

# A spectral Galerkin method for the solution of reaction-diffusion equations on metric graphs

Anna Weller

Department of Mathematics and Computer Science, Division of Mathematics

University of Cologne

weller@math.uni-koeln.de

## Abstract

We investigate a spectral solution approach for reaction-diffusion equations on graphs interpreted as topological space (*metric graphs*). Of special interest is the numerical computation of eigenfunctions of the negative second order derivative acting on each edge. Remarkably, it is possible to give an explicit characterization of these eigenfunctions and corresponding eigenvalues. Moreover, for equilateral graphs, we will show how to efficiently compute an arbitrary lower part of the spectrum using a very useful relationship to the graph Laplacian matrix of the underlying combinatorial graph. Finally, we can use the basis of eigenfunctions in a Galerkin approach to solve various PDEs on metric graphs, where we here focus on reaction-diffusion equations. This problem is motivated by a recent collaboration with the Institute of Geophysics and Meteorology of the University of Cologne on the modeling of protein distribution in Alzheimer's disease together with the University Hospital Cologne<sup>1</sup>. Part of my numerical results are joint work with Prof. Dr. Mark Ainsworth (Brown University) and with Chong-Son Droege (University of Cologne).

## References

- [1] M. Ainsworth, A. Weller, *A spectral Galerkin method for the solution of partial differential equations on metric graphs*, in preparation.
- [2] M. Ainsworth, A. Weller, *A spectral Galerkin method for the solution of reaction-diffusion equations on metric graphs*, Oberwolfach Reports, Workshop Report 36, 2021.
- [3] A. Kunoth, A. Weller, T. Yilmaz, *A computational brainsphere model for the simulation of Alzheimer's Disease*. Snapshots of modern mathematics from Oberwolfach, in preparation.
- [4] A. Weller, G.N. Bischof, P. Schlüter, N. Richter, J. Dronse, Ö. Onur, J. Kukolja, B. Neumaier, A. Kunoth, Y. Shao, T. van Eimeren, A. Drzezga, *Finding new communities: A principle of neuronal network reorganization in Alzheimers disease*. Brain Connectivity, 11(3):225-238, 2021. DOI: 10.1089/brain.2020.0889.

---

<sup>1</sup>This work was supported by the Excellence Initiative of the University of Cologne.