

Wound algebraic groups and their compactifications

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This serie of lectures deals with algebraic groups defined over an arbitrary field k [4]. We will begin by revisting basics of the theory, e.g. Weil restriction, quotients,... An algebraic k -group G is anisotropic (resp. wound) if it does not carry any k -subgroup isomorphic to \mathbb{G}_m (resp. \mathbb{G}_m or \mathbb{G}_a).

If G is reductive, Borel and Tits have shown that the two notions coincide; furthermore if k is perfect this is equivalent for G to admit a projective compactification G^c such that $G(k) = G^c(k)$ [1]. A related (equivalent) condition is that $G(k[[t]]) = G(k((t)))$ and this is Prasad's viewpoint on the result [8]. We are interested in the generalization of that statement in the following two directions.

1) **The case of an imperfect field.** This includes unipotent groups [2] and pseudo-reductive groups [3]. The main result is Gabber's compactification theorem [6] constructing for an arbitrary G a $G \times G$ -equivariant compactification G^c such that for any separable extension F/k we have $G(F) = G^c(F)$ if and only if G_F is wound.

2) **Group schemes over a ring A .** In the paper [5] we extended the notion of wound group schemes in that setting and this does not involve classification results. More precisely we defined a notion of index and residue for an element in $G(A((t))) \setminus G(A[[t]])$ which connects those elements with subgroup schemes isomorphic to $\mathbb{G}_{a,A}$ or $\mathbb{G}_{m,A}$. In the case of a reductive group G over a field k it provides a kind of stratification of $G(k((t)))$ related with the theory of affine grassmannians.

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