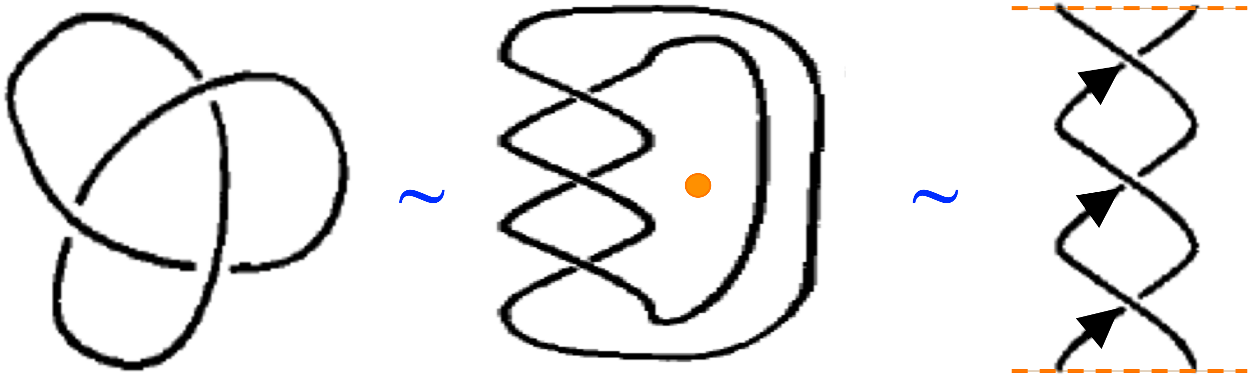


Braid words

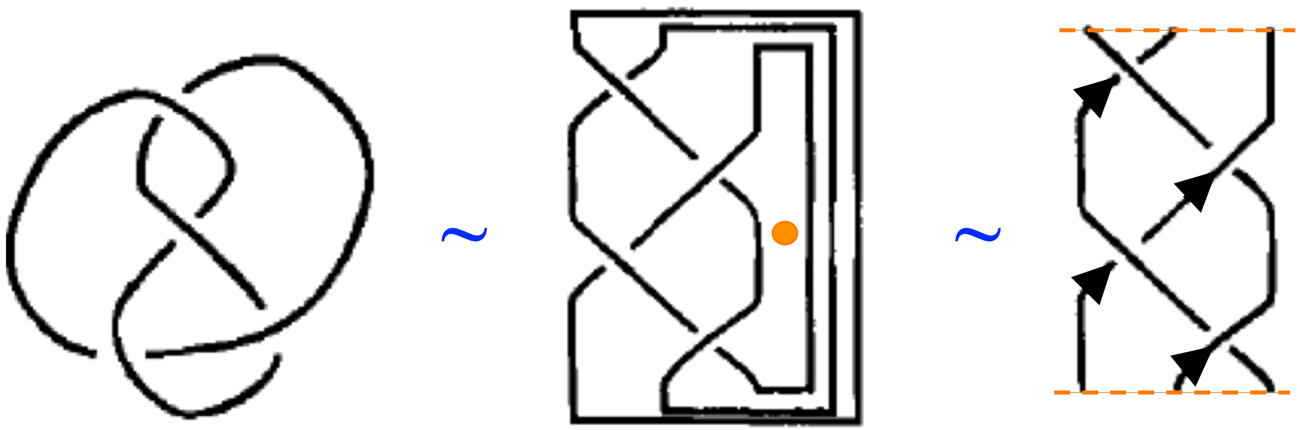
Hopf link:



Trefoil knot:



F^8 knot:

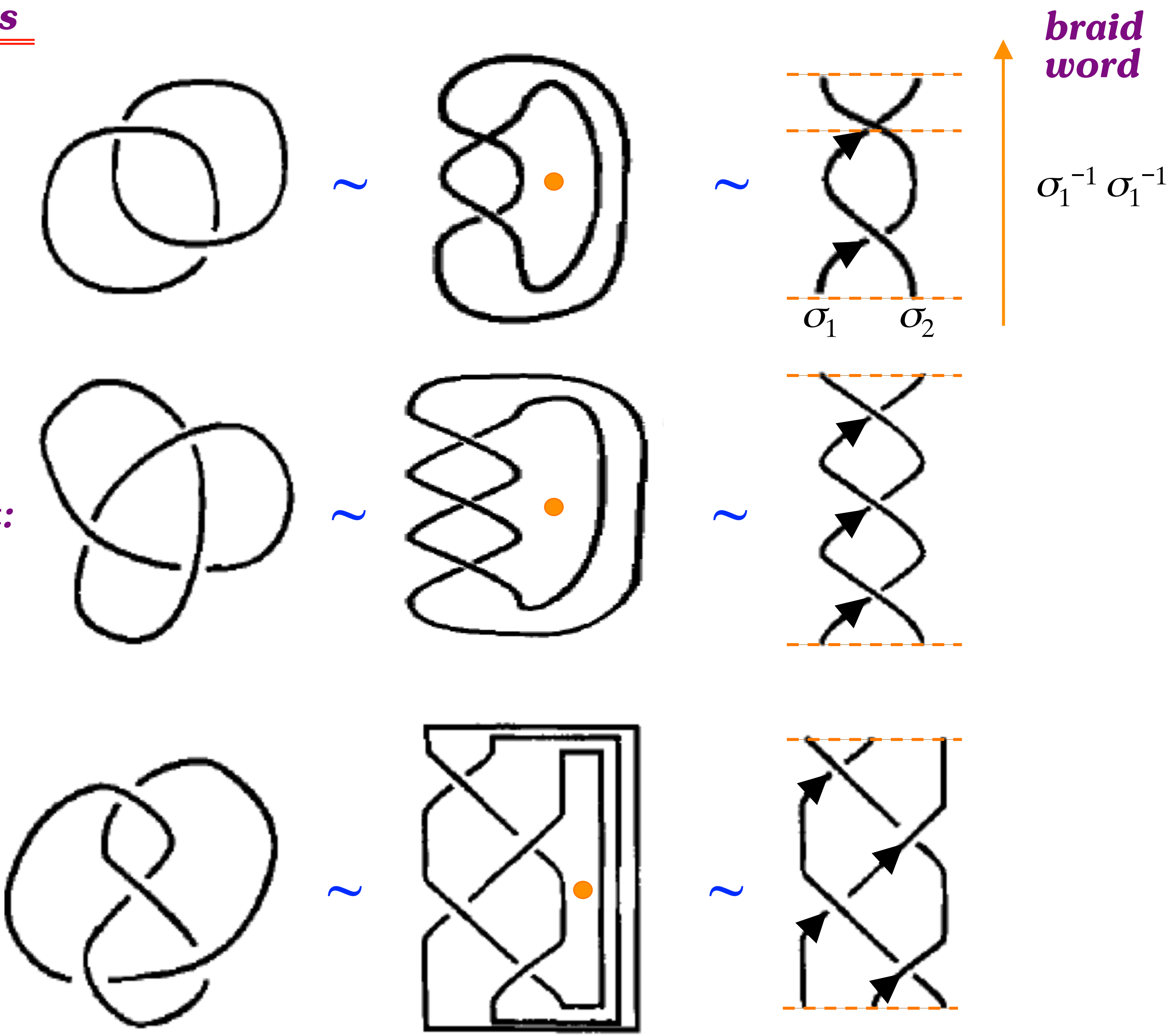


Braid words

Hopf link:

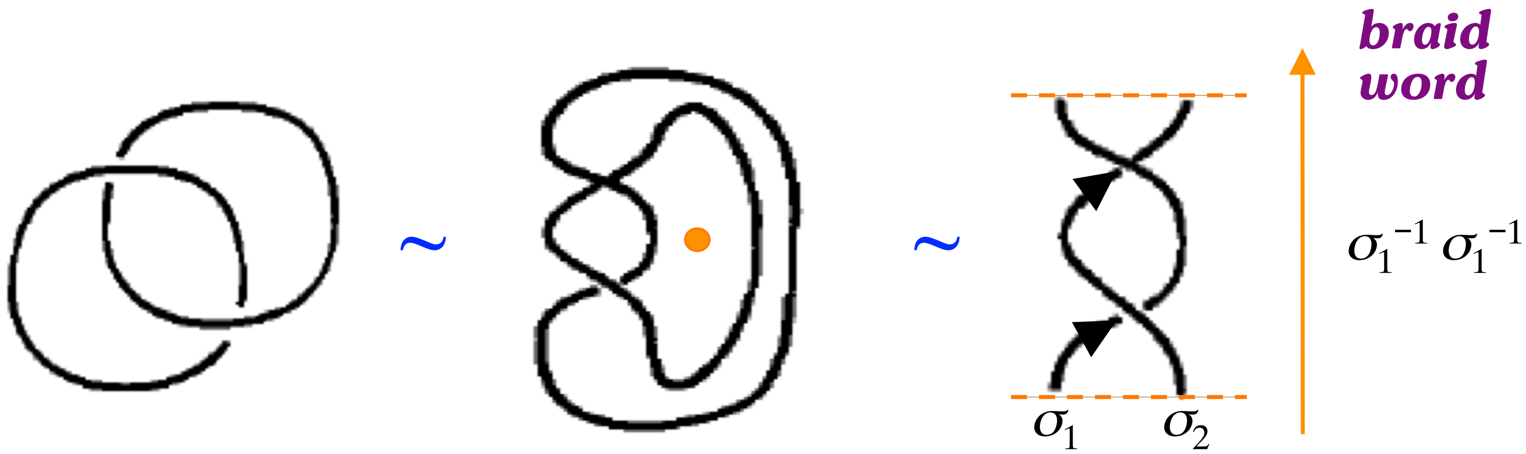
Trefoil knot:

F^8 knot:

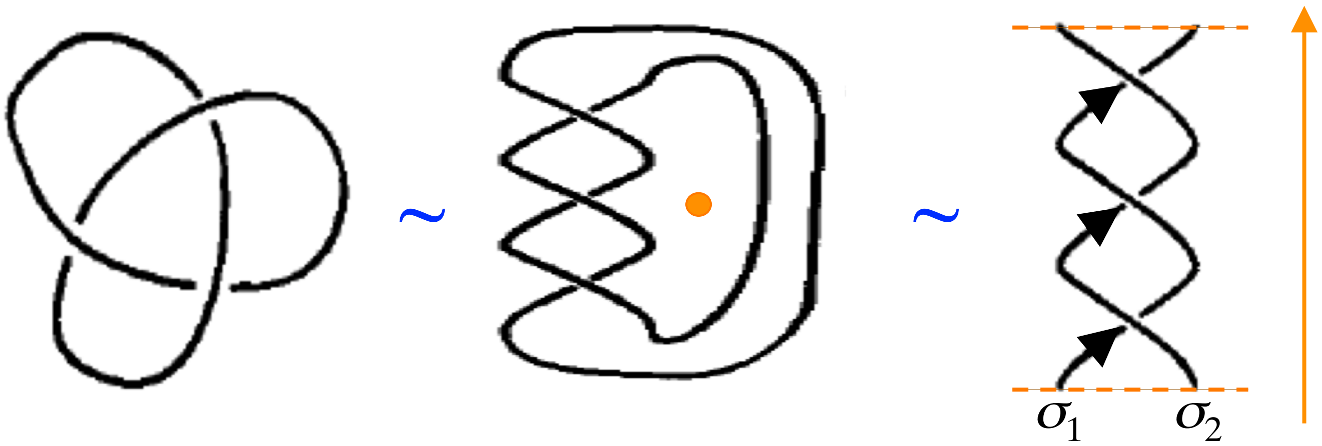


Braid words

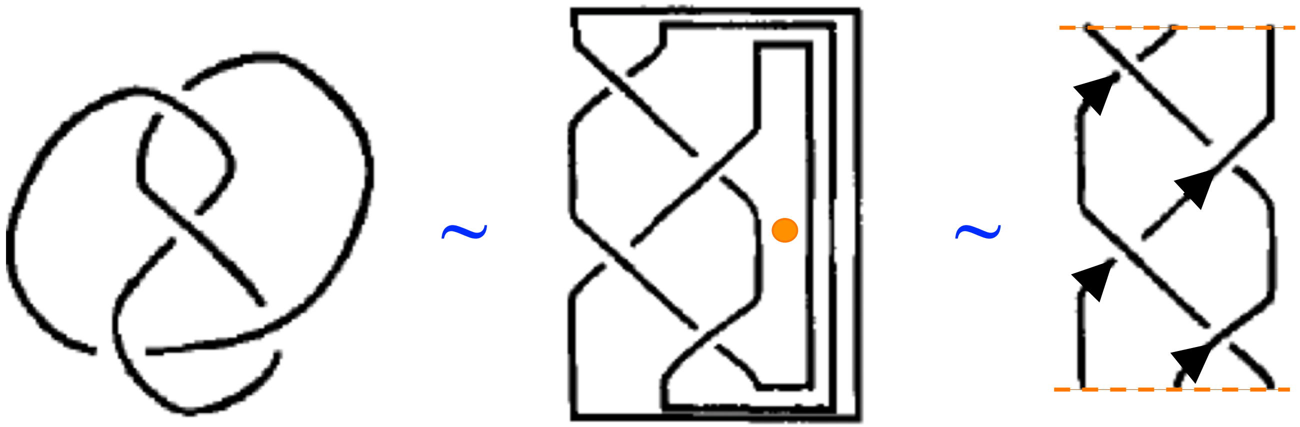
Hopf link:



Trefoil knot:



F^{8} *knot:*

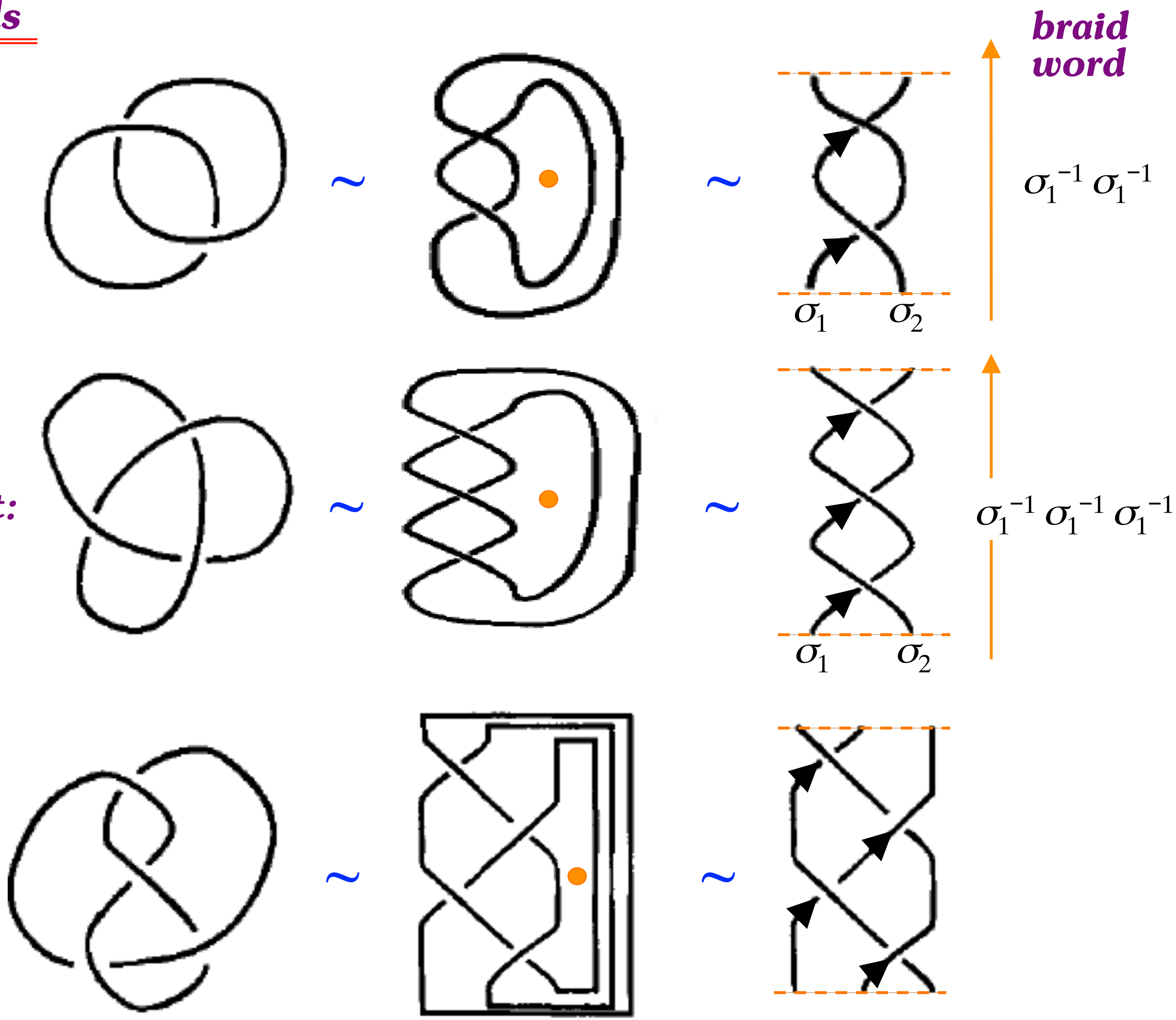


Braid words

Hopf link:

Trefoil knot:

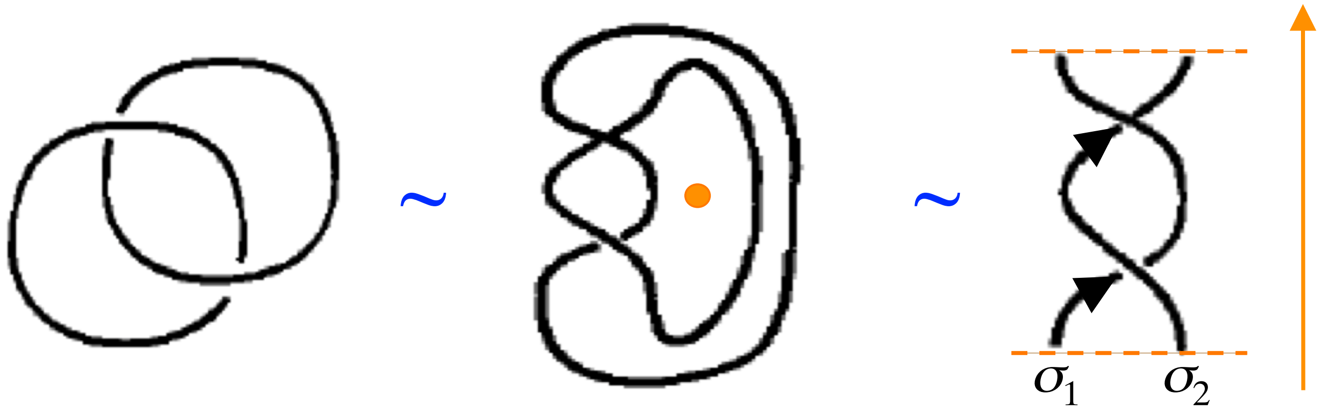
F^8 knot:



Braid words

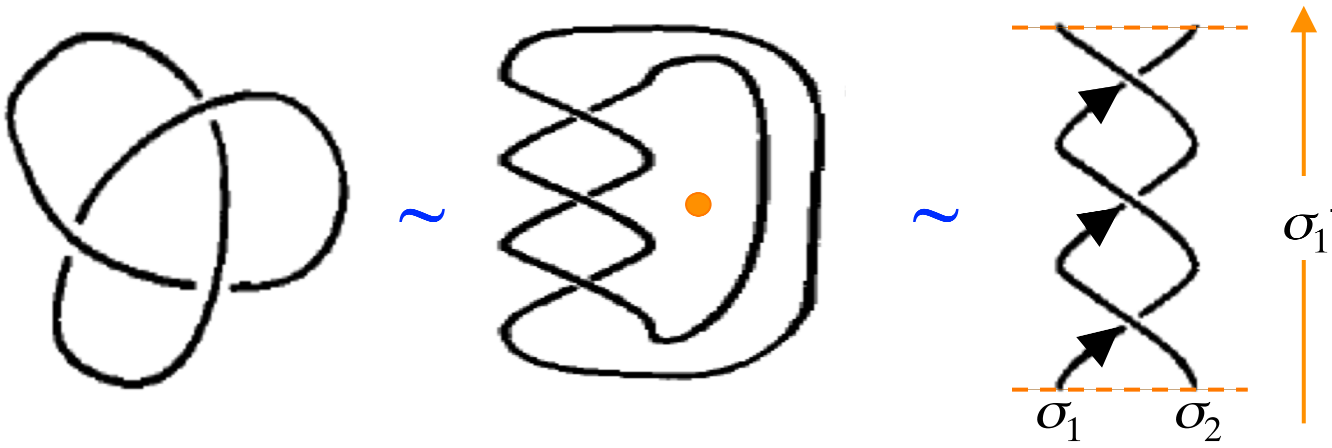
braid word

Hopf link:



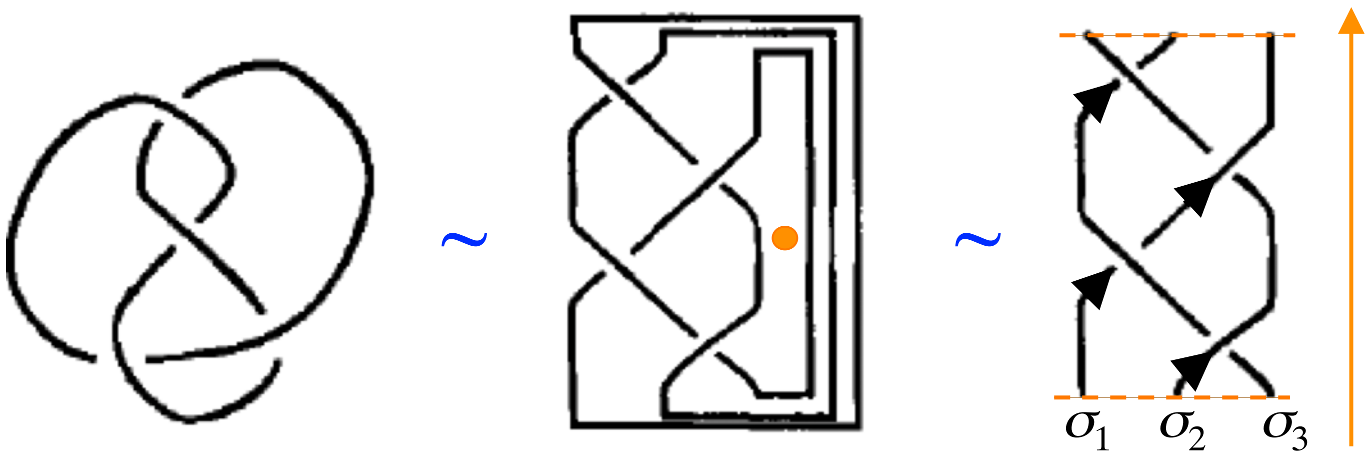
$\sigma_1^{-1}\,\sigma_1^{-1}$

Trefoil knot:

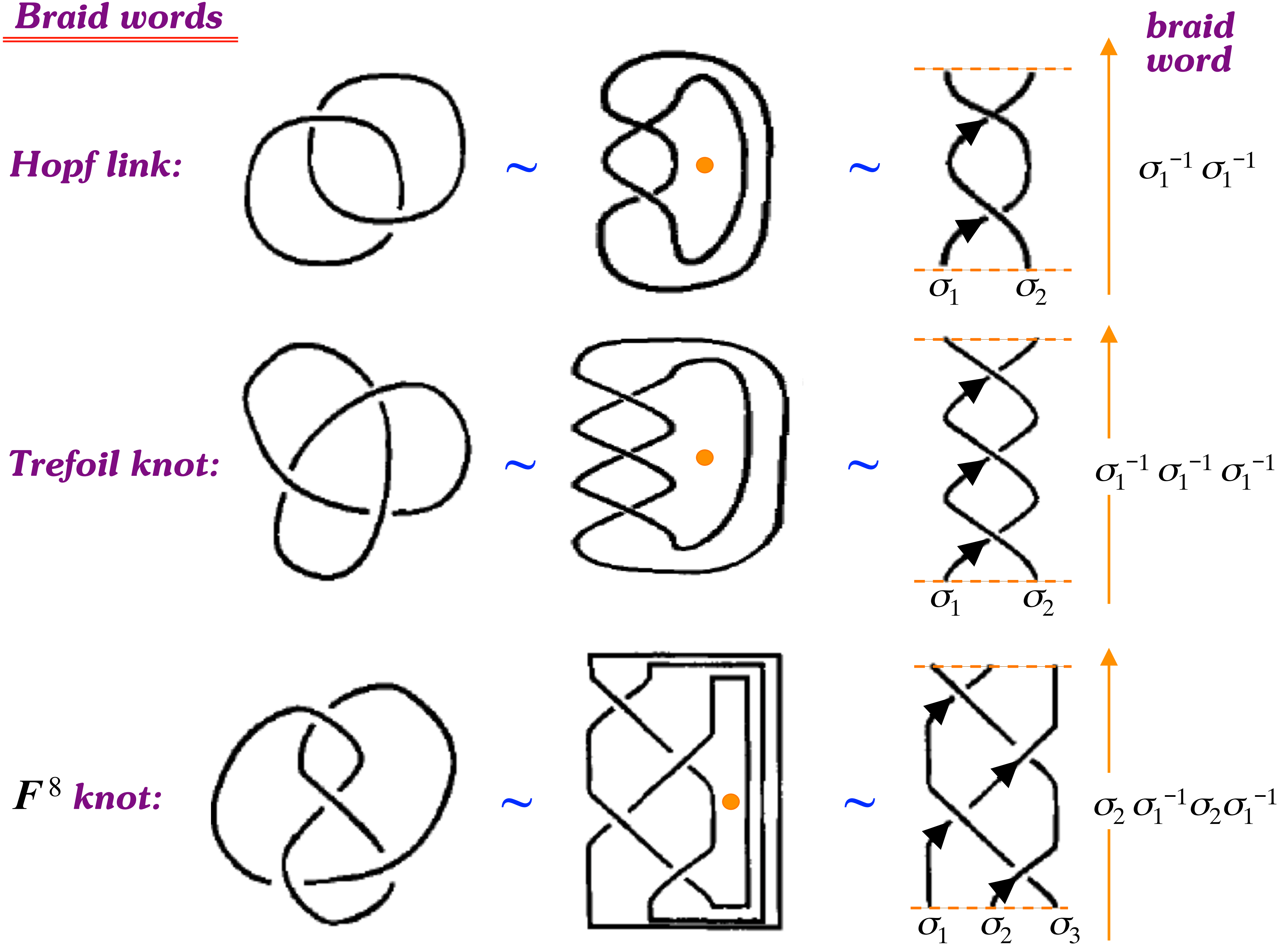


$\sigma_1^{-1}\,\sigma_1^{-1}\,\sigma_1^{-1}$

F^8 *knot:*

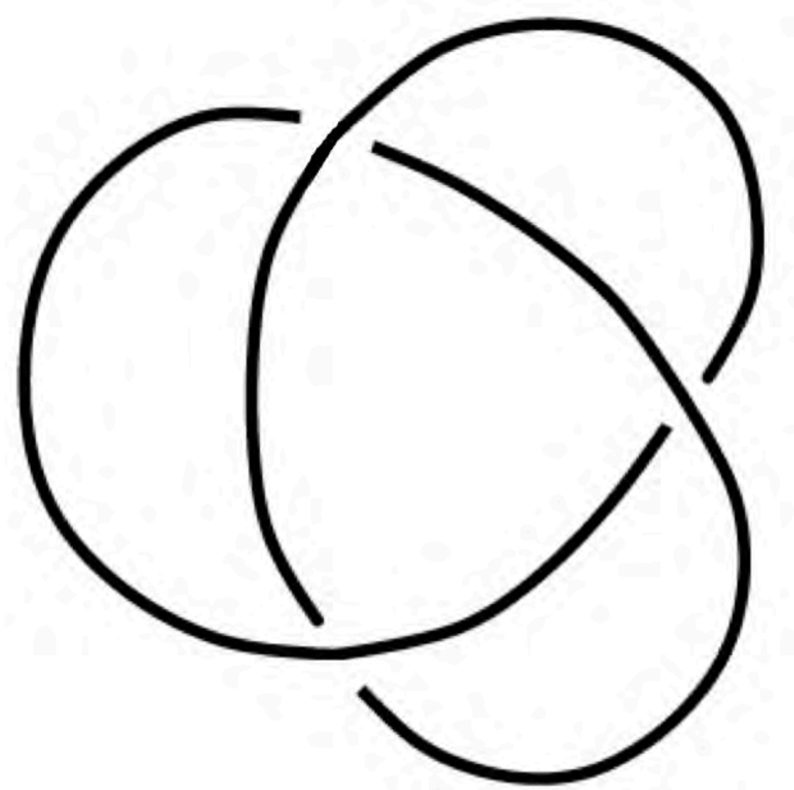


Braid words

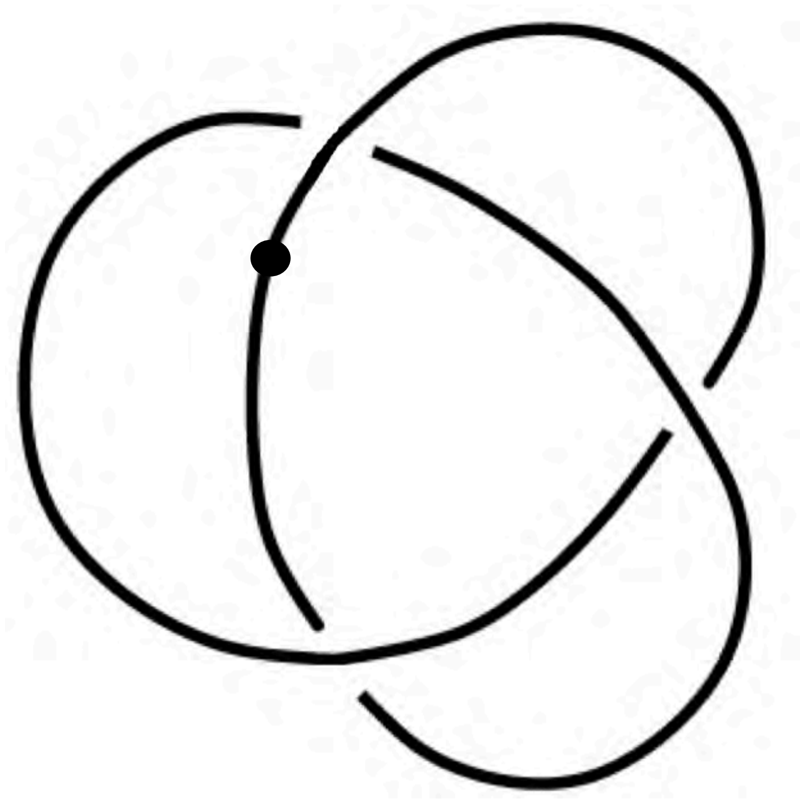


braid word
Hopf link:
~
~
σ_1 σ_2
$\sigma_1^{-1}\sigma_1^{-1}$
Trefoil knot:
~
~
σ_1 σ_2
$\sigma_1^{-1}\sigma_1^{-1}\sigma_1^{-1}$
F^8 knot:
~
~
σ_1 σ_2 σ_3
$\sigma_2\sigma_1^{-1}\sigma_2\sigma_1^{-1}$

Gauss code: an example



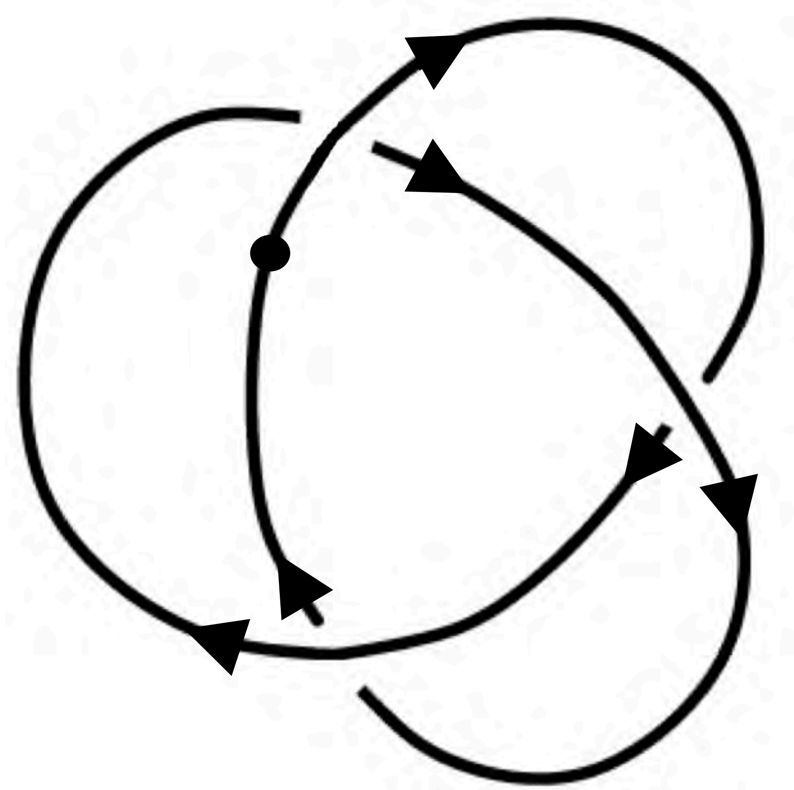
Gauss code: an example



- **Rule:**

i) fix an origin on the knot, (not on a crossing site);

Gauss code: an example

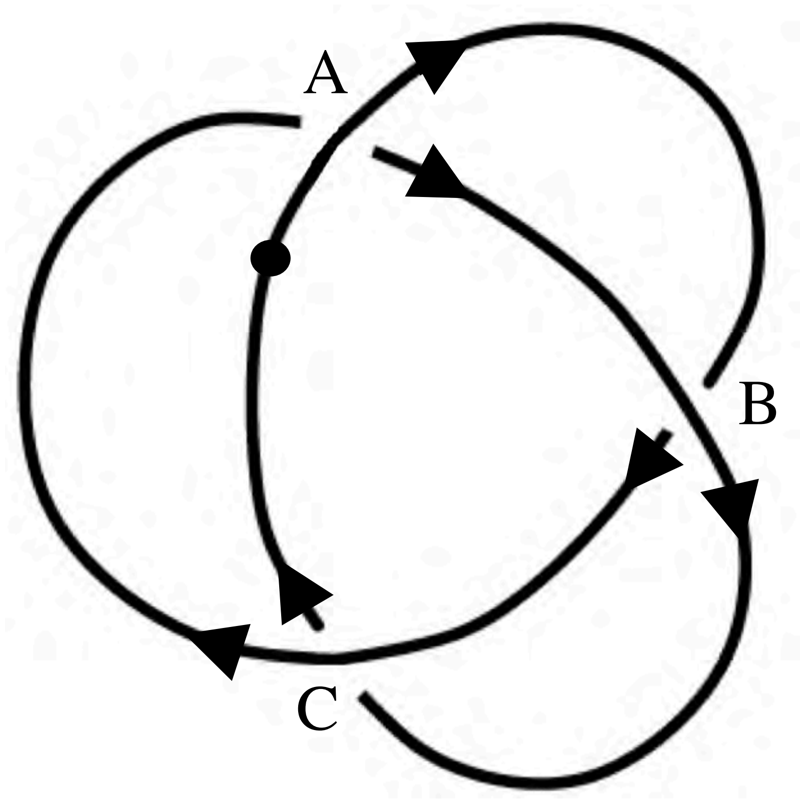


- ***Rule:***

i) ***fix an origin on the knot, (not on a crossing site);***

ii) ***orient the knot;***

Gauss code: an example



- ***Rule:***

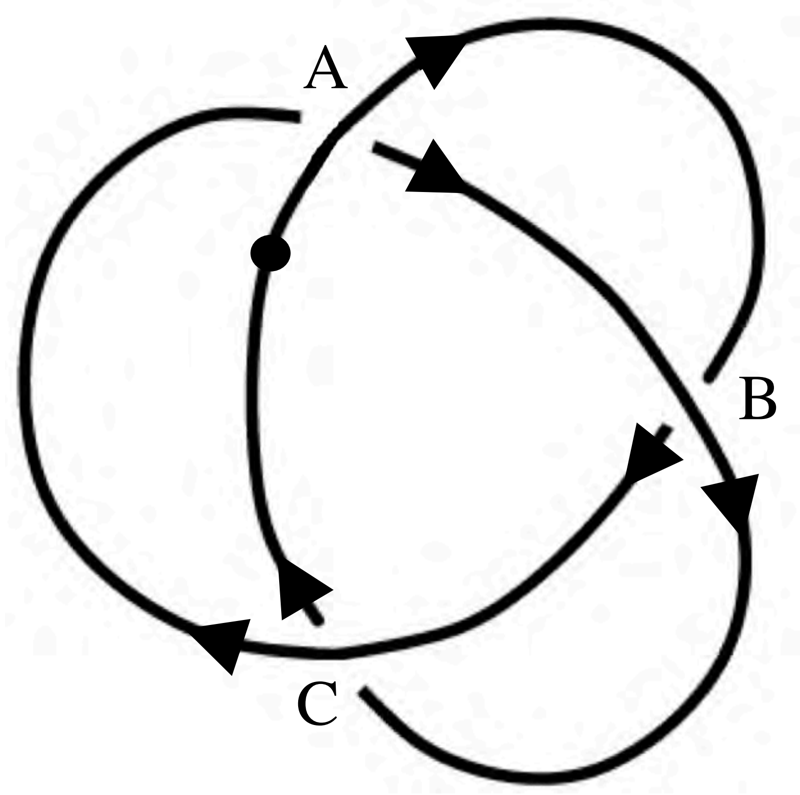
i) fix an origin on the knot,
(not on a crossing site);

ii) orient the knot;

iii) assign a letter (or a number) to each crossing site
in alphabetical order according to orientation;

Gauss code: an example

A, –B, C, –A, B, –C.



- ***Rule:***

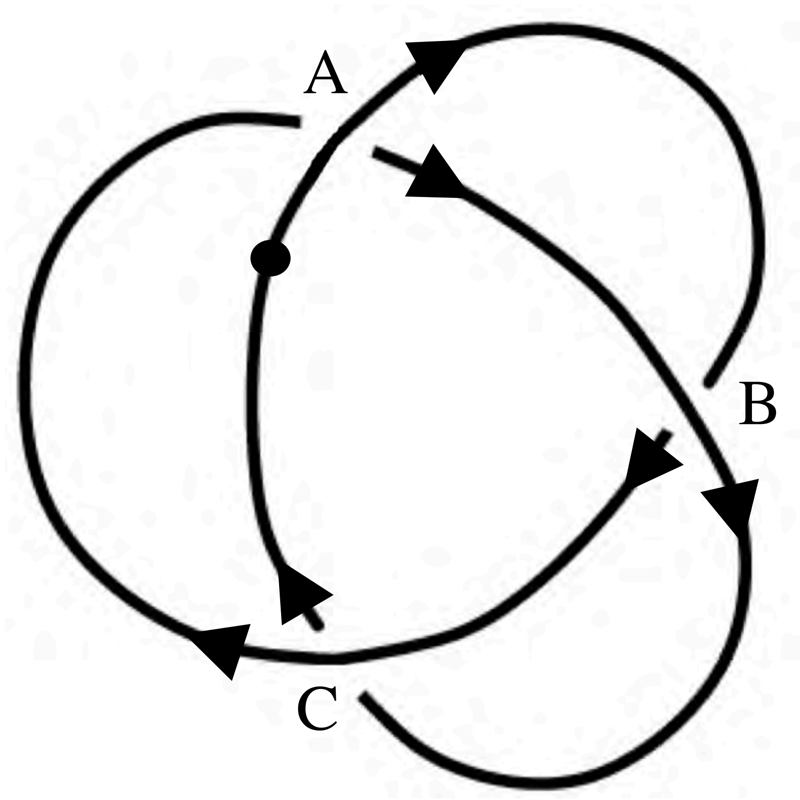
i) fix an origin on the knot,
(not on a crossing site);

ii) orient the knot;

iii) assign a letter (or a number) to each crossing site
in alphabetical order according to orientation;

iv) list all the signed letters in sequence all the way around the knot,
positive for an overpass, negative for an undrepass;

Gauss code: an example



Gauss code: {A –B C –A B –C}

- ***Rule:***

i) fix an origin on the knot,
(not on a crossing site);

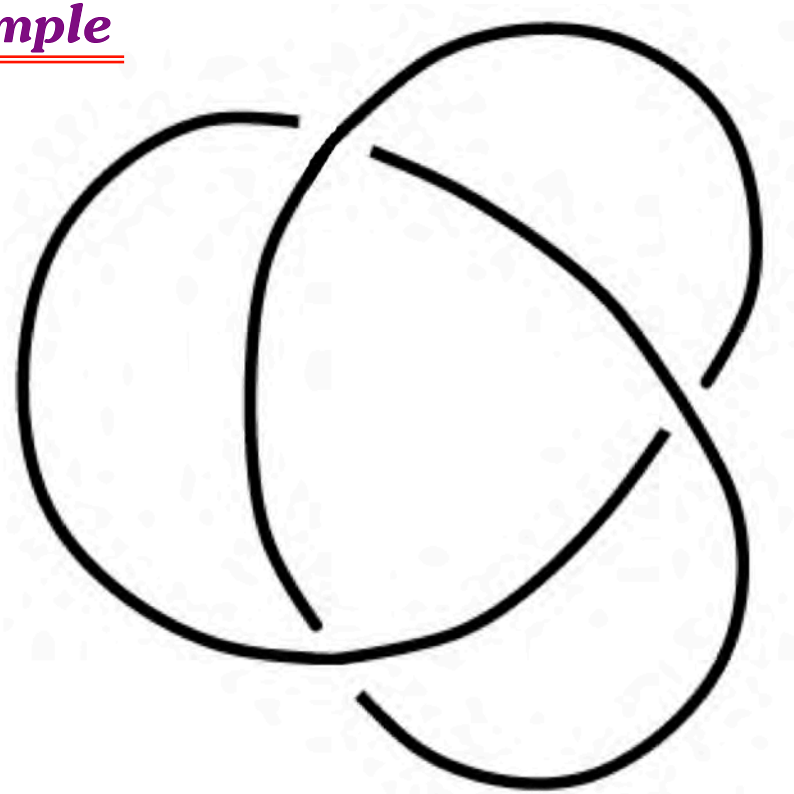
ii) orient the knot;

iii) assign a letter (or a number) to each crossing site
in alphabetical order according to orientation;

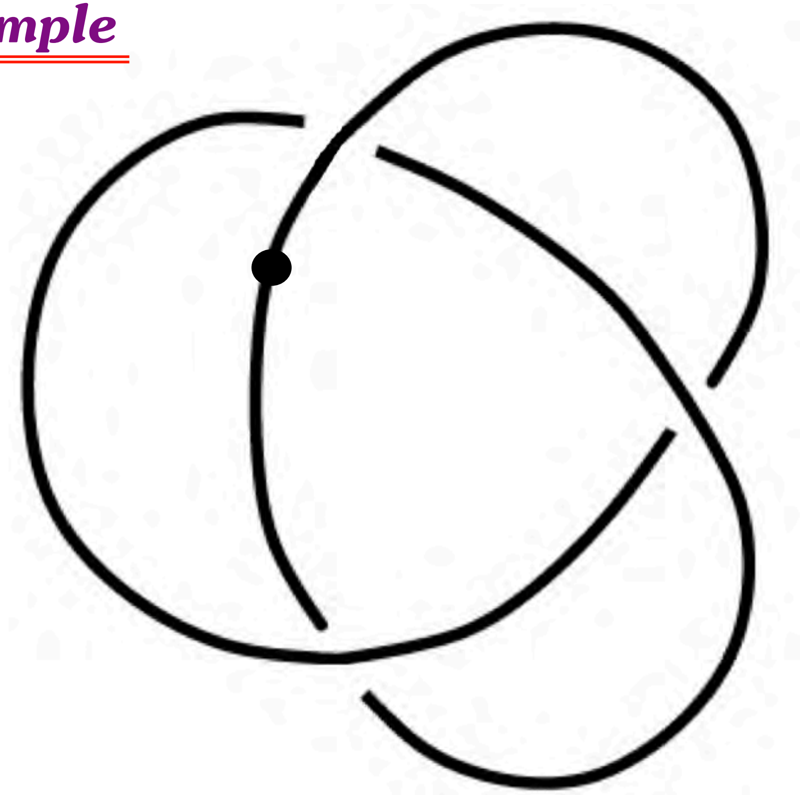
iv) list all the signed letters in sequence all the way around the knot,
positive for an overpass, negative for an undrepass;

v) the set of ordered sequence is the Gauss code.

DT (Dowker-Thistlethwaite) code: an example



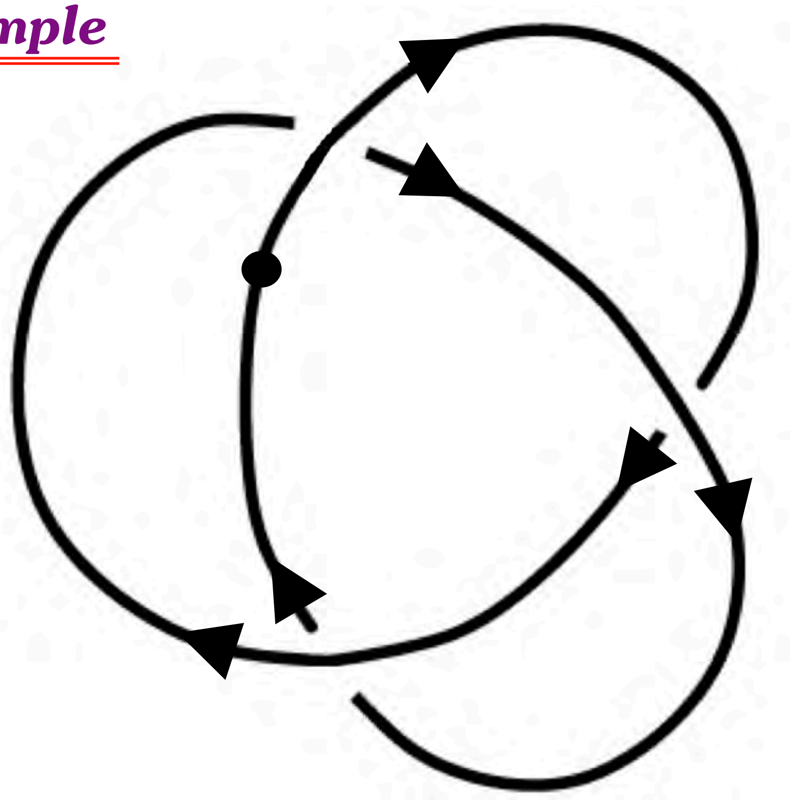
DT (Dowker-Thistlethwaite) code: an example



- ***Rule:***

i) *fix an origin on the knot,*
(not on a crossing site);

DT (Dowker-Thistlethwaite) code: an example

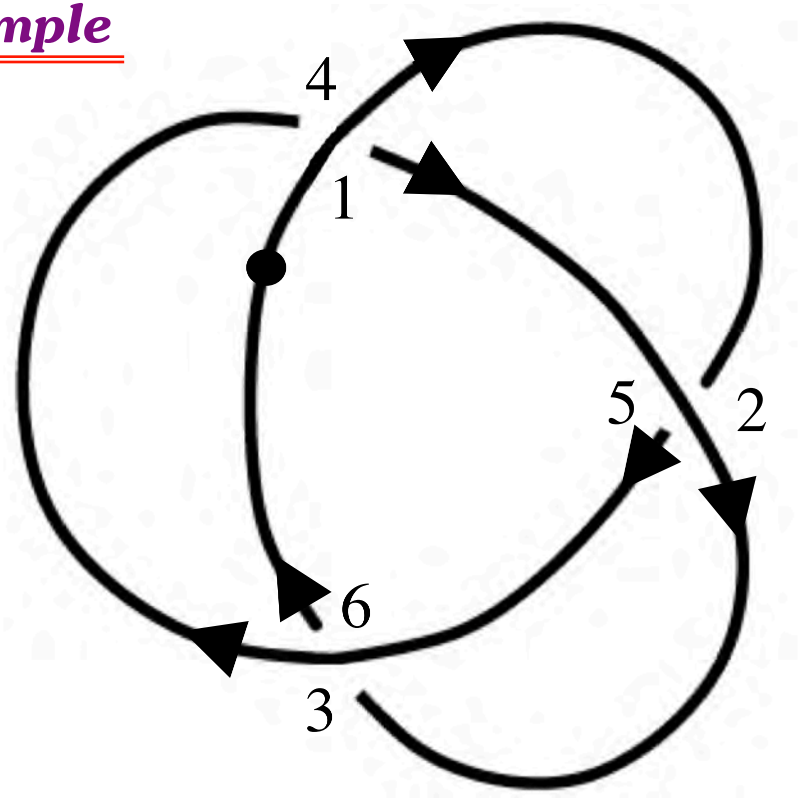


- **Rule:**

i) fix an origin on the knot, (not on a crossing site);

ii) orient the knot;

DT (Dowker-Thistlethwaite) code: an example



- ***Rule:***

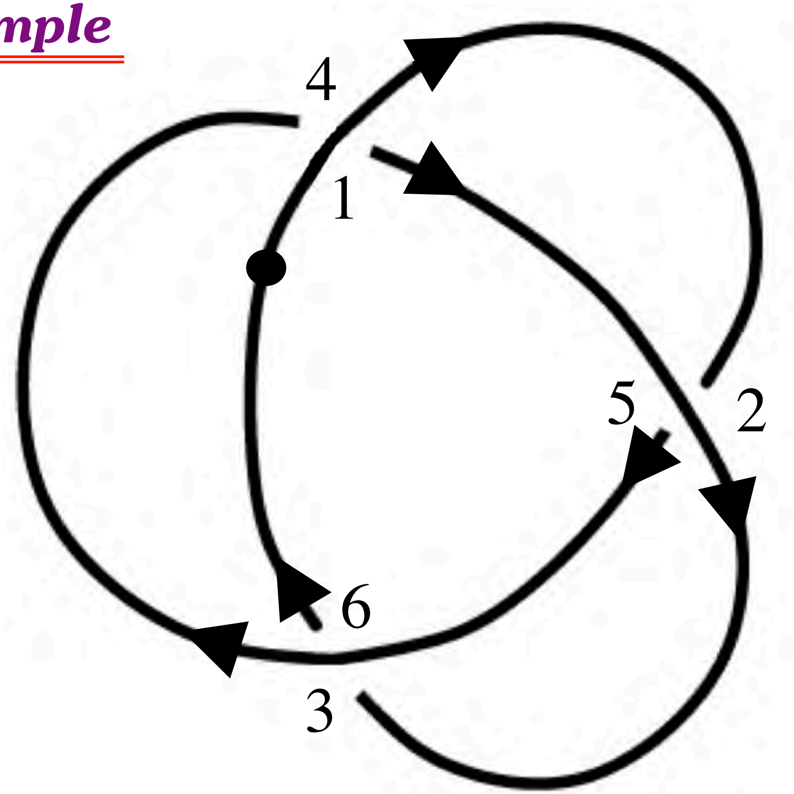
i) fix an origin on the knot,
(not on a crossing site);

ii) orient the knot;

iii) assign a number to each crossing site
in increasing order according to orientation, all around the knot;

DT (Dowker-Thistlethwaite) code: an example

{ 1 2 3 4 5 6 }



- ***Rule:***

i) fix an origin on the knot,
(not on a crossing site);

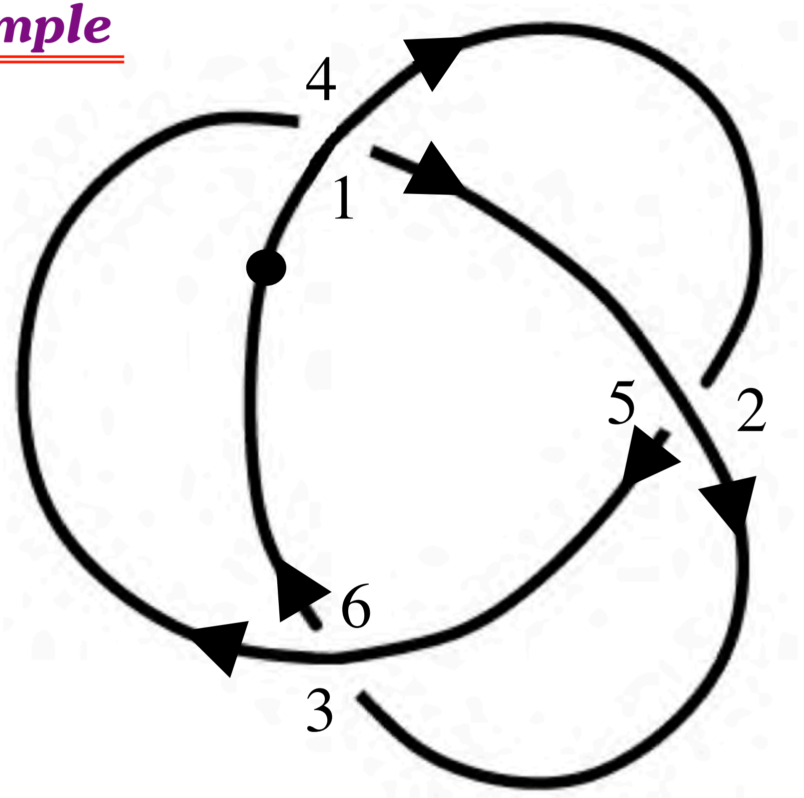
ii) orient the knot;

iii) assign a number to each crossing site
in increasing order according to orientation, all around the knot;

iv) list the sequence of numbers by assigning a negative sign to even
overpasses;

DT (Dowker-Thistlethwaite) code: an example

$$\{\,1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\,\} \longrightarrow \left\{\begin{matrix} 1 & 3 & 5 \\ 4 & 6 & 2 \end{matrix}\right\}$$



- ***Rule:***

i) fix an origin on the knot, (not on a crossing site);

ii) orient the knot;

iii) assign a number to each crossing site in increasing order according to orientation, all around the knot;

iv) list the sequence of numbers by assigning a negative sign to even overpasses;

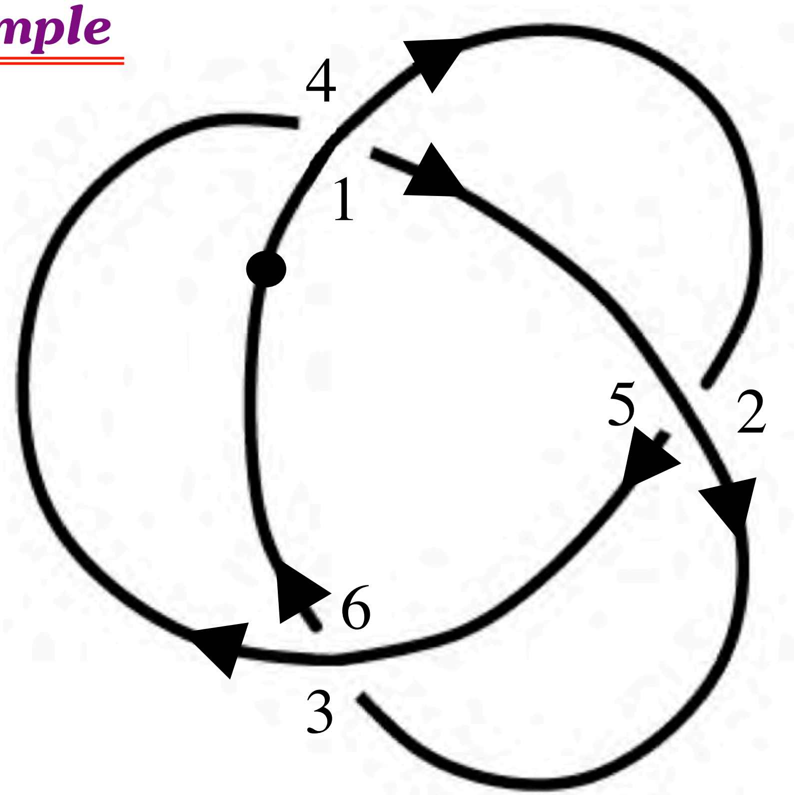
iv) starting from the odd numbers group odd and even numbers assigned to each crossing in separate ordered sequences, by placing the odd sequence above the even sequence;

DT (Dowker-Thistlethwaite) code: an example

$$\{1\ 2\ 3\ 4\ 5\ 6\} \rightarrow \begin{Bmatrix} 1 & 3 & 5 \\ 4 & 6 & 2 \end{Bmatrix}$$

$$\downarrow$$

DT code: [4 6 2]



- ***Rule:***

i) fix an origin on the knot,
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in increasing order according to orientation, all around the knot;

iv) list the sequence of numbers by assigning a negative sign to even overpasses;

iv) starting from the odd numbers group odd and even numbers assigned to each crossing in separate ordered sequences, by placing the odd sequence above the even sequence;

v) the set of even sequence represents the DT code.

DT (Dowker-Thistlethwaite) code: a worked-out example

- Remark: mirror knots have same DT code

DT (Dowker-Thistlethwaite) code: a worked-out example

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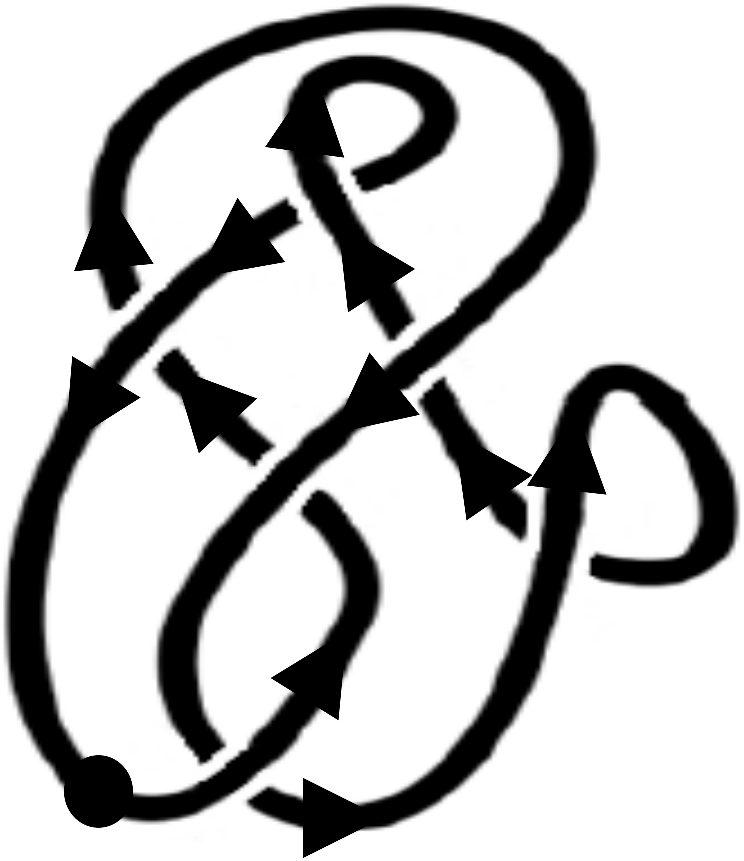
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*

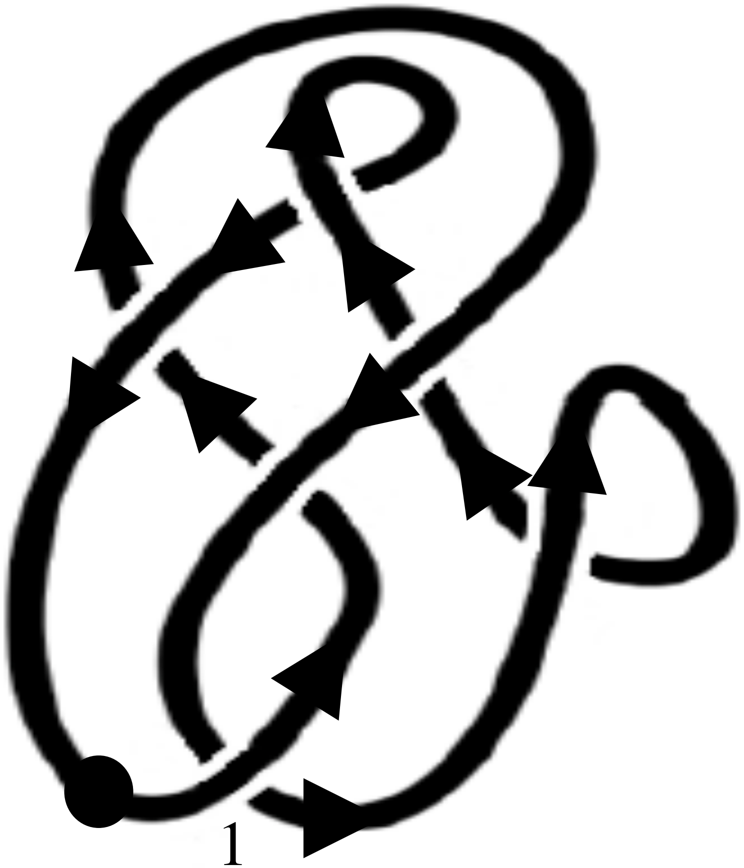
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



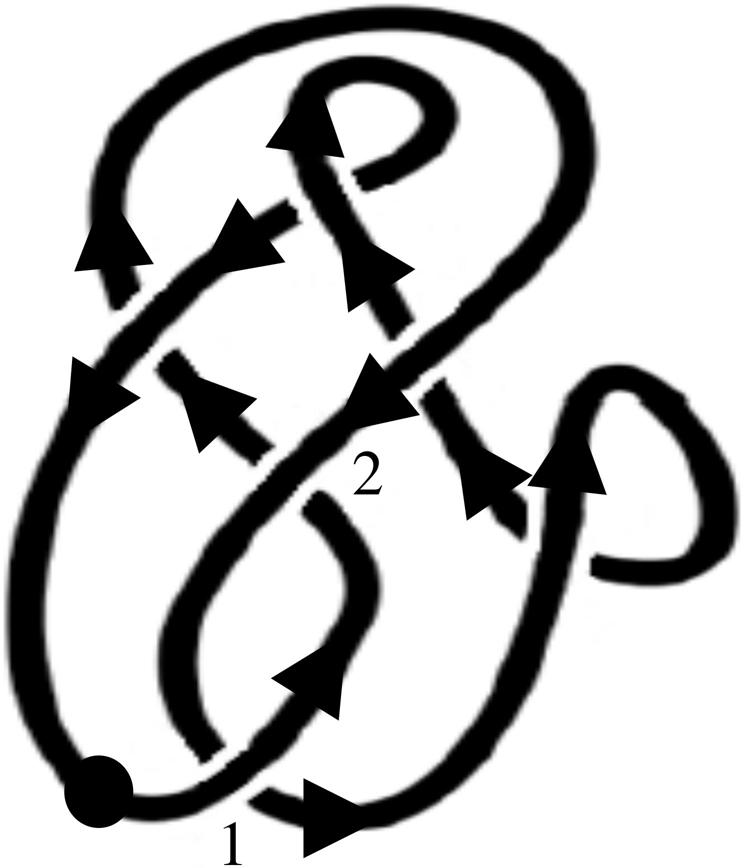
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



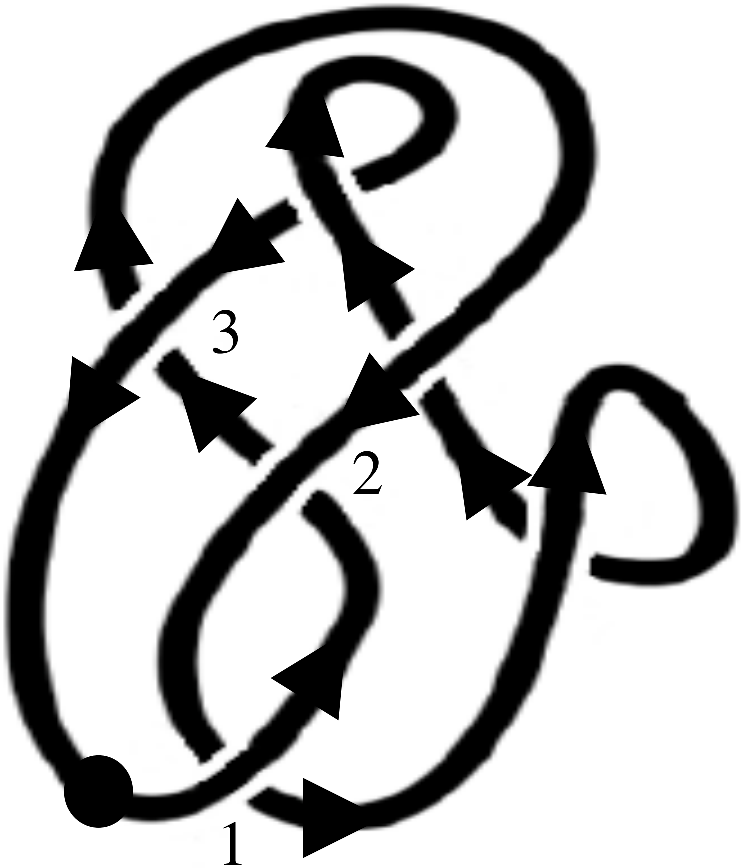
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



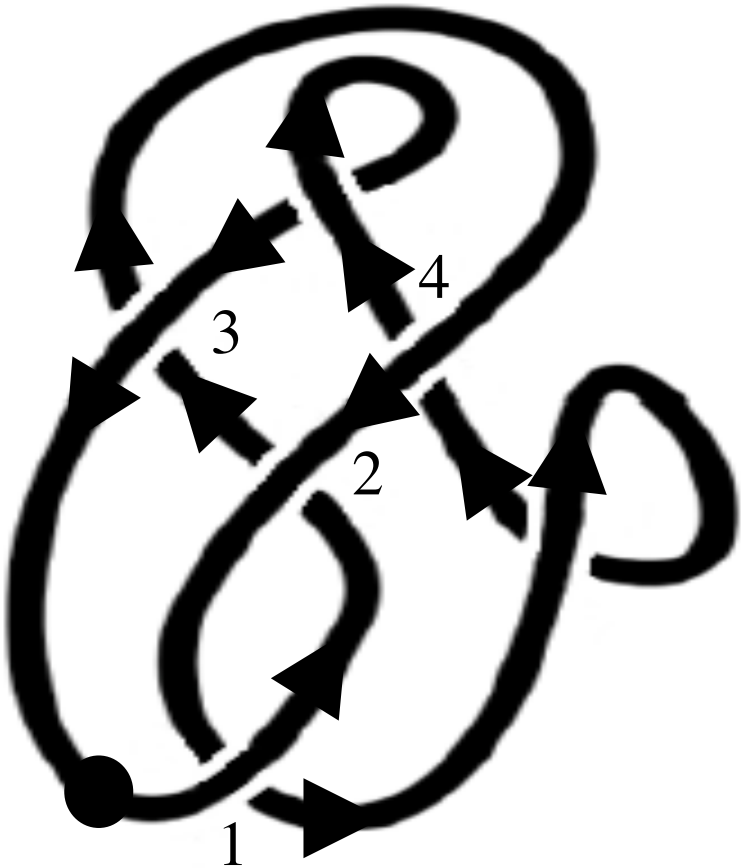
DT (Dowker-Thistlethwaite) code: a worked-out example

- ***Remark: mirror knots have same DT code***



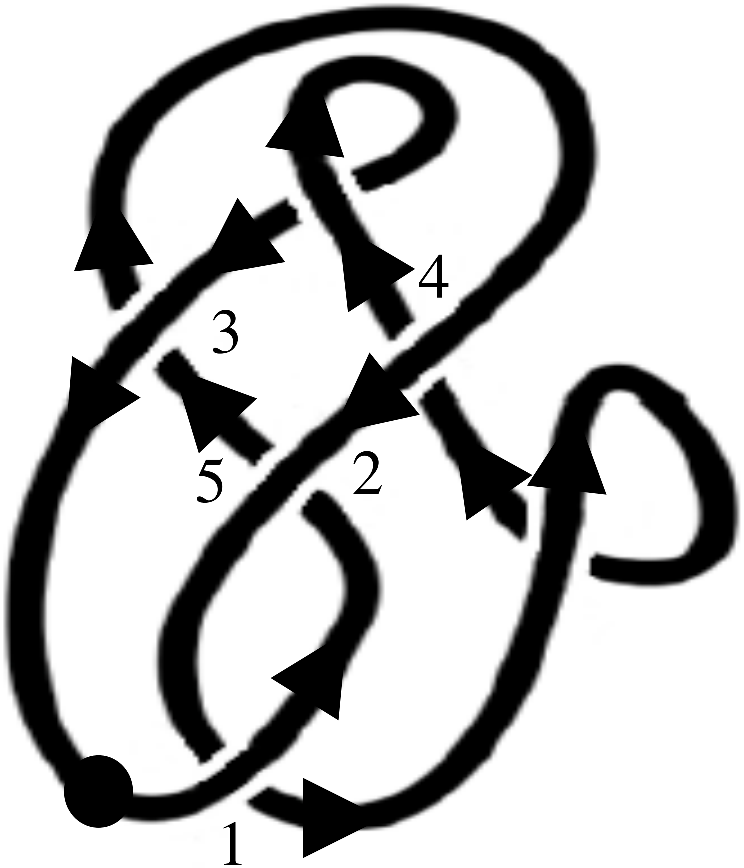
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



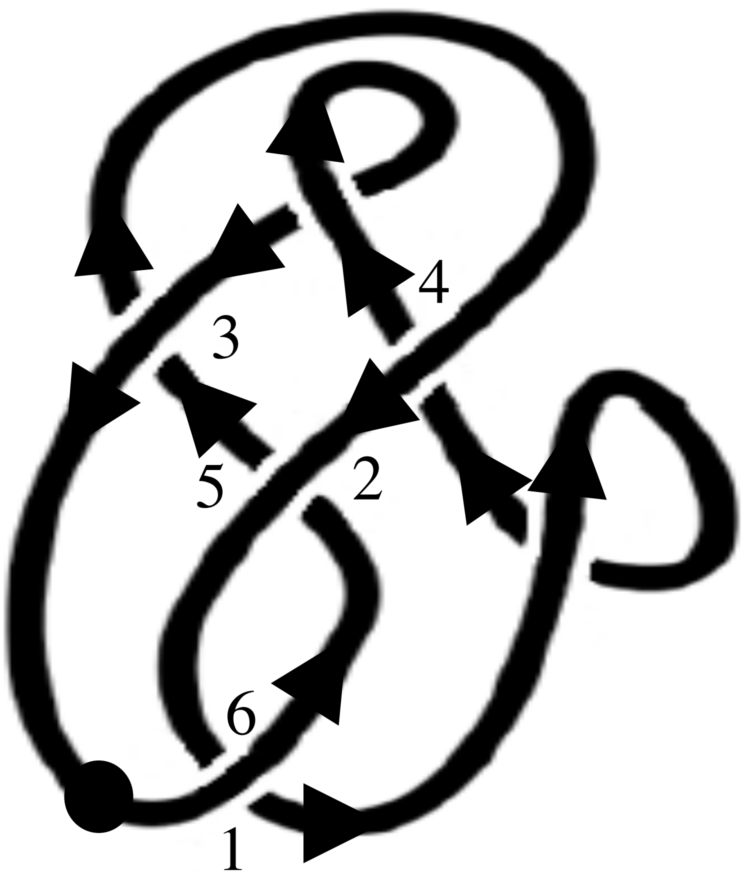
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



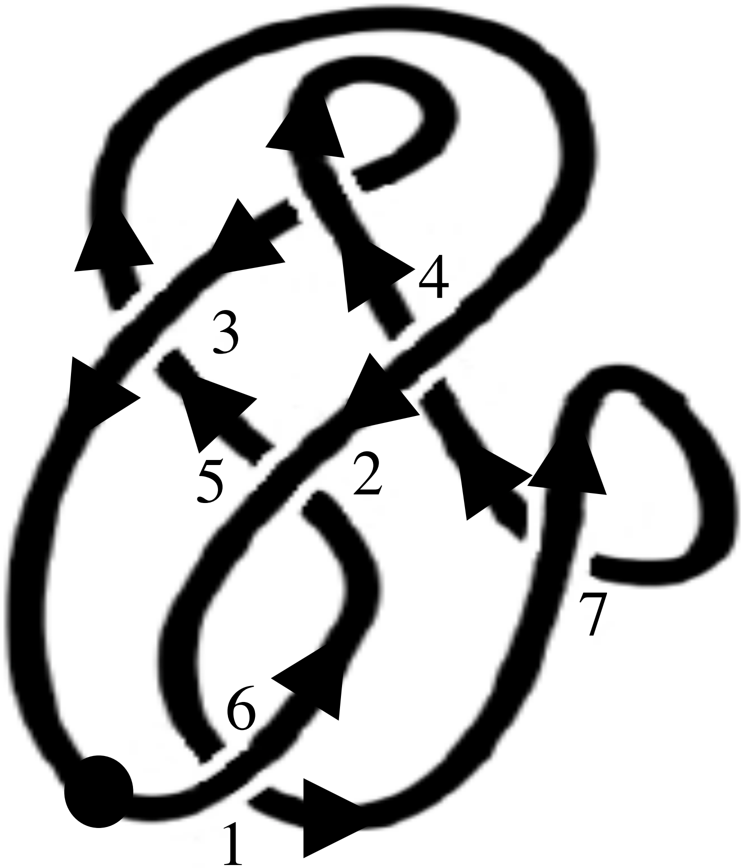
DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*



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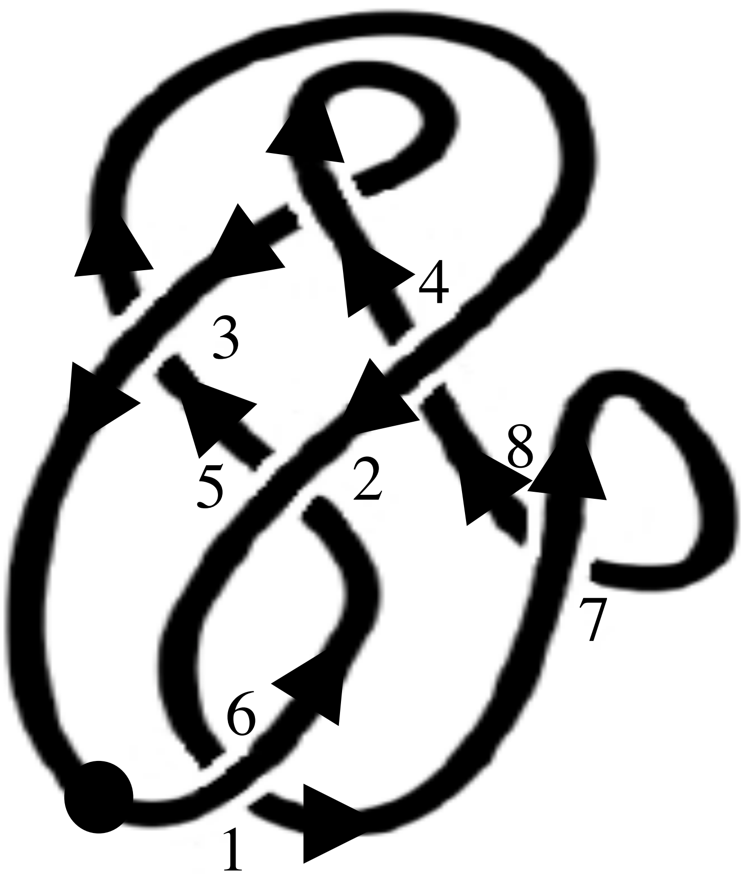

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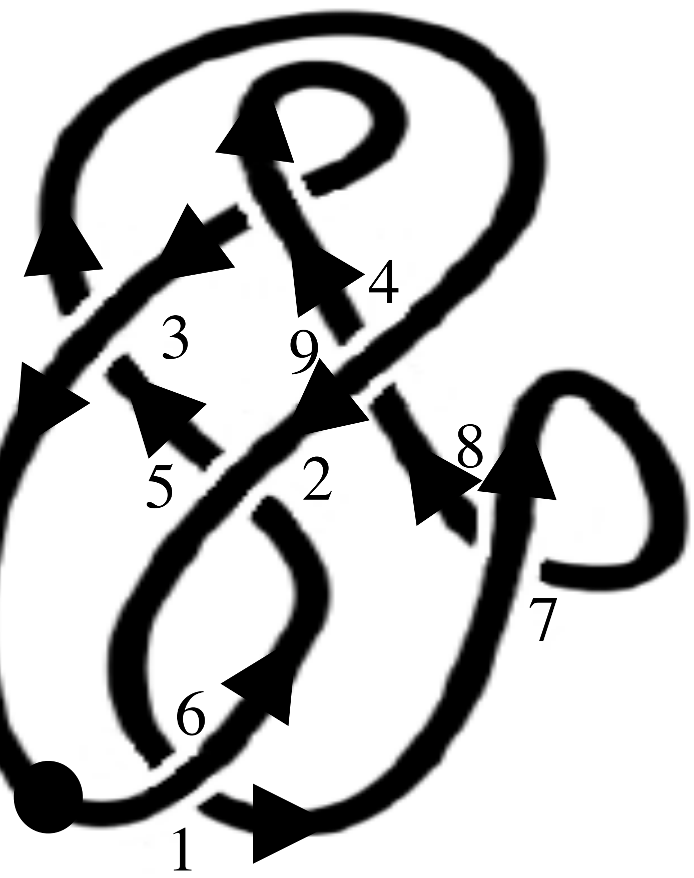

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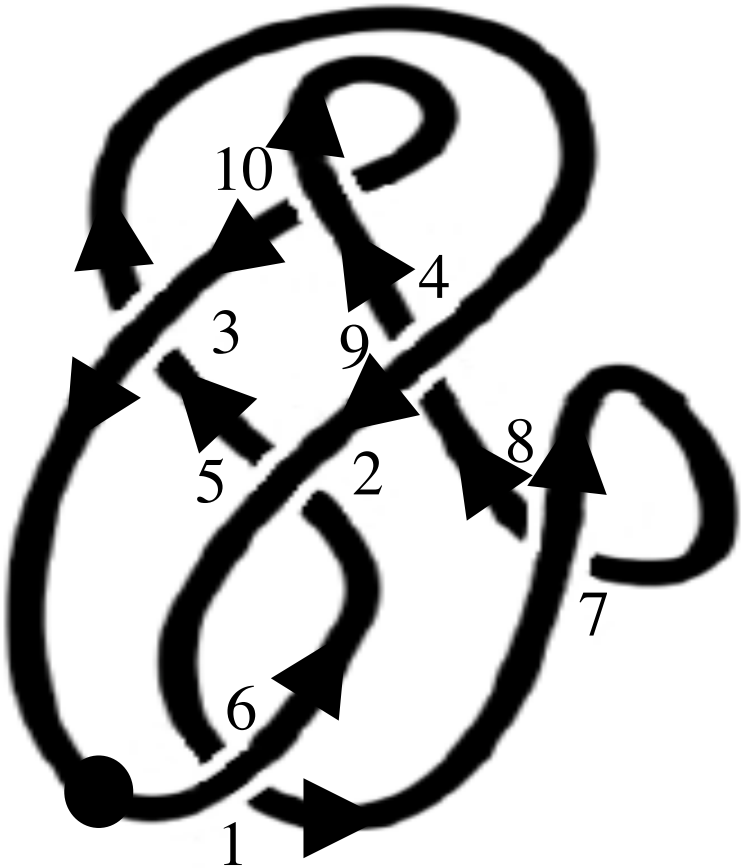

DT (Dowker-Thistlethwaite) code: a worked-out example

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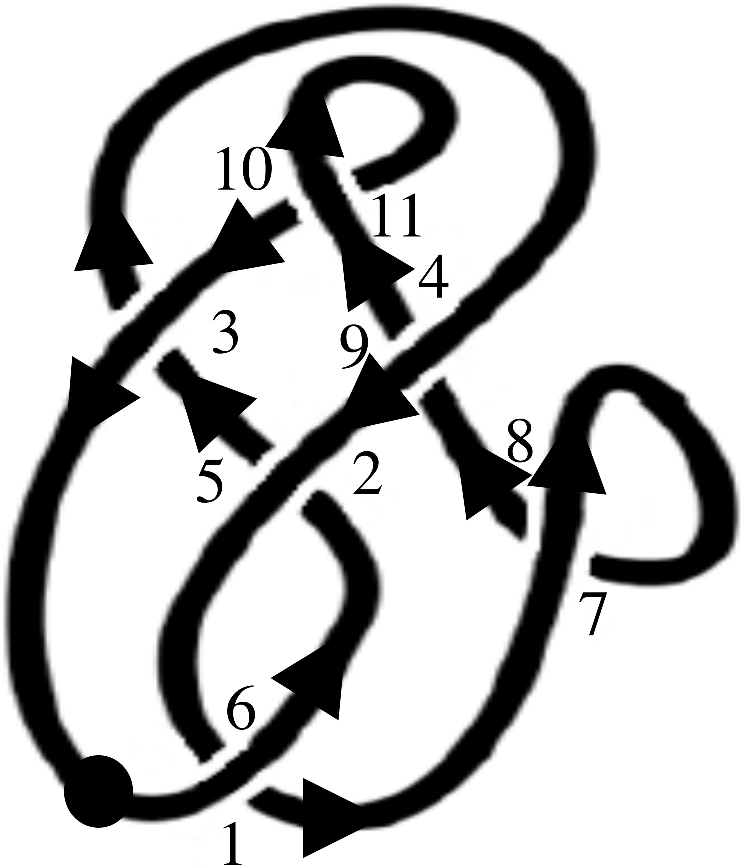

DT (Dowker-Thistlethwaite) code: a worked-out example

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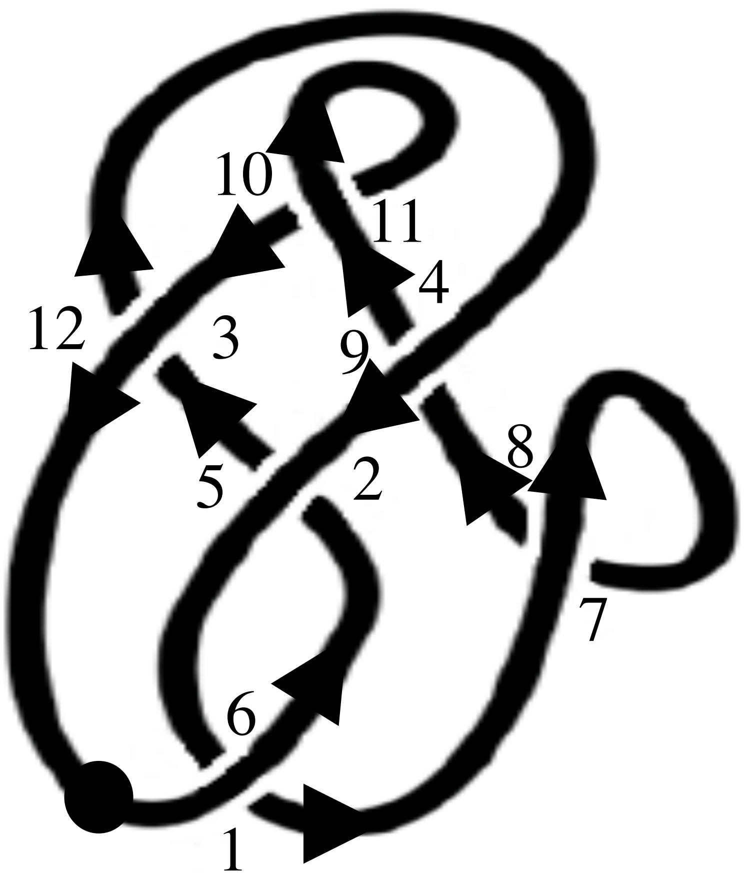
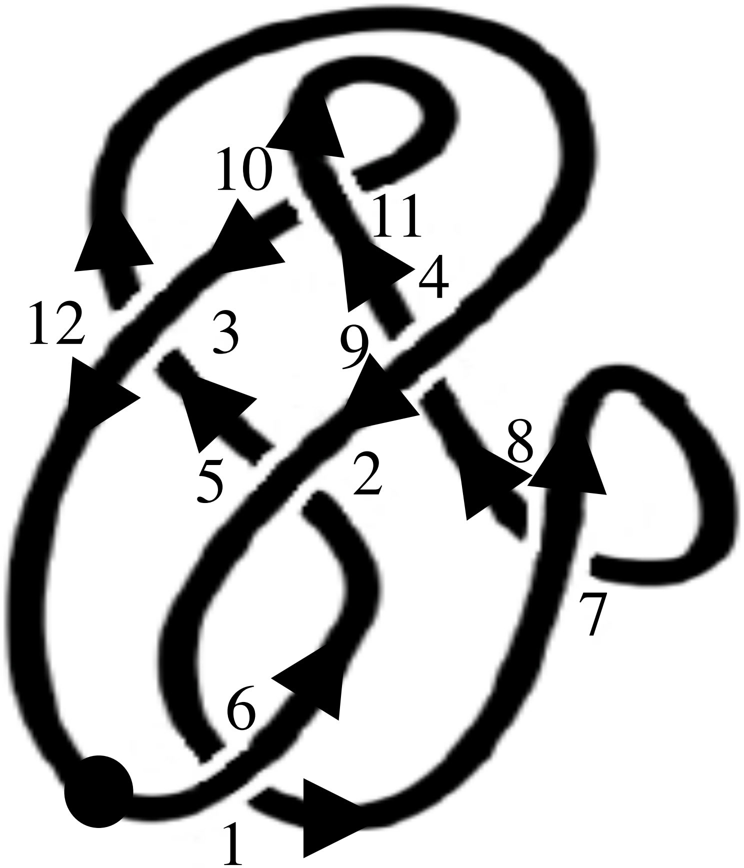


DT (Dowker-Thistlethwaite) code: a worked-out example

- *Remark: mirror knots have same DT code*

{1 2 3 4 5 6 7 8 9 10 11 12}



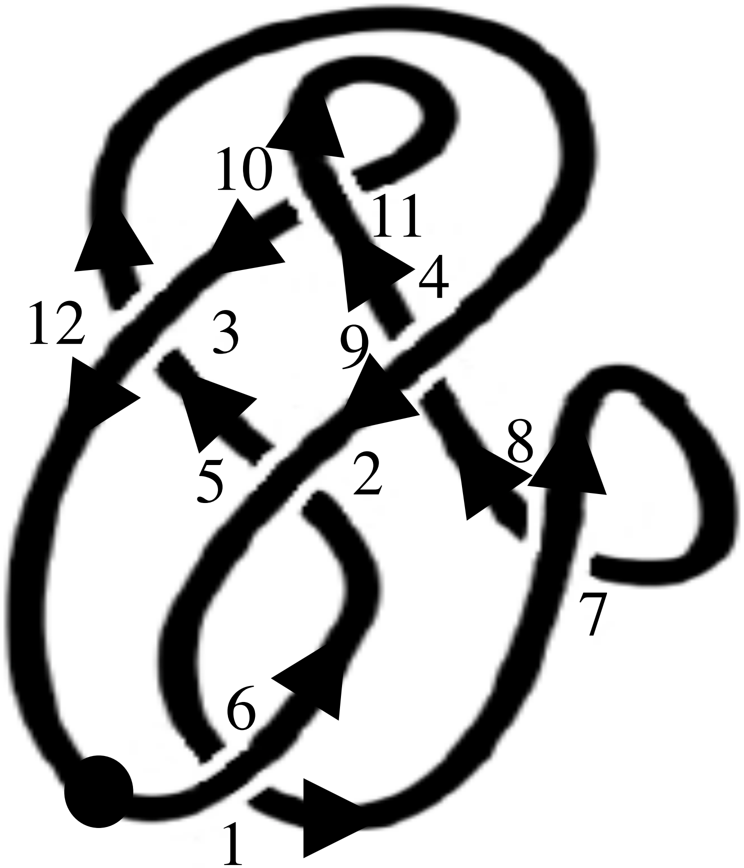
DT (Dowker-Thistlethwaite) code: a worked-out example

- ***Remark: mirror knots have same DT code***

{1 2 3 4 5 6 7 8 9 10 11 12}

↓

1	3	5	7	9	11
6	–12	2	8	–4	–10



DT (Dowker-Thistlethwaite) code: a worked-out example

- ***Remark: mirror knots have same DT code***

{1 2 3 4 5 6 7 8 9 10 11 12}

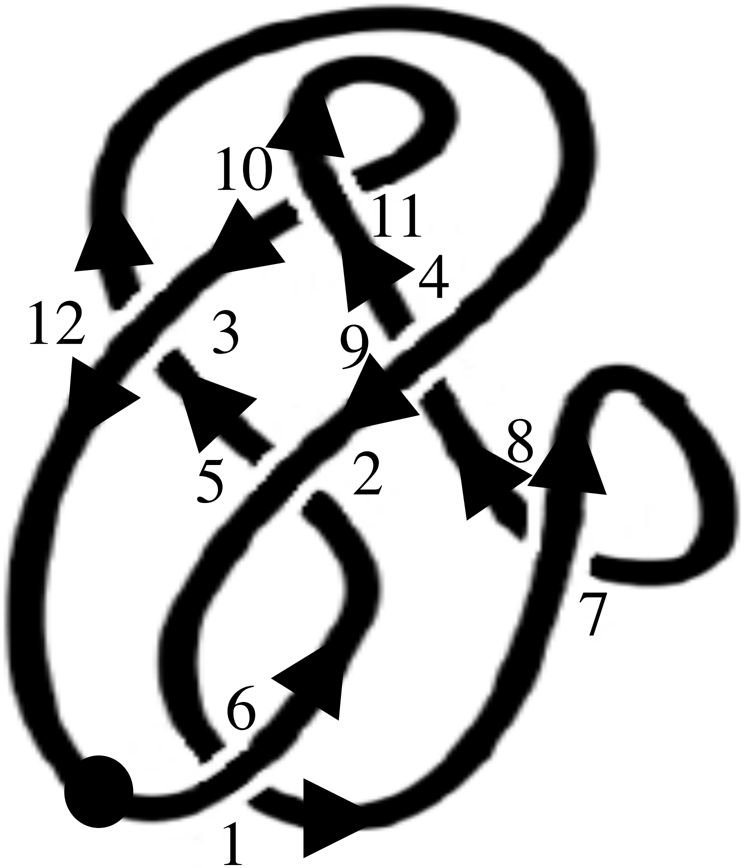
↓

1	3	5	7	9	11
6	–12	2	8	–4	–10

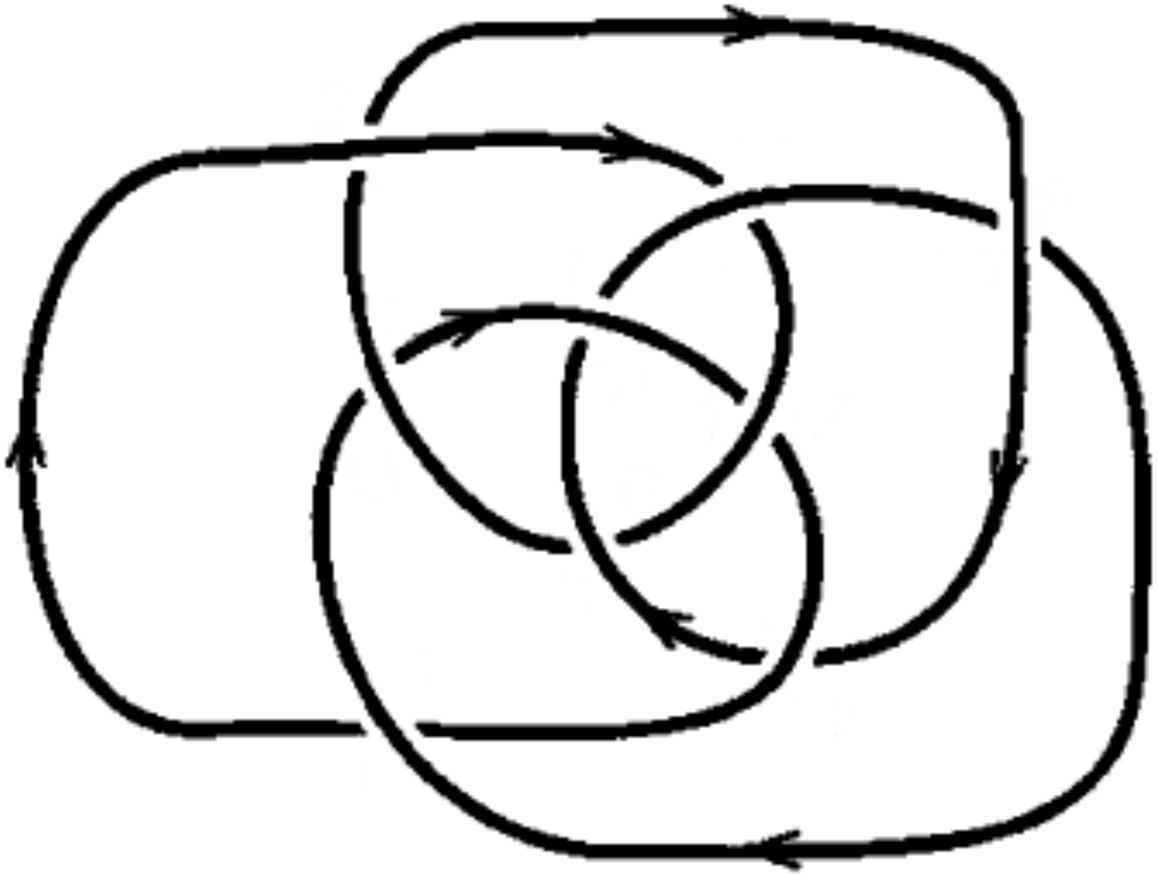
↓

DT code:

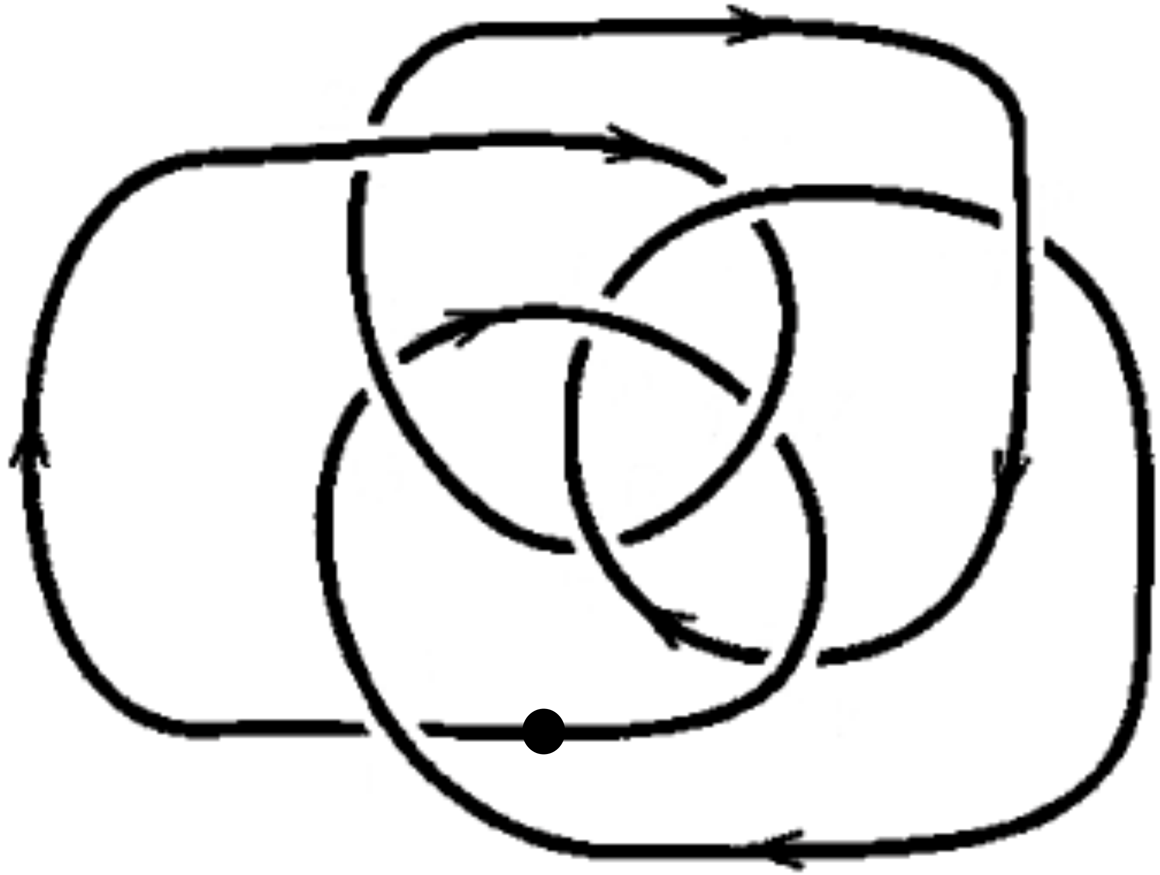
[6 –12 2 8 –4 –10]



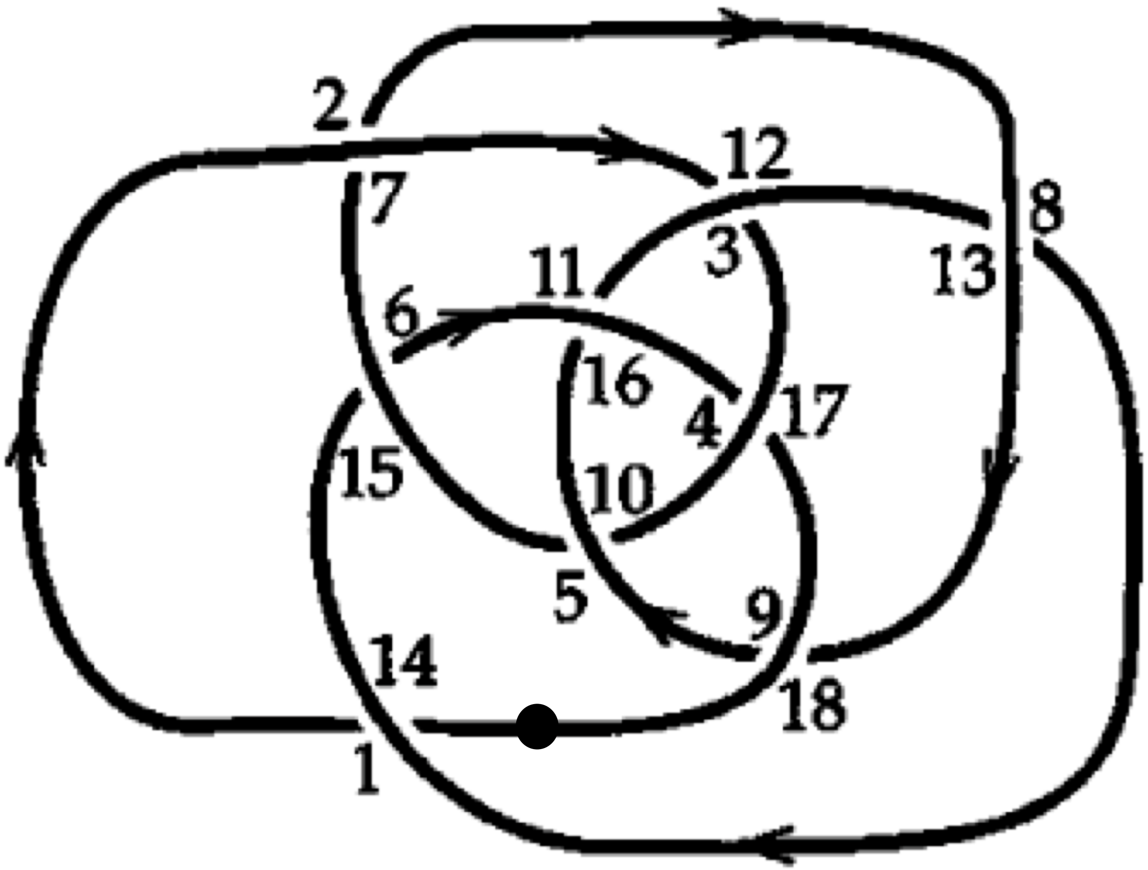
Exercise: determine the DT code of this knot



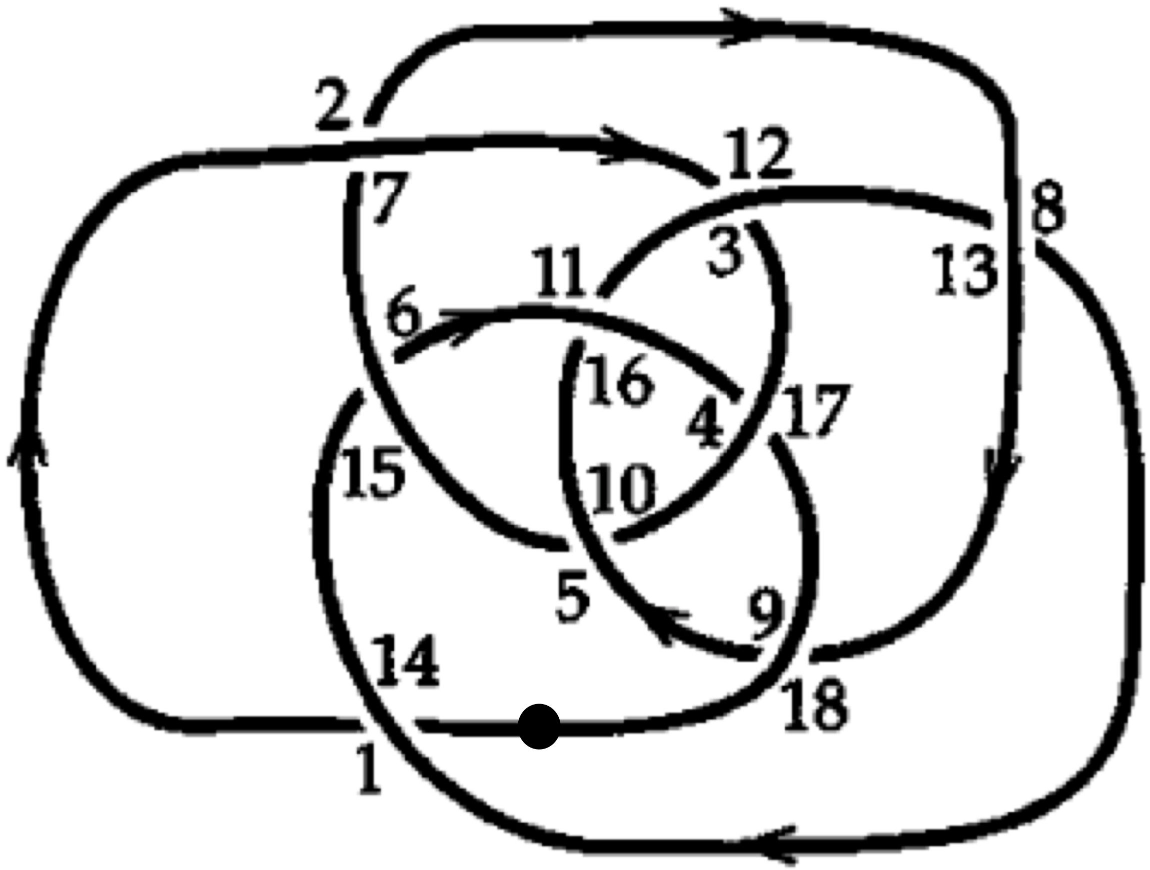
Exercise: determine the DT code of this knot



Exercise: determine the DT code of this knot



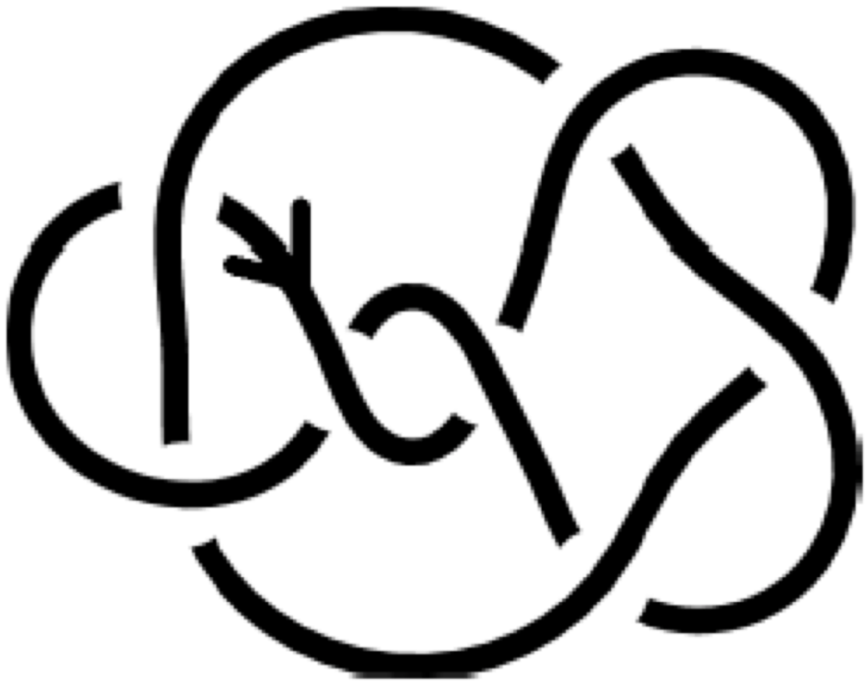
Exercise: determine the DT code of this knot



DT code: [14 12 10 2 18 16 8 6 4]

… and what about these?

(a)



(b)

Jones polynomial $V(\boldsymbol{K})$

- ***Skein relations:***

(V.1) $V(\circlearrowleft) = 1$

Jones polynomial $V(\boldsymbol{K})$

- ***Skein relations:***

$V(\boldsymbol{K})$:

(V.1) $V\Big($  $\Big) = 1$

(V.2) $\tau^{-1} V\Big($  $\Big) - \tau V\Big($  $\Big) = \left(\tau^{\frac{1}{2}} - \tau^{-\frac{1}{2}}\right) V\Big($  $\Big)$

+   −   =

Jones polynomial $V(\boldsymbol{K})$

- ***Skein relations:***

$V(\boldsymbol{K})$:

(V.1) $V(\circlearrowleft) = 1$

(V.2) $\tau^{-1} V\left(\underset{+}{\times}\right) - \tau V\left(\underset{-}{\times}\right) = \left(\tau^{\frac{1}{2}} - \tau^{-\frac{1}{2}}\right) V\left(\underset{=}{)(}\right)$

- ***Left-handed trefoil knot*** $\boldsymbol{T}^{L}$ **:**

$$V(\boldsymbol{T}^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$

$\boldsymbol{T}^{L}$: ***unknot*** $\boldsymbol{H}_{-}$

Jones polynomial $V(K)$

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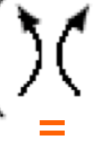
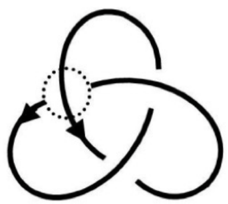
$V(K)$:

(V.1) $V($  $) = 1$

(V.2) $\tau^{-1} V($  $) - \tau V($  $) = \left(\tau^{\frac{1}{2}} - \tau^{-\frac{1}{2}}\right) V($  $)$

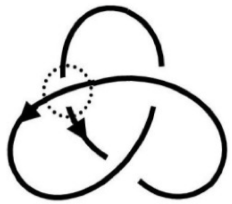
- ***Left-handed trefoil knot*** T^L **:**

$$V(T^L) = \tau^{-1} + \tau^{-3} - \tau^{-4}$$



T^L

:



unknot

H_-

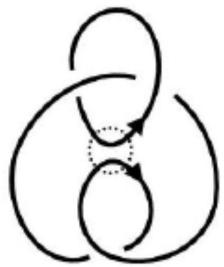
- ***Figure-of-eight knot*** F^8 **:**

$$V(F^8) = \tau^{-2} - \tau^{-1} + 1 - \tau + \tau^2$$

F^8

:

unknot



H_-

Jones polynomials of first knots

By replacing τ (dummy variable) by t, we have:

Knot type	*Jones polynomial*
3_1	$-t^{-4}+t^{-3}+t^{-1}$

Jones polynomials of first knots

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3_1	$-t^{-4}+t^{-3}+t^{-1}$
4_1	$t^{-2}-t^{-1}+1-t+t^2$
5_1	$-t^{-7}+t^{-6}-t^{-5}+t^{-4}+t^{-2}$
5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$

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5_1	$-t^{-7}+t^{-6}-t^{-5}+t^{-4}+t^{-2}$
5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$
6_1	$t^{-4}-t^{-3}+t^{-2}-2t^{-1}+2-t+t^2$
6_2	$t^{-5}-2t^{-4}+2t^{-3}-2t^{-2}+2t^{-1}-1+t$
6_3	$-t^{-3}+2t^{-2}-2t^{-1}+3-2t+2t^2-t^3$

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5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$
6_1	$t^{-4}-t^{-3}+t^{-2}-2t^{-1}+2-t+t^2$
6_2	$t^{-5}-2t^{-4}+2t^{-3}-2t^{-2}+2t^{-1}-1+t$
6_3	$-t^{-3}+2t^{-2}-2t^{-1}+3-2t+2t^2-t^3$
7_1	$-t^{-10}+t^{-9}-t^{-8}+t^{-7}-t^{-6}+t^{-5}+t^{-3}$
7_2	$-t^{-8}+t^{-7}-t^{-6}+2t^{-5}-2t^{-4}+2t^{-3}-t^{-2}+t^{-1}$
7_3	$t^2-t^3+2t^4-2t^5+3t^6-2t^7+t^8-t^9$
7_4	$t-2t^2+3t^3-2t^4+3t^5-2t^6+t^7-t^8$
7_5	$-t^{-9}+2t^{-8}-3t^{-7}+3t^{-6}-3t^{-5}+3t^{-4}-t^{-3}+t^{-2}$
7_6	$-t^{-6}+2t^{-5}-3t^{-4}+4t^{-3}-3t^{-2}+3t^{-1}-2+t$
…	…

Jones polynomials of first knots

By replacing τ (dummy variable) by t, we have:

Knot type	Jones polynomial	Code
3_1	$-t^{-4}+t^{-3}+t^{-1}$ →	$\{-4\}(-1+1\ 0+1)$
4_1	$t^{-2}-t^{-1}+1-t+t^2$	
5_1	$-t^{-7}+t^{-6}-t^{-5}+t^{-4}+t^{-2}$	
5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$	
6_1	$t^{-4}-t^{-3}+t^{-2}-2t^{-1}+2-t+t^2$	
6_2	$t^{-5}-2t^{-4}+2t^{-3}-2t^{-2}+2t^{-1}-1+t$	
6_3	$-t^{-3}+2t^{-2}-2t^{-1}+3-2t+2t^2-t^3$	
7_1	$-t^{-10}+t^{-9}-t^{-8}+t^{-7}-t^{-6}+t^{-5}+t^{-3}$	
7_2	$-t^{-8}+t^{-7}-t^{-6}+2t^{-5}-2t^{-4}+2t^{-3}-t^{-2}+t^{-1}$	
7_3	$t^2-t^3+2t^4-2t^5+3t^6-2t^7+t^8-t^9$	
7_4	$t-2t^2+3t^3-2t^4+3t^5-2t^6+t^7-t^8$	
7_5	$-t^{-9}+2t^{-8}-3t^{-7}+3t^{-6}-3t^{-5}+3t^{-4}-t^{-3}+t^{-2}$	
7_6	$-t^{-6}+2t^{-5}-3t^{-4}+4t^{-3}-3t^{-2}+3t^{-1}-2+t$	
…	…	

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4_1	$t^{-2}-t^{-1}+1-t+t^2$	$\longrightarrow$	$\{-2\}(+1-1+1-1+1)$
5_1	$-t^{-7}+t^{-6}-t^{-5}+t^{-4}+t^{-2}$		
5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$		
6_1	$t^{-4}-t^{-3}+t^{-2}-2t^{-1}+2-t+t^2$		
6_2	$t^{-5}-2t^{-4}+2t^{-3}-2t^{-2}+2t^{-1}-1+t$		
6_3	$-t^{-3}+2t^{-2}-2t^{-1}+3-2t+2t^2-t^3$		
7_1	$-t^{-10}+t^{-9}-t^{-8}+t^{-7}-t^{-6}+t^{-5}+t^{-3}$		
7_2	$-t^{-8}+t^{-7}-t^{-6}+2t^{-5}-2t^{-4}+2t^{-3}-t^{-2}+t^{-1}$		
7_3	$t^2-t^3+2t^4-2t^5+3t^6-2t^7+t^8-t^9$		
7_4	$t-2t^2+3t^3-2t^4+3t^5-2t^6+t^7-t^8$		
7_5	$-t^{-9}+2t^{-8}-3t^{-7}+3t^{-6}-3t^{-5}+3t^{-4}-t^{-3}+t^{-2}$		
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…	…		

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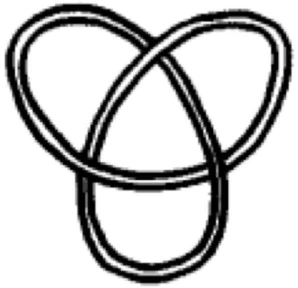
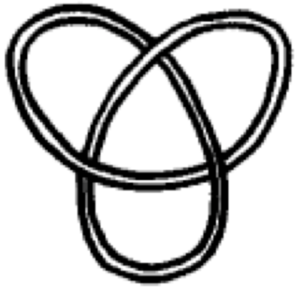
5_1	$-t^{-7}+t^{-6}-t^{-5}+t^{-4}+t^{-2}$	→	… …
5_2	$-t^{-6}+t^{-5}-t^{-4}+2t^{-3}-t^{-2}+t^{-1}$		
6_1	$t^{-4}-t^{-3}+t^{-2}-2t^{-1}+2-t+t^2$		
6_2	$t^{-5}-2t^{-4}+2t^{-3}-2t^{-2}+2t^{-1}-1+t$		
6_3	$-t^{-3}+2t^{-2}-2t^{-1}+3-2t+2t^2-t^3$		
7_1	$-t^{-10}+t^{-9}-t^{-8}+t^{-7}-t^{-6}+t^{-5}+t^{-3}$		
7_2	$-t^{-8}+t^{-7}-t^{-6}+2t^{-5}-2t^{-4}+2t^{-3}-t^{-2}+t^{-1}$		
7_3	$t^2-t^3+2t^4-2t^5+3t^6-2t^7+t^8-t^9$		
7_4	$t-2t^2+3t^3-2t^4+3t^5-2t^6+t^7-t^8$		
7_5	$-t^{-9}+2t^{-8}-3t^{-7}+3t^{-6}-3t^{-5}+3t^{-4}-t^{-3}+t^{-2}$		
7_6	$-t^{-6}+2t^{-5}-3t^{-4}+4t^{-3}-3t^{-2}+3t^{-1}-2+t$		
…	…		

Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

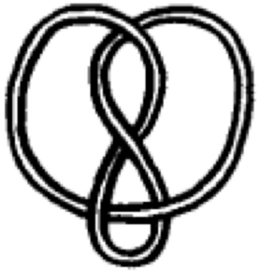
T^L **:** 

Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

$\boldsymbol{T}^{L}$ **:**

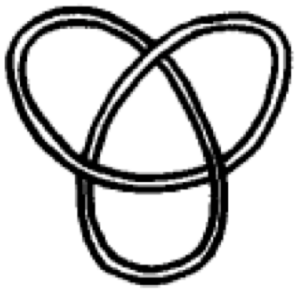


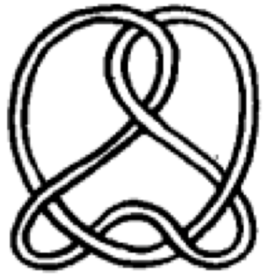
$\boldsymbol{F}^{8}$ **:**



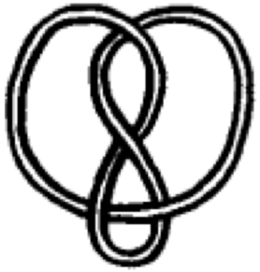
Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

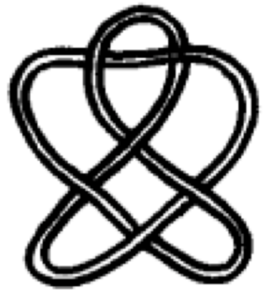
T ^{L} **:**

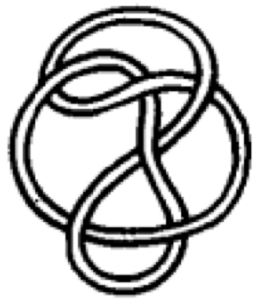




F ^{8} **:**

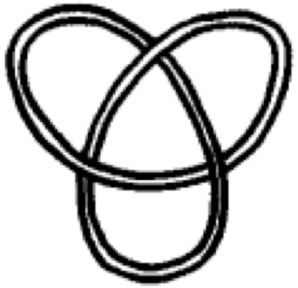




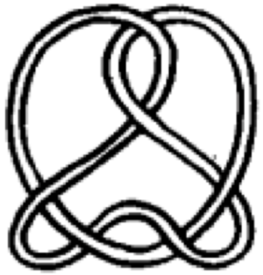


Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

T^{L} :

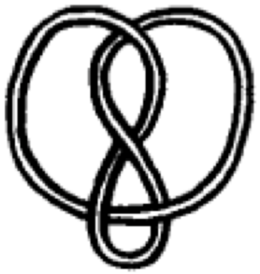


3_1

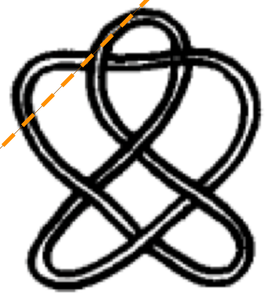


6_1

F^{8} :

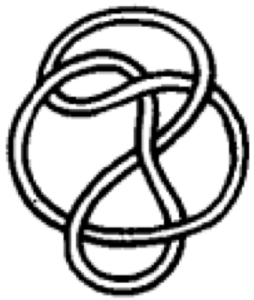


4_1



6_2

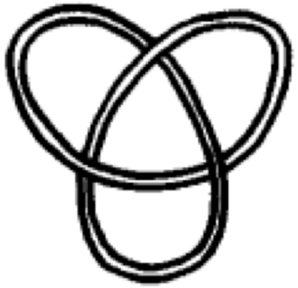
- Alexander-Briggs notation (knot/link type)



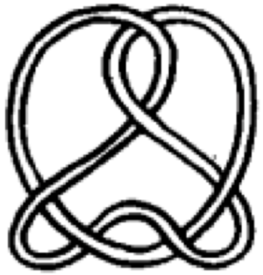
6_3

Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

T^L:

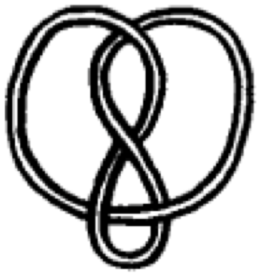


3_1 3

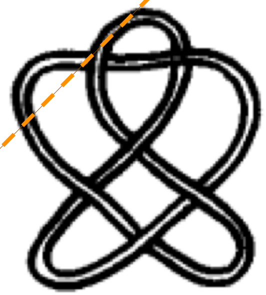


6_1 42

F^8:

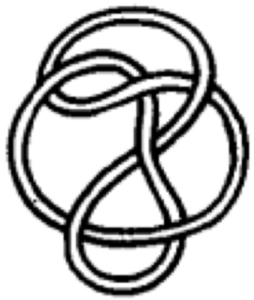


4_1 22



6_2 312

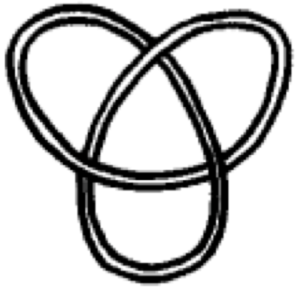
- *Alexander-Briggs notation (knot/link type)*
- *Conway notation*



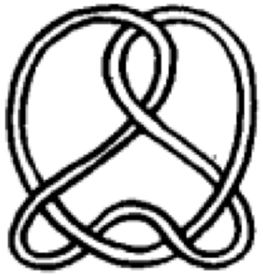
6_3 2112

Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

T^L :

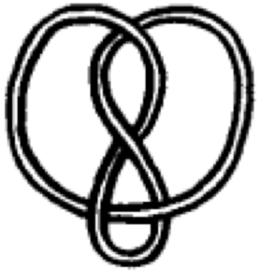


3_1 3
0.0

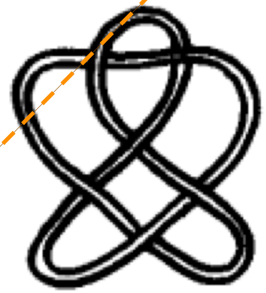


6_1 42
3.16396322

F^8 :

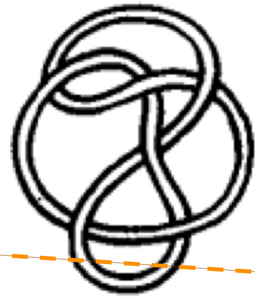


4_1 22
2.02988321



6_2 312
4.40083251

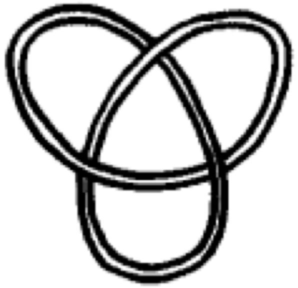
- Alexander-Briggs notation (knot/link type)
- Conway notation
- hyperbolic volume



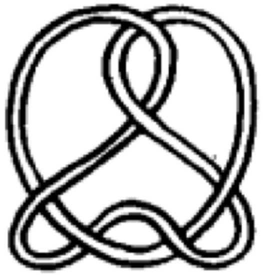
6_3 2112
5.69302109

Alexander-Briggs notation (up to 9 crossings – Adams, 1994)

T^L :

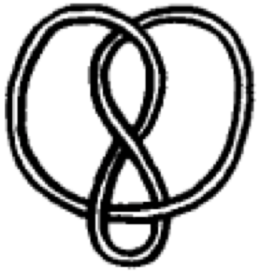


3_1 3
0.0
{−4} (−1 1 0 1)

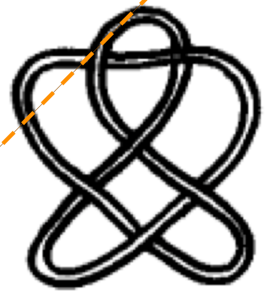


6_1 42
3.16396322
{−4} (1 −1 1 −2 2 −1 1)

F^8 :

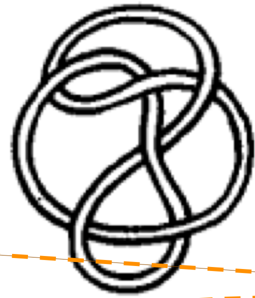


4_1 22
2.02988321
{−2} (1 −1 1 −1 1)



6_2 312
4.40083251
{−5} (1 −2 2 −2 2 −1 1)

- ***Alexander-Briggs notation (knot/link type)***
- ***Conway notation***
- ***hyperbolic volume***
- ***Jones notation (minimum degree & coefficients)***



6_3 2112
5.69302109
{−3} (−1 2 −2 3 −2 2 −1)

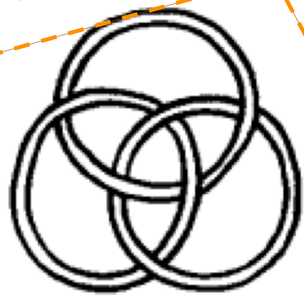
Alexander-Briggs notation (up to 10 crossings)

# of link components

8^2_{15} 22,2,2−

3.66386237

$\{\frac{-5}{2}\}$ (−1 1 −2 1 −1 1 −1)



6^3_2 .1

7.32772475

$\{\frac{-6}{2}\}$ (−1 3 −2 4 −2 3 −1)

8^2_{16} 211,2,2−

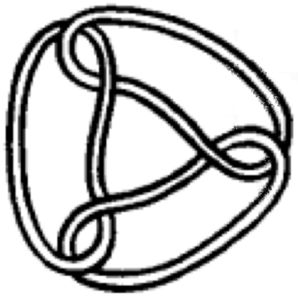
5.33348956

$\{\frac{1}{2}\}$ (−2 2 −2 2 −2 1 −1)

6^3_3 2,2,2−

0.0

$\{\frac{-8}{2}\}$ (1 0 1 0 2)



6^3_1 2,2,2

5.33348956

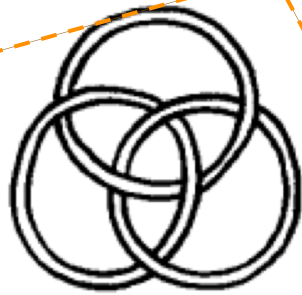
$\{\frac{2}{2}\}$ (1 −2 3 −1 3 −1 1)

… …

Alexander-Briggs notation (up to 10 crossings)

# of link components

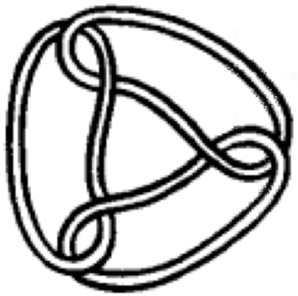
8^2_{15} 22,2,2−
3.66386237
$\{\frac{-5}{2}\}$ (−1 1 −2 1 −1 1 −1)



6^3_2 .1
7.32772475
$\{\frac{-6}{2}\}$ (−1 3 −2 4 −2 3 −1)

8^2_{16} 211,2,2−
5.33348956
$\{\frac{1}{2}\}$ (−2 2 −2 2 −2 1 −1)

6^3_3 2,2,2−
0.0
$\{\frac{-8}{2}\}$ (1 0 1 0 2)



6^3_1 2,2,2
5.33348956
$\{\frac{2}{2}\}$ (1 −2 3 −1 3 −1 1)

… …

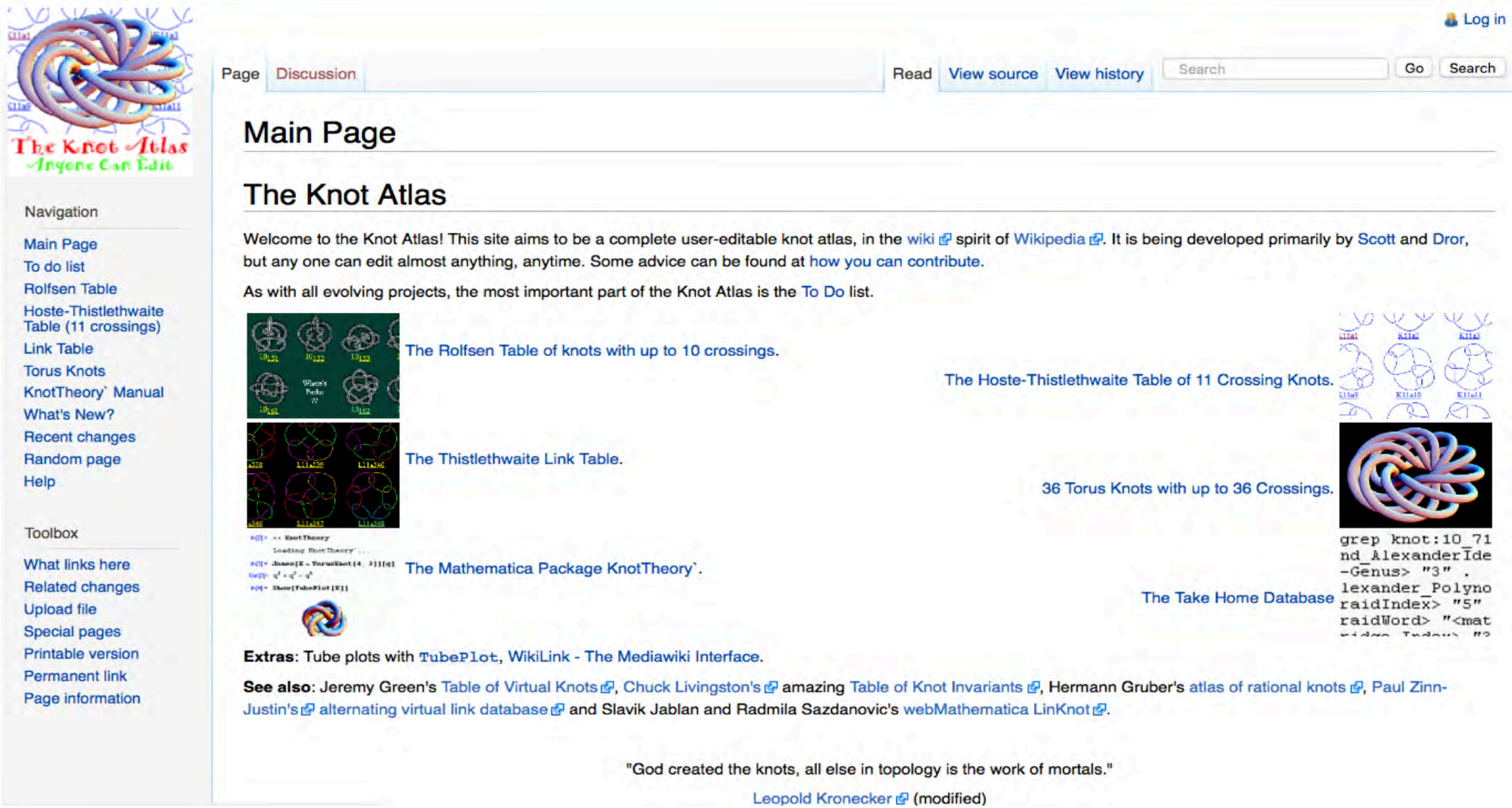
Alexander-Briggs notation up to 10 crossings; then DT code of type "Kc_{min}a123" or "Kc_{min}n123" in lexicographical order

Online databases - 1

Data bases of knots and invariants:

- Knot Atlas by Dror Bar Natan and Scott Morrisson

http://katlas.math.toronto.edu/wiki/Main_Page



Log in

Page | Discussion | Read | View source | View history | Search | Go | Search

The Knot Atlas — Anyone Can Edit

Navigation

- Main Page
- To do list
- Rolfsen Table
- Hoste-Thistlethwaite Table (11 crossings)
- Link Table
- Torus Knots
- KnotTheory` Manual
- What's New?
- Recent changes
- Random page
- Help

Toolbox

- What links here
- Related changes
- Upload file
- Special pages
- Printable version
- Permanent link
- Page information

Main Page

The Knot Atlas

Welcome to the Knot Atlas! This site aims to be a complete user-editable knot atlas, in the wiki spirit of Wikipedia. It is being developed primarily by Scott and Dror, but any one can edit almost anything, anytime. Some advice can be found at how you can contribute.

As with all evolving projects, the most important part of the Knot Atlas is the To Do list.

The Rolfsen Table of knots with up to 10 crossings.

The Hoste-Thistlethwaite Table of 11 Crossing Knots.

The Thistlethwaite Link Table.

36 Torus Knots with up to 36 Crossings.

The Mathematica Package KnotTheory`.

The Take Home Database

Extras: Tube plots with `TubePlot`, WikiLink - The Mediawiki Interface.

See also: Jeremy Green's Table of Virtual Knots, Chuck Livingston's amazing Table of Knot Invariants, Hermann Gruber's atlas of rational knots, Paul Zinn-Justin's alternating virtual link database and Slavik Jablan and Radmila Sazdanovic's webMathematica LinKnot.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

Online databases - 2

Data bases of knots and invariants:

- KnotInfo by Chuck Livingston

http://www.indiana.edu/~knotinfo/

Table of Knot Invariants

Please try **LinkInfo:Table of Link Invariants** and give us feedback.

Please Cite KnotInfo	Visitor List	Full Database Download
Calculators: Concordance, Homology	Unknown Values (last updated: 14 June 2014)	Acknowledgments

Build a Knot Table. Welcome to KnotInfo. Check the desired boxes in the sections below and then click SUBMIT on the page to produce your desired table of knots. If you do not know the name of a particular knot you are interested in, KnotFinder can help you.

Preferences (Select invariants to hide.)

New advanced search feature is now available! [Advanced Search]

Select knots you want tabulated. **[Advanced Search]**
Specify crossing numbers. The letters a and n designate alternating and nonalternating knots. *12 crossing knots are grouped.*

3-6	7	8	9	10
11a	11n	12a (1-200)	12a (201-400)	12a (401-600)
12a (601-800)	12a (801-1000)	12a (1001-1200)	12a (1200-1288)	12n (1-200)
12n (201-400)	12n (401-600)	12n (601-800)	12n (801-888)	All

Names and descriptions. Please select the naming and notational descriptions desired. *Names are linked to diagrams.*

Name	Name Rank	Alternating	DT Name
DT Notation	DT Rank	Classical Conway Name	Conway Notation
Gauss Notation	PD Notation	Braid Notation	Two-Bridge Notation
Fibered	Monodromy	Tetrahedral Census Name	

Three-Dimensional Invariants.

Arc Index	Braid Index	Braid Length	Bridge Index
Crosscap Number	Crossing Number	Determinant	Morse-Novikov Number
Nakanishi Index	Polygon Index	Seifert Matrix	Small or Large
Super Bridge Index	Symmetry Type	Three Genus	Thurston-Bennequin Number
Tunnel Number	Turaev Genus	Unknotting Number	Width

Concordance and Four-Dimensional Invariants.

Arf Invariant	Four-Ball Crossing Number	Smooth Concordance Genus	Topological Concordance Genus
Smooth Concordance Order	Topological Concordance Order	Algebraic Concordance Order	Smooth Four Genus
Topological Four Genus	Smooth 4D Crosscap Number	Topological 4D Crosscap Number	Rasmussen Invariant
Ozsvath-Szabo Tau-Invariant	Signature	Signature Function	Smooth Concordance Crosscap Number
Topological Concordance Crosscap Number			

Positivity.

Positive Braid	Positive	Strongly Quasipositive	Quasipositive
Positive Braid Notation	Positive PD notation	Strongly Quasipositive Braid Notation	Quasipositive Braid Notation

Polynomial Invariants.

A-Polynomial	Alexander Polynomial	Conway Polynomial	HOMFLY Polynomial
Jones Polynomial	Kauffman Polynomial	Khovanov Polynomial	Khovanov Torsion Polynomial
Show polynomials as coefficient vectors			

Hyperbolic Invariants.

Volume	Maximum Cusp Volume	Longitude Length	Meridian Length
Longitude Translation	Meridian Translation	Other Short Geodesics	Full Symmetry Group
Chern-Simons Invariant			

Diagrams and Other Information. Check if you want to see small diagrams (linked to larger figures) and if you want the table to include links to the Knot Atlas site.

Diagram	Knot Atlas Page

Submission. Click to submit query.

KnotInfo was created and is maintained by Chuck Livingston with the assistance of Jae Choon Cha. Please send your comments to Chuck Livingston: livingst@indiana.edu. Knotinfo is partially supported by Indiana University and by the NSF. Privacy Notice

Online databases - 2

- KnotFinder powered by Knotscape on KnotInfo website
http://www.indiana.edu/~knotinfo/knotfinder.php

Input: DT notation for prime knots up to 13 crossings

Output: Classical name

- KnotSketcher by Jiho Kim Java program

http://www.indiana.edu/~knotinfo/homelinks/knotsketcher.html

Input: Drawing

Output: DT name

Knot theory software - 1

KNOT THEORY
D. Bar Natan

http://katlas.math.toronto.edu/wiki/The_Mathematica_Package_KnotTheory

Input:

- Any notation you can think of ...

Can compute:

- Almost anything you can come up with in classical knot theory
- Khovanov_homology-J. Green, other A. Shumakovitch KhoHo

Cotton Seed.

- Heegaard Floer by Jean Marie Droz (other: J. Baldwin, W.D. Gillam)

Knot theory software - 2

KNOTSCAPE

Jim Hoste & Morween Thistlethwaite

With the input from Bruce Ewing, Ken Millett, Ken Stephenson and Jeff Weeks (SnapPea, new version SnapPy by M. Culler and N. Dunfield)

- http://pzacad.pitzer.edu/~jhoste/HosteWebPages/kntscp.html

INPUT:

- Braid
- Dowker/Thistlethwaite code (Gauss code)
- by selecting from the knot tables- classical name
- or by drawing with a mouse

KNOTSCAPE

Jim Hoste & Morween Thistlethwaite

- http://pzacad.pitzer.edu/~jhoste/HosteWebPages/kntscp.html

OUPUT:

- find the knot (or its summands with respect to connected sum) in the tables (provided it is truly a 16 crossing knot or less!),
- draw a pretty picture of it,
- compute its HOMFLY, Kauffman, Jones, or Alexander polynomial,
- compute representations of its fundamental group into the symmetric group on five letters, or
- compute various hyperbolic invariants a la Snap Pea.

LinKnot

Slavik V Jablan & Sazdanovic Radmila

http://www.mi.sanu.ac.rs/vismath/linknot/

LinKnot

by S. Jablan, R. Sazdanovic

including

KNOT 2000

by M.Ochiai, N.Imafuji

and

KnotTheory

+

some functions from the program

KhoHo

by A. Shumakovitch

+

some functions from the program

SnapPy

by M. Culler and N. Dunfield

New version, updated 20.02.2011, working in *Mathematica* 5.2, 6.0, 7.0, and 8.0,

with more than 100 new functions,

working also with virtual knots and links, hyperbolic volume of knots and links, Tutte polynomials of knots and links (see the program Tutte80.nb in the directory *Tutte Program*), and knots and polyhedra (see the program POLYKNOTS80.nb)

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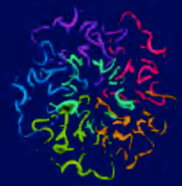

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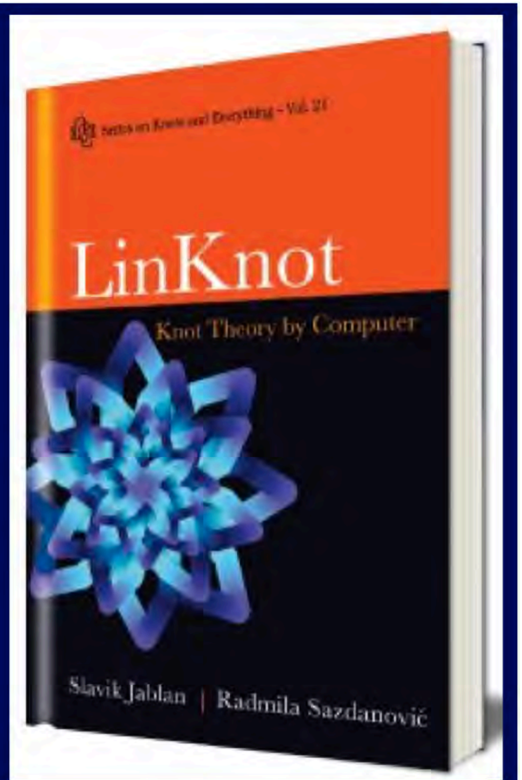

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Alexander-Briggs tabulation vs. alternative tabulations

6_1 42

3.16396322

{−4} (1 −1 1 −2 2 −1 1)

Alexander-Briggs tabulation vs. alternative tabulations

6_1 42
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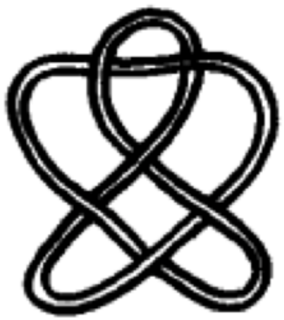


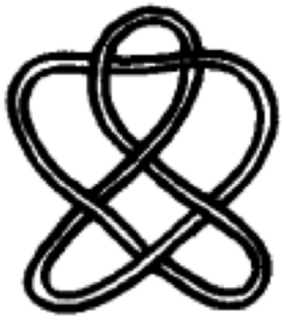
6_2 312
4.40083251

{−5} (1 −2 2 −2 2 −1 1)

Alexander-Briggs tabulation vs. alternative tabulations

6_1 42
3.16396322

{−4} (1−1 1 −2 2 −1 1)



6_2 312
4.40083251

{−5} (1 −2 2 −2 2 −1 1)

6_3
5.69302109

{−3} (−1 2 −2 3 −2 2 −1)

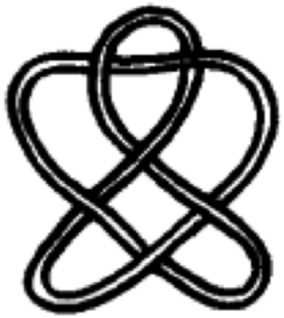
Alexander-Briggs tabulation vs. alternative tabulations

6_1 42

3.16396322

{−4} (1 −1 1 −2 2 −1 1)

?



6_2 312

4.40083251

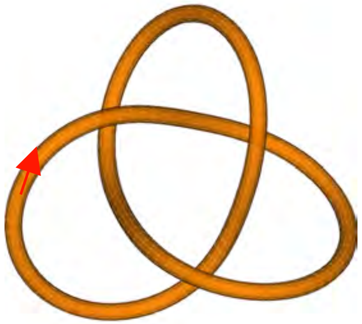
{−5} (1 −2 2 −2 2 −1 1)

6_3

5.69302109

{−3} (−1 2 −2 3 −2 2 −1)

Knot tightening software



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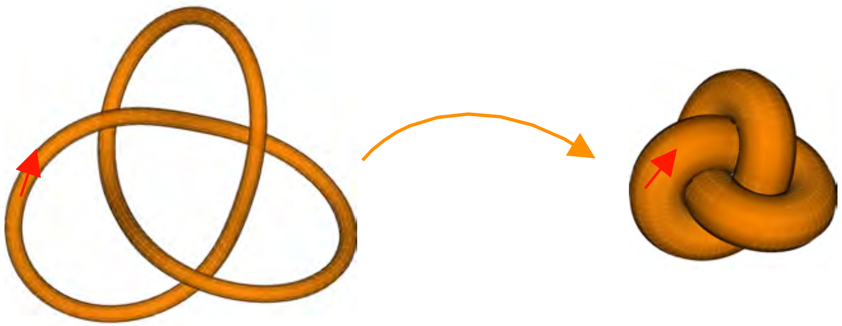
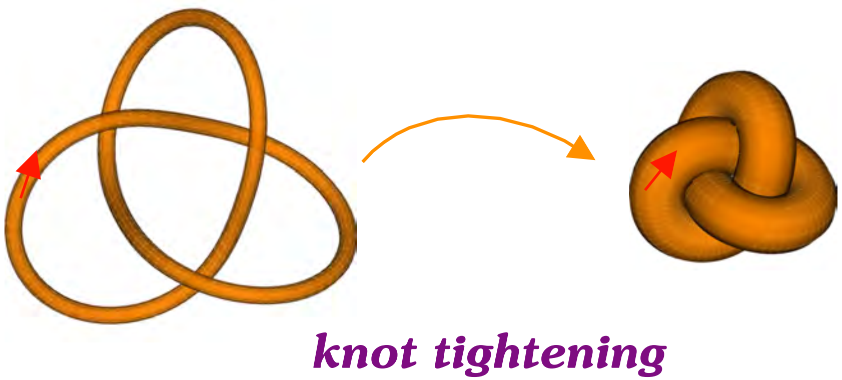
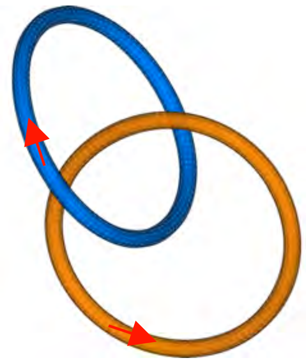


knot tightening

Knot tightening software





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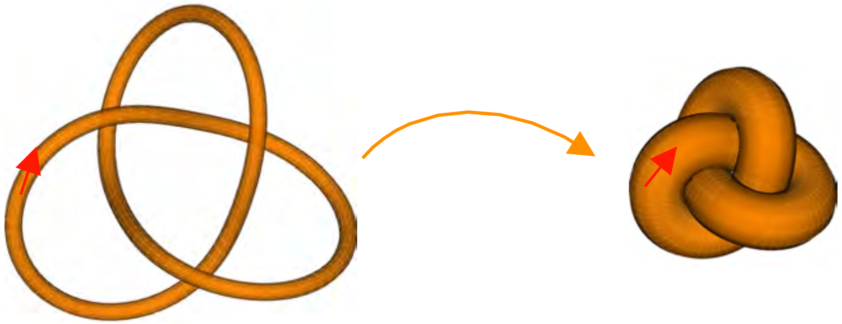

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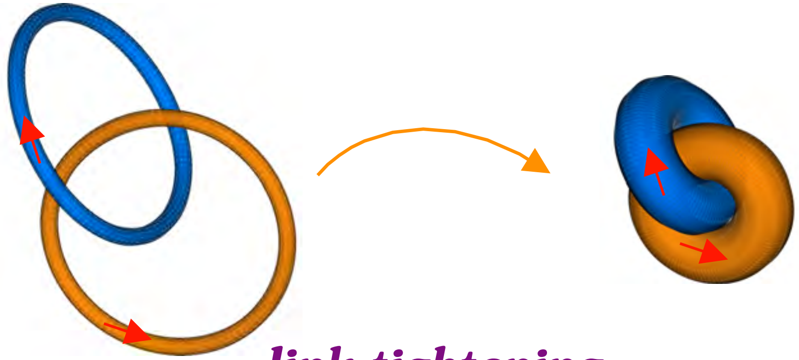
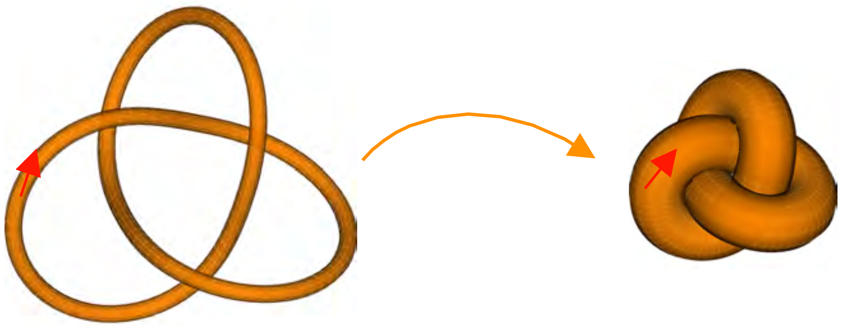
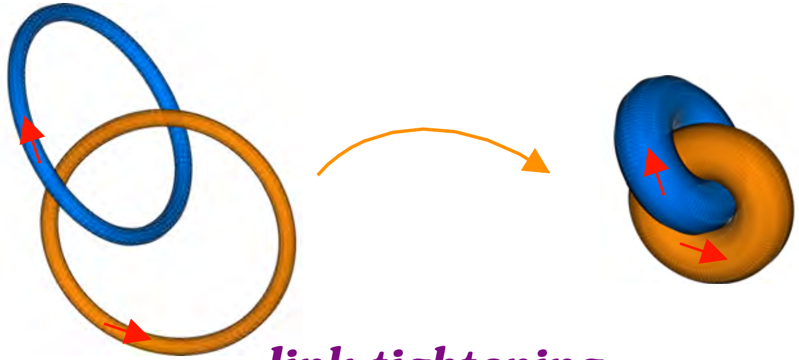


link tightening

Knot tightening software

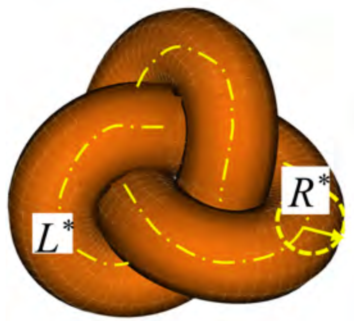


knot tightening



link tightening

- ***Ropelength:*** $\lambda = \frac{L_{\min}}{R_{\max}} = \frac{L^*}{R^*}$



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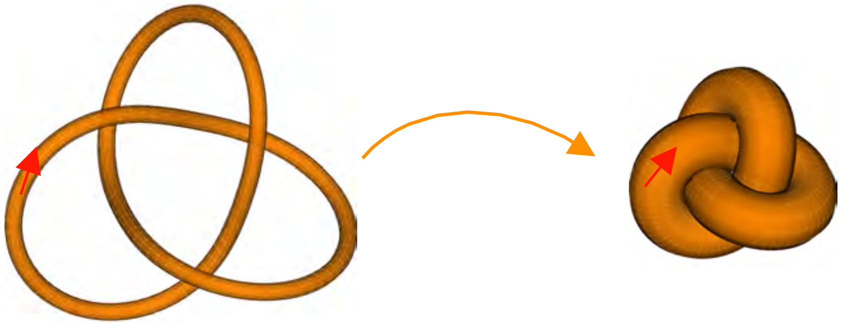

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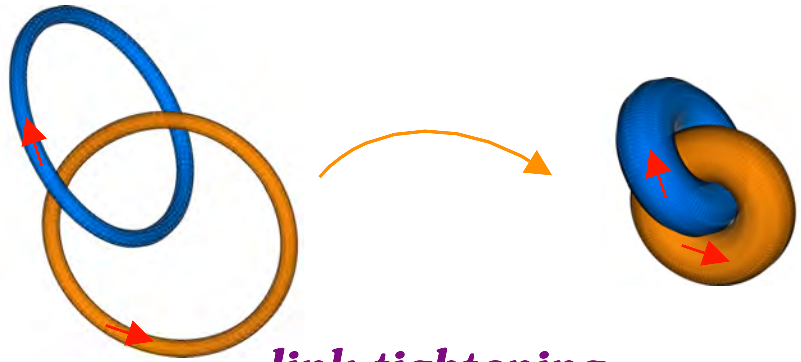

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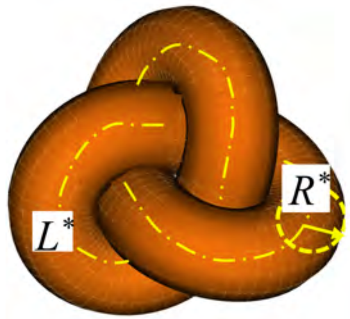


- SONO by Piotr Pieransky

  http://etacar.put.poznan.pl/piotr.pieranski/

- Ridgerunner by Jason Cantarella

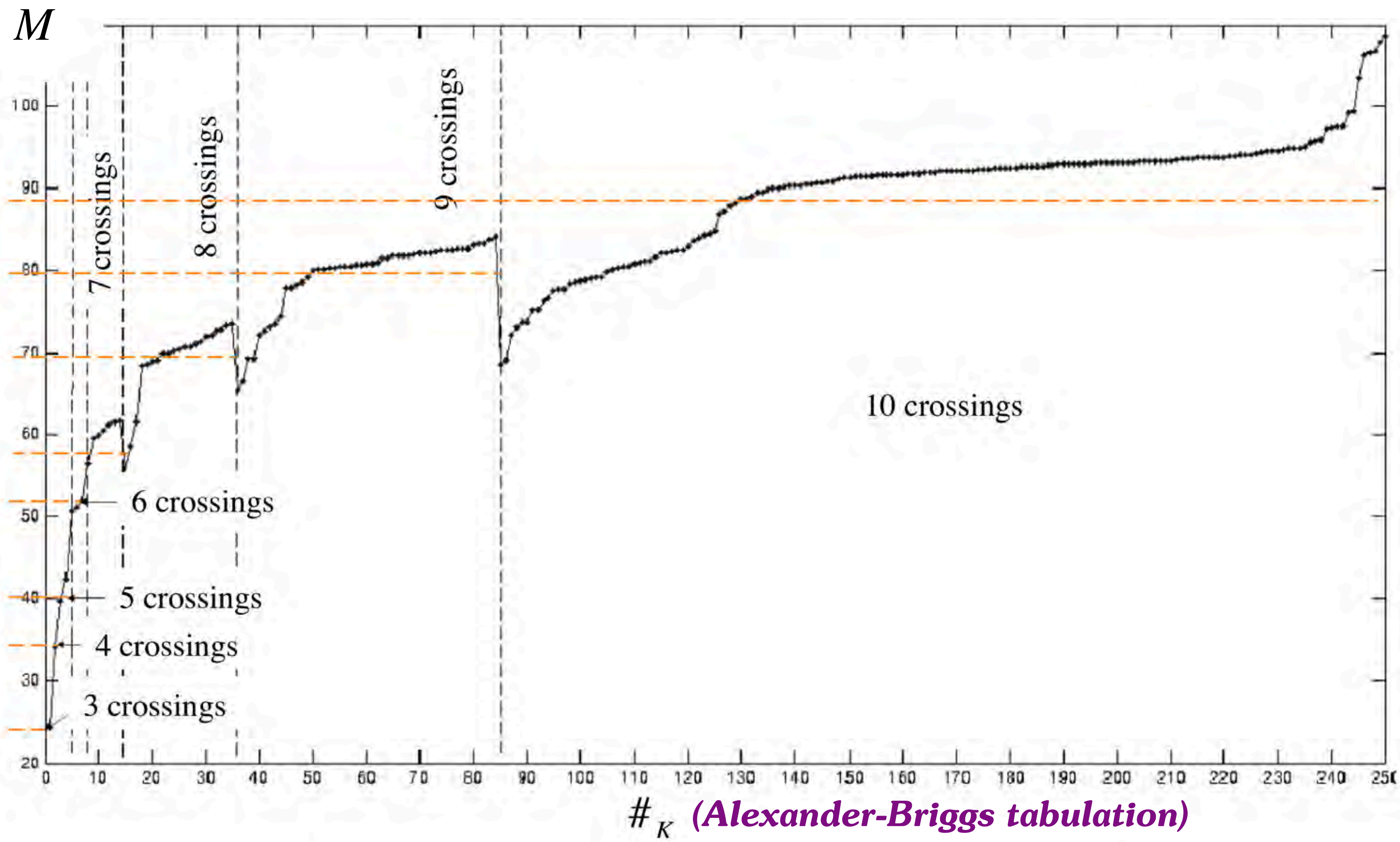
  http://www.jasoncantarella.com/webpage/index.php?title=Software

Tight knots and groundstate energy spectrum

M = *magnetic/elastic energy (in non-dimensional units)*

Tight knots and groundstate energy spectrum

M = magnetic (elastic) energy (in non-dimensional units)



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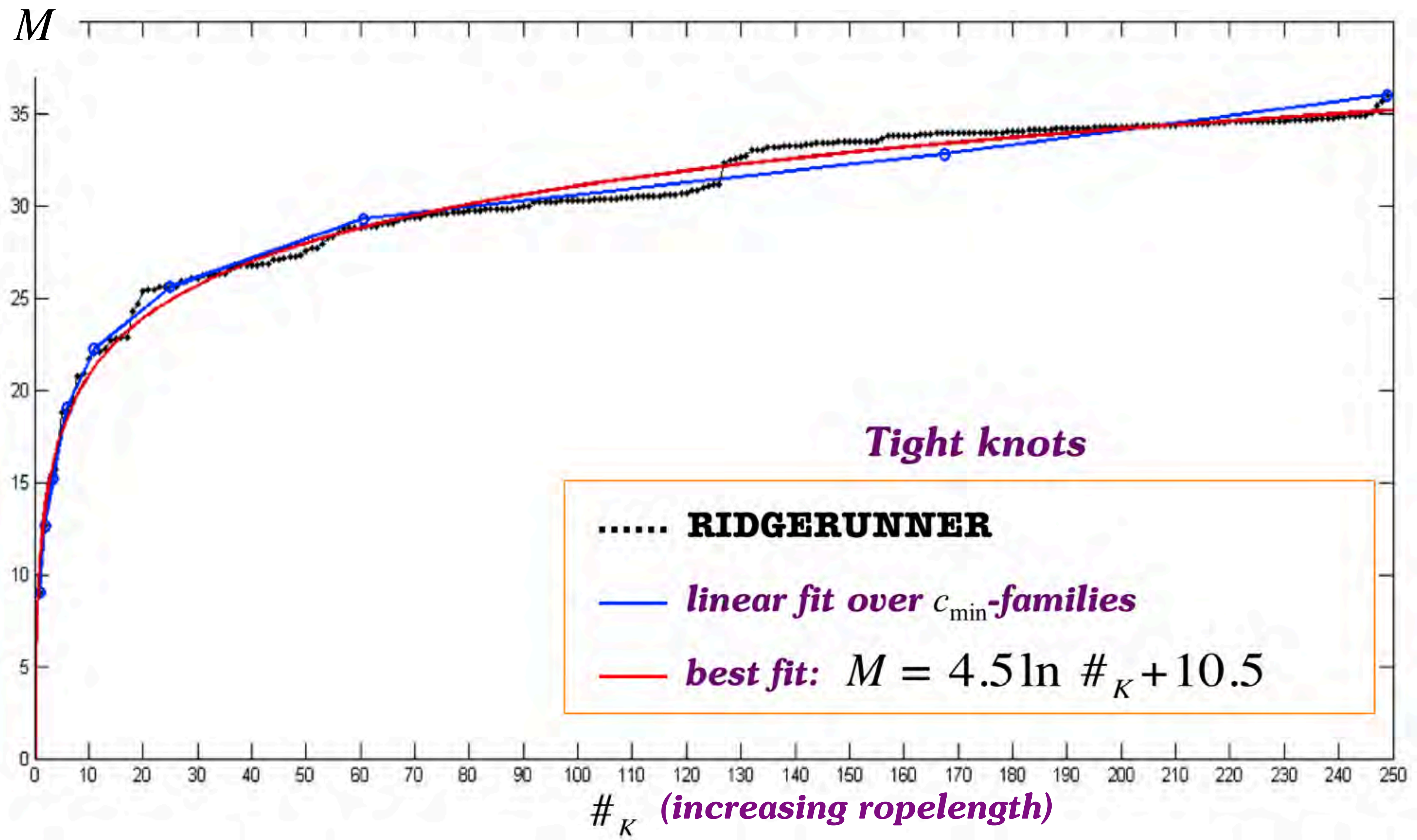

(Ricca & Maggioni, J Phys A 2014)

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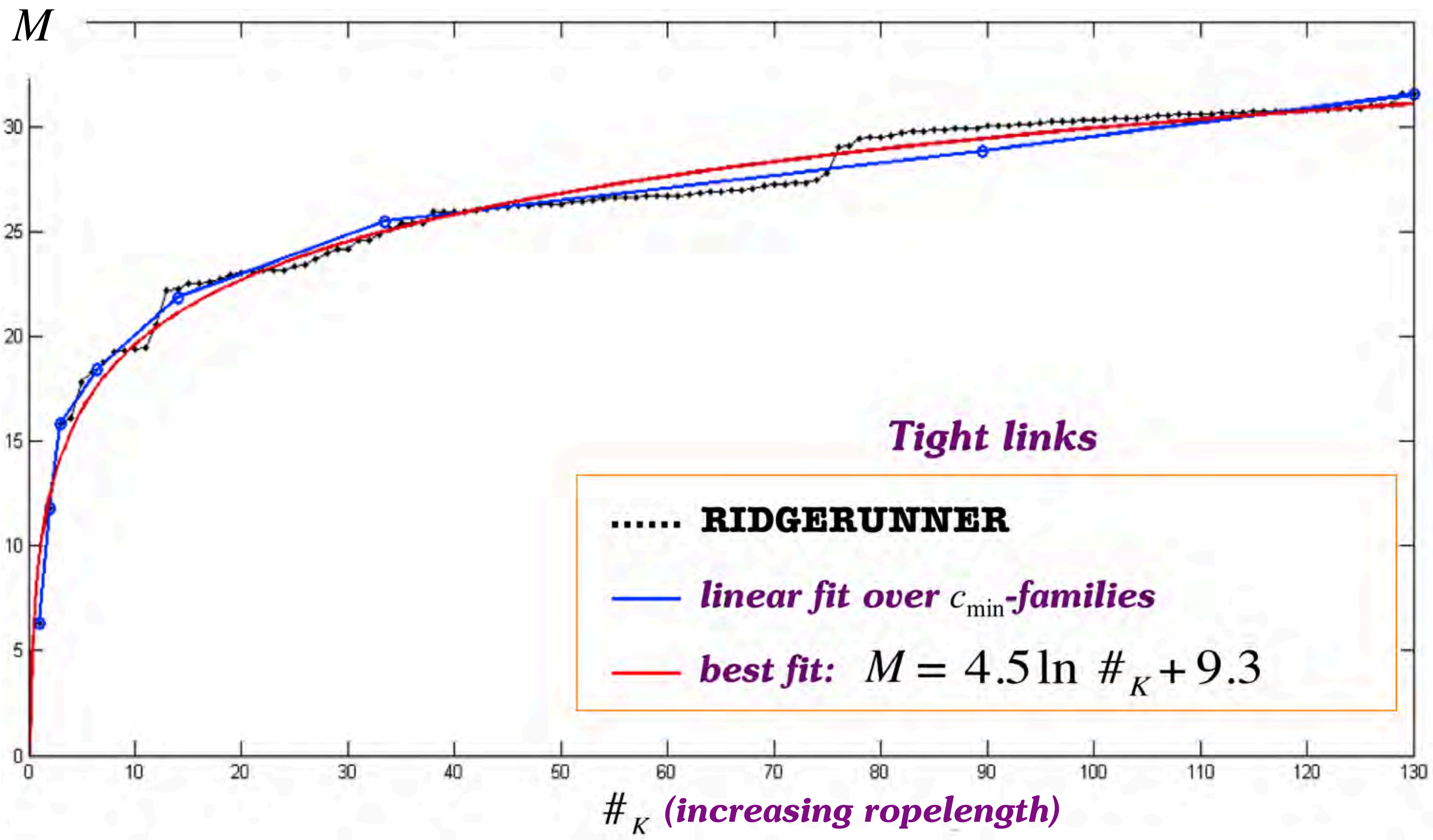

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Knot visualization, calculator and software explorer

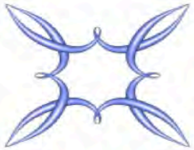
Visualization, exploration and experimentation:

KNOTPLOT

by Robert Scharein

http://knotplot.com/

- Visualize and manipulate knots in three and four dimensions.



The KnotPlot Site

Welcome to the KnotPlot Site!

Here you will find a collection of knots and links, viewed from a (mostly) mathematical perspective. Nearly all of the images here were created with KnotPlot, an elaborate program to visualize and manipulate mathematical knots in three and four dimensions.

DOWNLOAD KNOTPLOT FOR WINDOWS, MACOSX AND LINUX

 **Knot Pictures** 

Check out the mathematical knots (**M**) page as well to see more knot pictures. Or try some of the following examples to see some knots in a different light. The pages marked with New! have been updated or created as of May 2006. Those marked with an **M** have at least one MPEG animation.

Various Picture Galleries

- Knots as radiating tubes **M** New!
- More knots as radiating tubes New!
- Decorative knots: on white, on black, goth New!
- Antoine's Necklace (a fractal link)
- Moiré pictures — not knot like, but still rendered with KnotPlot New!
- Ashley knots
- Nifty Knots with Fancy Lighting
- A knot zoo
- Torus knots and links, arranged by crossing number
- Brunnian Links **M**
- Linking Spheres in three or more dimensions
- PDF and PostScript examples
- Knot Carpets
- 3D Stereoscopic Pictures (page 1, page 2, page 3, page 4, page 5, page 6)
- Celtic Knots
- Hyperbolic knots
- Complex knots New!
- Interactive knot viewer (randomly chosen knot, requires Java): smooth knots minimal-stick knots equilateral minimal-stick knots crazy knots.
- VRML Models
- 3D Surface Models
- Perko pair knots
- Moebius strip knots
- Coney
- Project Aum
- Stick numbers for minimal stick knots (data only, no pictures)
- Some excellent references on knot theory.
- Some favourite figures from my thesis (PDF and PostScript)
- Symmetric projections of minimal-stick knots
- Links to other peoples' knot sites.
- The KnotServer (experimental, still not too useful), 3D interactive viewer (which can save 3D models).

Knot visualization, calculator and software explorer

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- Ashley knots
- Nifty Knots with Fancy Lighting
- A knot zoo
- Torus knots and links, arranged by crossing number
- Brunnian Links M
- Linking Spheres in three or more dimensions
- PDF and PostScript examples
- Knot Carpets
- 3D Stereoscopic Pictures (page 1, page 2, page 3, page 4, page 5, page 6)
- Celtic Knots
- Hyperbolic knots
- Complex knots New!
- Interactive knot viewer (randomly chosen knot, requires Java): smooth knots minimal-stick knots equilateral minimal-stick knots crazy knots.
- VRML Models
- 3D Surface Models
- Perko pair knots
- Moebius strip knots
- Coney
- Project Aum
- Stick numbers for minimal stick knots (data only, no pictures)
- Some excellent references on knot theory.
- Some favourite figures from my thesis (PDF and PostScript)
- Symmetric projections of minimal-stick knots
- Links to other peoples' knot sites.
- The KnotServer (experimental, still not too useful), 3D interactive viewer (which can save 3D models).

KnotPlot *(Scharein, 1998-2011)*

- ***Database:***
  - *Knot catalogue of first 384 knots and links (knots up to 10 crossings, links up to 4 components and 9 crossings);*
  - *Geometric and topological properties such as average crossing number, writhe, ropelength, linking number, bridge number, DT code, Alexander and HOMFLYPT polynomial;*

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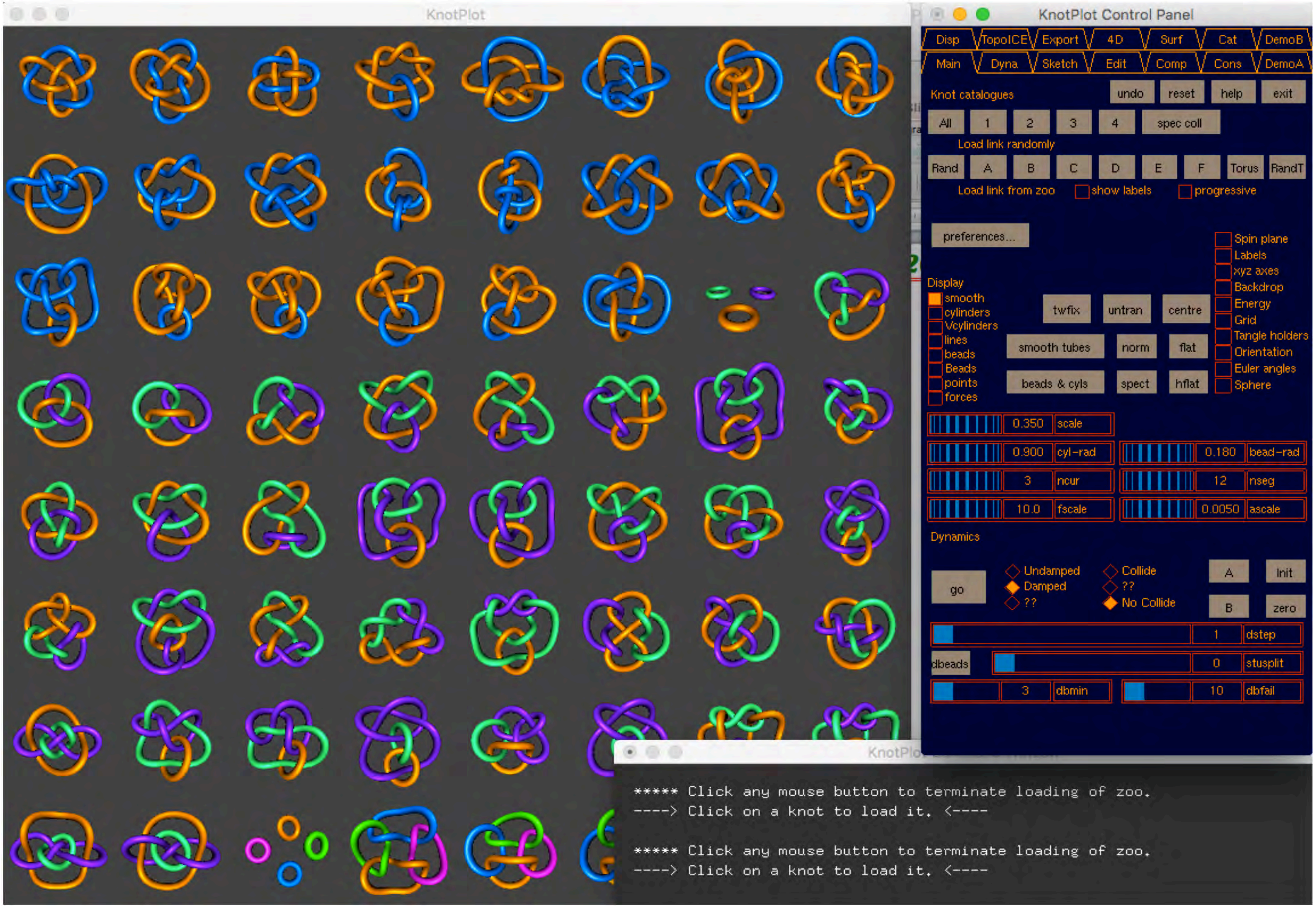
- ***Knot dynamics:***
  - *Energy relaxation by applied forces;*
  - *Interactive manipulation of knots by local and global actions.*

KnotPlot *(Scharein, 1998-2011)*



Selected references

Books:

An elementary mathematical introduction:

- Adams, C.C. 1994 *The Knot Book*. W.H. Freeman & Co., New York.

A comprehensive, non-mathematical collection:

- Ashley, C. 1944 *The Ashley Book of Knots*. Doubleday, New York.

An overall view of modern developments:

- Kauffman, L.H. 2001 *Knots and Physics*. World Scientific, Singapore.

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Online resources:

- KnotAtlas: *katlas.math.toronto.edu/wiki*
- KnotInfo: *indiana.edu/~knotinfo*
- KnotFinder: *indiana.edu/~knotinfo/knotfinder.php*
- KnotPlot: *knotplot.com*
- KnotScape: *pzacad.pitzer.edu/~jhoste/HosteWebPages/kntscp.html*
- LinKnot: *www.mi.sanu.ac.rs/vismath/linknot/index.html*

End of Course