

Stability Issues and Fundamental Forces

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*2024 ROBERTO PETRONZIO LECTURE
U. ROMA "TOR VERGATA", APRIL 5 2024*

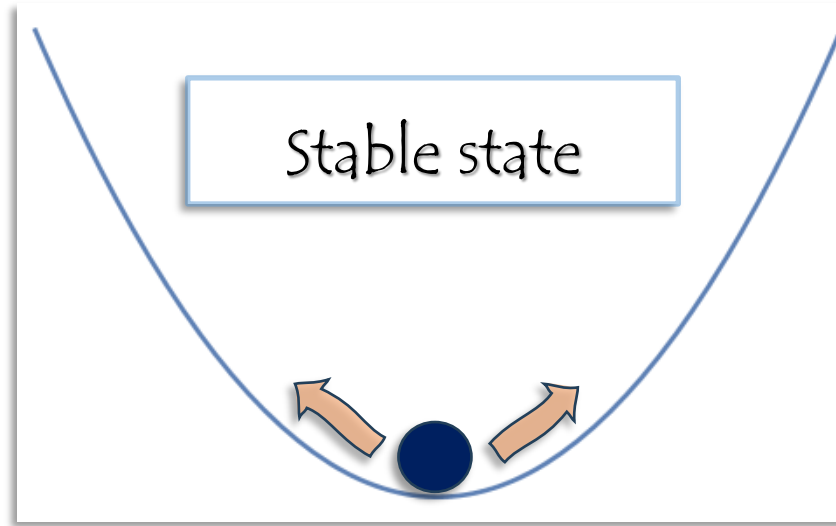
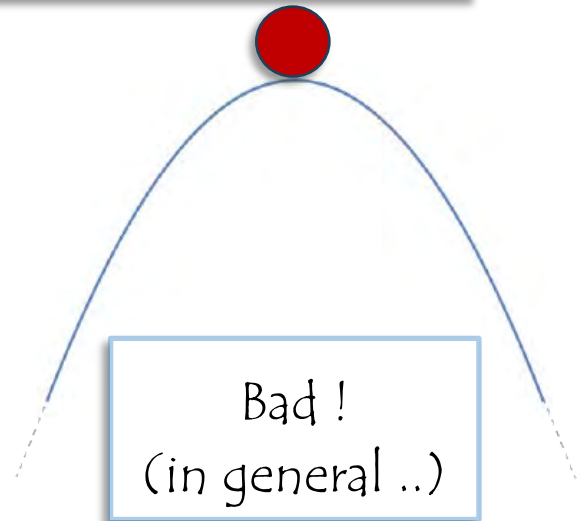
Stable vs Unstable Equilibria

Let us begin by considering a **point particle subject to gravity**, near the surface of the Earth, sliding on a "rail" in a vertical plane (**circle**)

STABLE and **UNSTABLE** equilibria

Lowest equilibrium point:

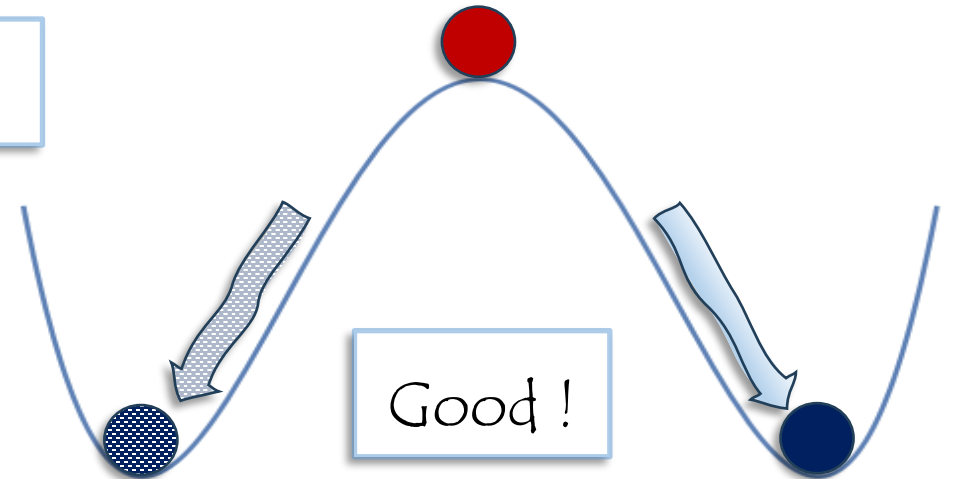
"GROUND STATE" or "VACUUM"



This type of cartoon is a recurrent tool, with the **point particle (order parameter)** representing the value of a suitable "potential" or **"ENERGY FUNCTION"**

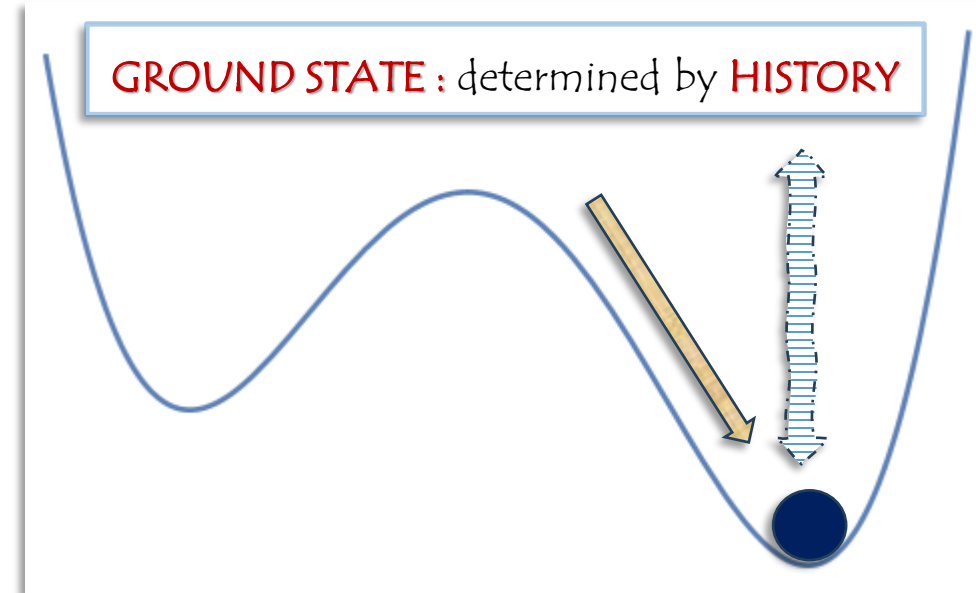
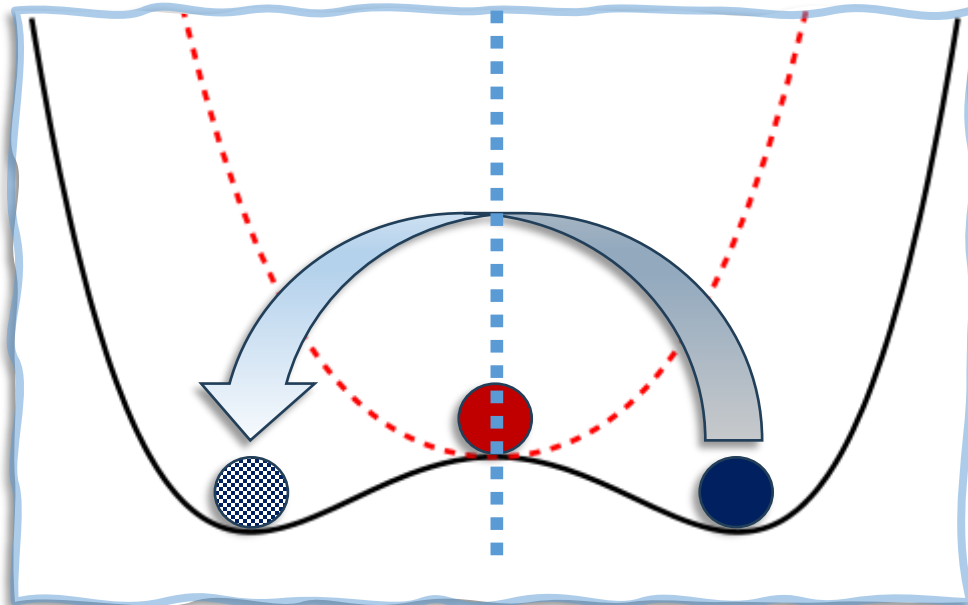
● , ● : order parameter

Unstable states



Spontaneous Symmetry Breaking, I

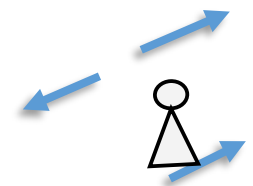
The "GROUND STATE" CAN BE LESS SYMMETRIC than the potential



UNDERLIES our understanding of a **WIDE RANGE OF PHENOMENA** :

- Condensed Matter Physics
- Fundamental Interactions
-
- The Universe

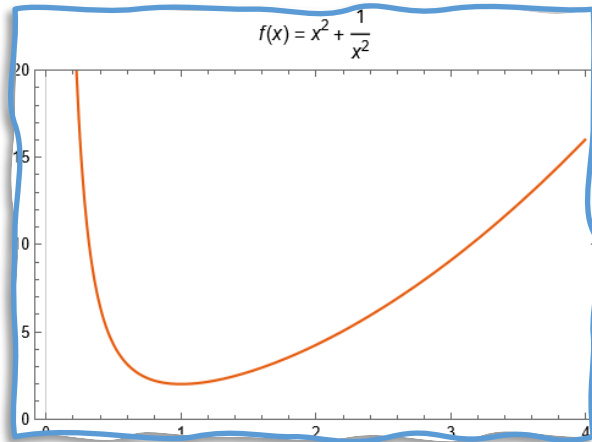
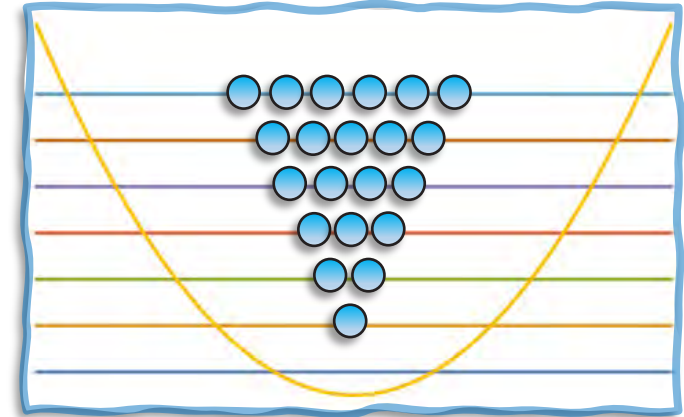
BUT: the mechanism generally leaves behind some "DEBRIS" → **DILUTE** it !



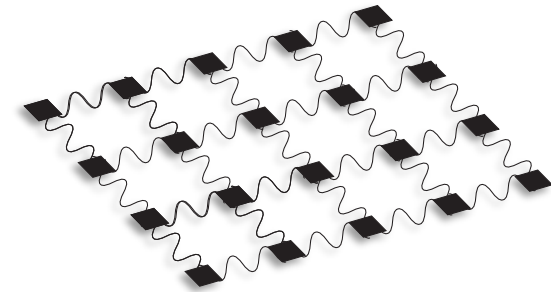
Quantum Mechanics

Harmonic Oscillator

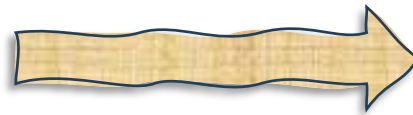
$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$
$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega$$



$$E_0 \geq \frac{\hbar^2}{8m \langle (\Delta x)^2 \rangle} + \frac{1}{2} m \omega^2 \langle (\Delta x)^2 \rangle \geq \frac{\hbar \omega}{2}$$
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

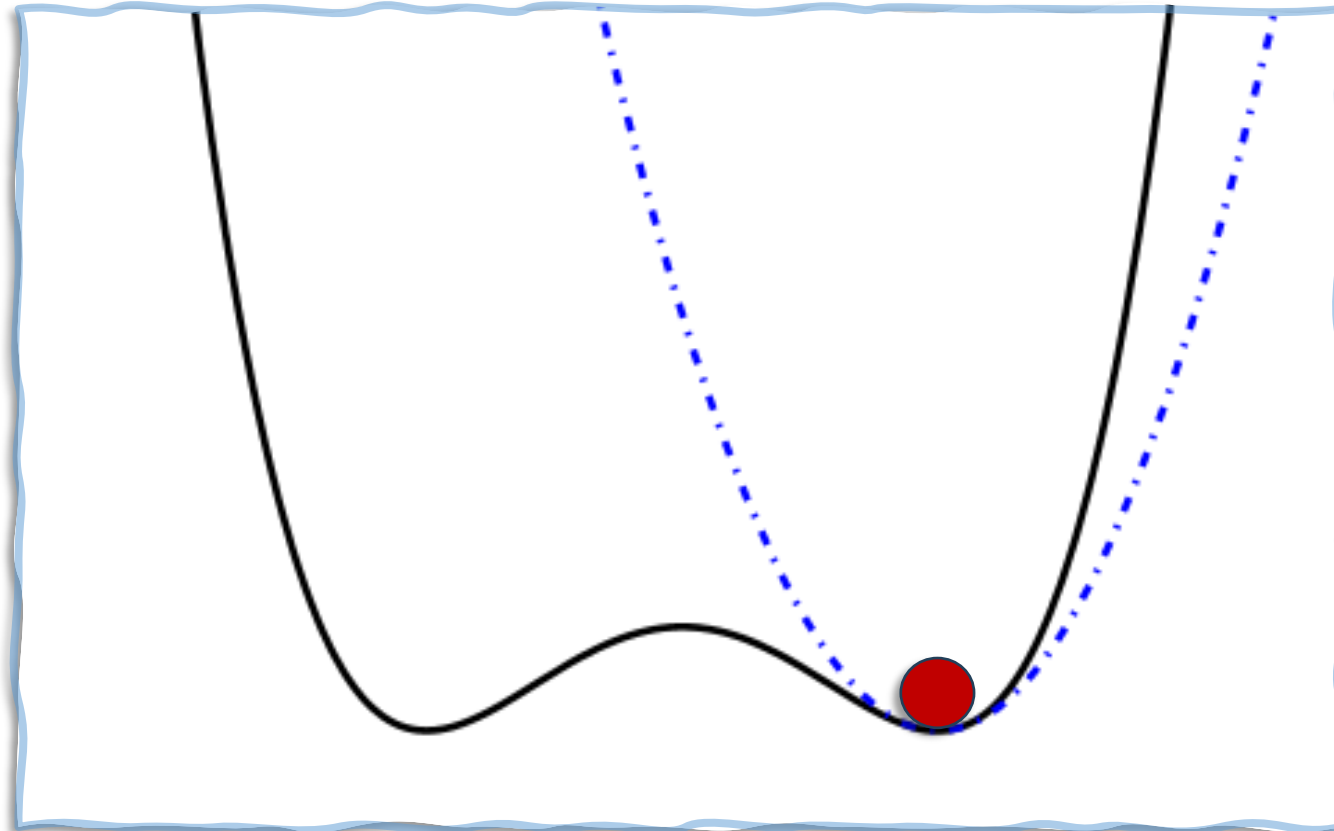


"Additive Spectrum" → QUANTA
BUT: WAVES originate from
COUPLED HARMONIC OSCILLATORS



Elementary Particles:
"QUANTA of WAVES"

Small Oscillations and Particle Masses



- Around **LOCAL MINIMA** of a potential $U(x)$, the **ORDER PARAMETER** can undergo **SMALL OSCILLATIONS** with

$$\omega^2 \sim U''(x_0)$$

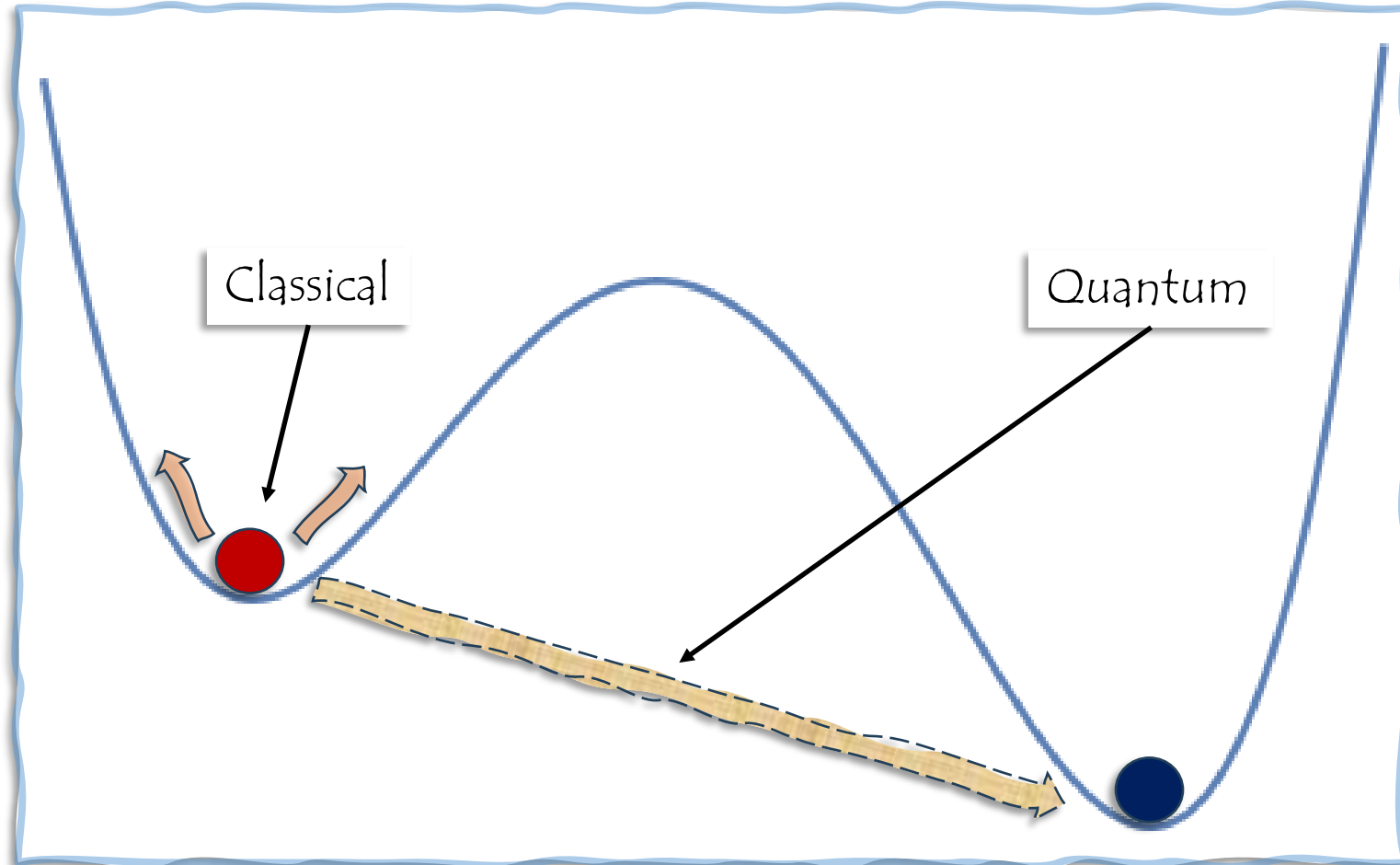
- For "**QUANTA** of **WAVES**" these **FREQUENCIES** are proportional to their **MASSES**:

$$\omega^2 \sim m^2$$

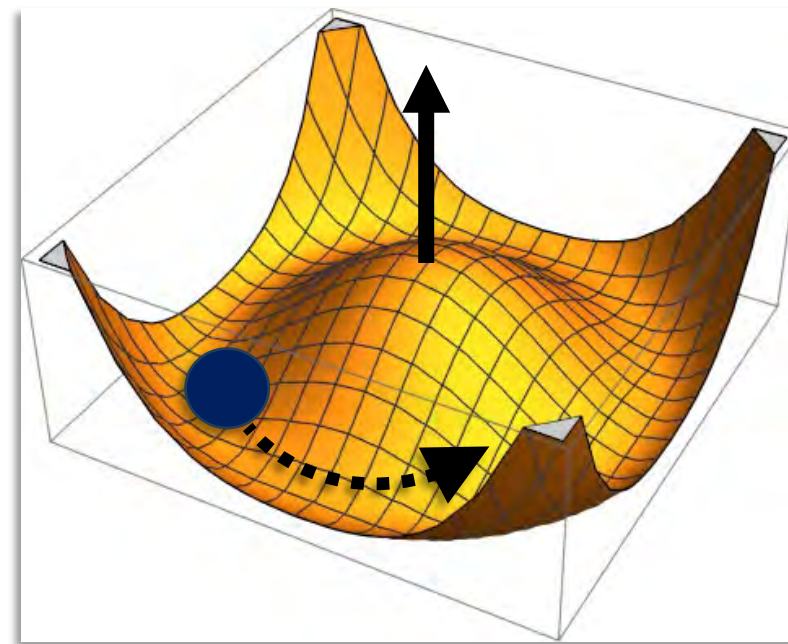
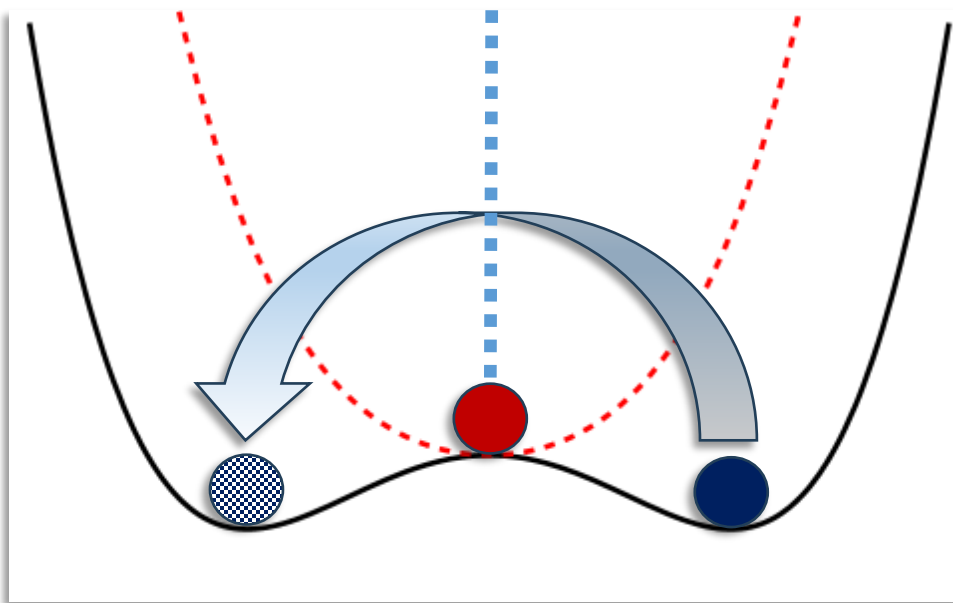
- **MASSES** → **SHORT-RANGE FORCES**
- ❖ **FORCES** → **INTERACTIONS**, generally **TURN** particles (quanta) into other types

Vacuum (In)Stability & Quantum Theory

In QUANTUM MECHANICS the VACUUM can be destabilized by TUNNELING

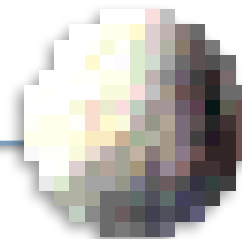


Spontaneous Symmetry Breaking, II



WITH a **CONTINUOUS BROKEN SYMMETRY** (in this case **ROTATIONS** about the vertical axis):

- **MOVING** along the valley "costs" **NO POTENTIAL ENERGY** → a **MASSLESS "Goldstone"** particle
- **ADDING** Electromagnetism → "**HIGGS (or B.E.H.)**" mechanism: **SHORT-RANGE VECTOR FORCES!**
- New types of "**DEFECTS**" MAY EXIST: **magnetic monopoles** → need to **DILUTE THEM**



Fundamental Interactions, I

LONG RANGE:

- GRAVITY [~ 1600] (only attractive):
- ELECTROMAGNETISM [~ 1800] (attractive or repulsive):

$$\begin{array}{l} - \frac{G_N m^2}{r_{12}^2} \hat{r}_{12} \\ \pm \frac{e^2}{r_{12}^2} \hat{r}_{12} \end{array}$$

- Long range
- $2 = 3 - 1$

SHORT RANGE: spontaneous symmetry breaking ; $A_\mu \equiv (\phi, A_i) \rightarrow$ MATRICES

- WEAK INTERACTIONS [~ 1900] ($\rightarrow W^+, W^-, Z$)
- STRONG INTERACTIONS [~ 1900] \rightarrow ["CONFINEMENT" \rightarrow residual "van der Waals-like" interaction]
- [HIGGS + (SELF)INTERACTIONS [~ 2010] : "SMOKING GUN" of "spontaneous symmetry breaking"]



STANDARD MODEL (1970's)

E.g. : Electromagnetism \rightarrow Electro - Weak

$$A_\mu \rightarrow \begin{pmatrix} (c + s) A_\mu + (c - s) Z_\mu & \sqrt{2} W_\mu^- \\ \sqrt{2} W_\mu^+ & (c - s) A_\mu - (c + s) Z_\mu \end{pmatrix}$$

$$c = \cos \theta_w, \quad s = \sin \theta_w \quad \left[\mathcal{M}_1 \mathcal{M}_2 \neq \mathcal{M}_2 \mathcal{M}_1 \right]$$

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STANDARD MODEL (1970's)

Fundamental Interactions, II

Terrestrial gravity

Celestial gravity

$$g = \frac{G_N M_T}{R_T^2}$$

$$G_{\mu\nu} \sim T_{\mu\nu}$$

(General Relativity)

Classical
(Macroscopic)



Electricity

Magnetism

$$F = -e(E + \frac{v}{c} \times B)$$

(Electromagnetism)

$$\alpha_e = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

Quantum
(Microscopic)

Weak Interactions

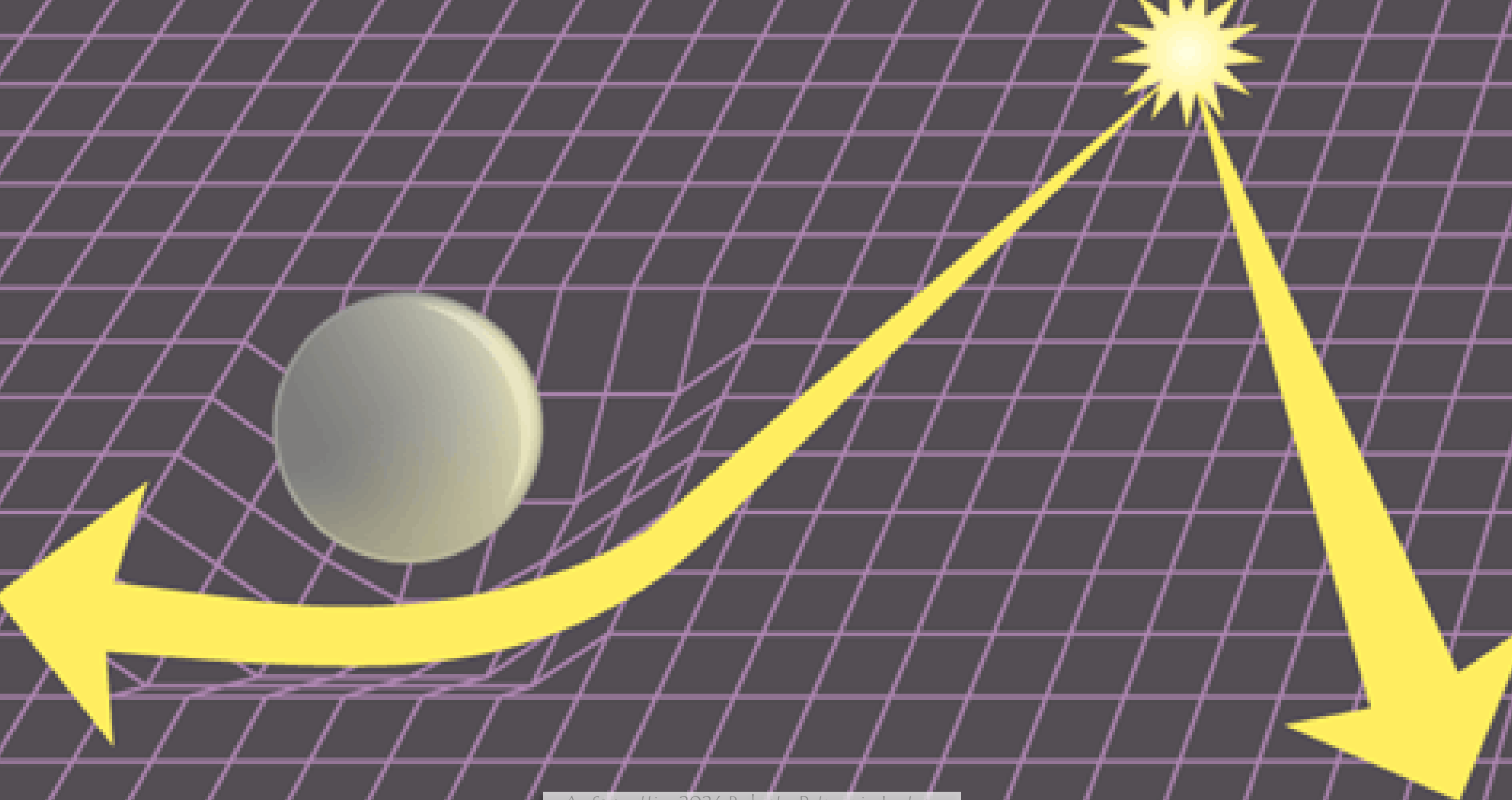
Strong Interactions

$$G_F = \frac{g^2}{p^2 + M^2} \Big|_{p=0}$$

(Electroweak theory)

$$\Gamma_p = \frac{g^4 M_p^5}{M_x^4}$$

(G.U.T's)



General Relativity

- **A CLASSICAL THEORY:** combines Newtonian gravity and Special Relativity

Einstein equations: Spacetime is **CURVED** by **MATTER & ENERGY**

- **DESCRIBES:** stars, planets and galaxies, and the **evolution of the Universe**

- ✓ **GRAVITY WAVES:** "Hertzian" waves, since 2015 (photons → gravitons?)
- ✓ **BLACK HOLES:** (Ultra)dense bodies: **CLASSICALLY NOTHING** can **ESCAPE**
- ✓ **GRAVITY** can be **ALSO REPULSIVE:** Λ (**COSMOLOGICAL CONSTANT**)

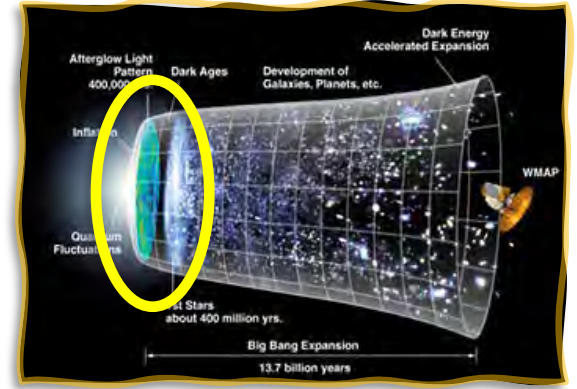
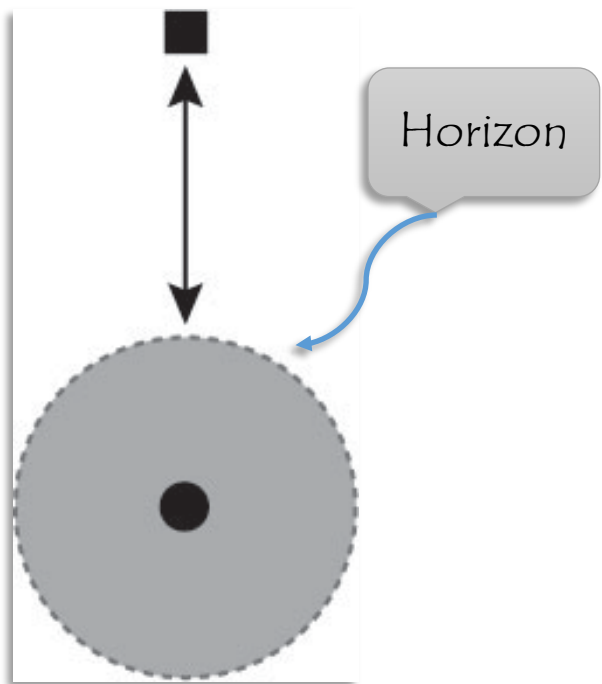
- **E.M. + Q.M.:** interactions among (anti)electrons e^\pm : $\alpha_e = \frac{e^2}{\hbar c} \sim \frac{1}{137}$

$$\frac{e^2}{r_{12}^2} \quad \text{vs} \quad \frac{G_N m^2}{r_{12}^2}$$



$$\alpha_G = \frac{G_N m^2}{\hbar c} \rightarrow \frac{G_N E^2}{\hbar c^5}$$

α_G grows $\sim E^2$!



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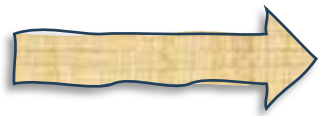
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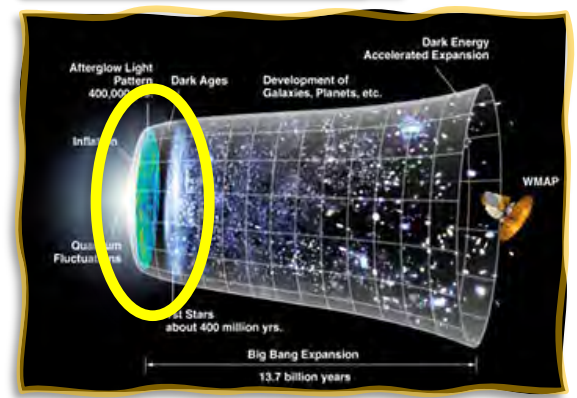
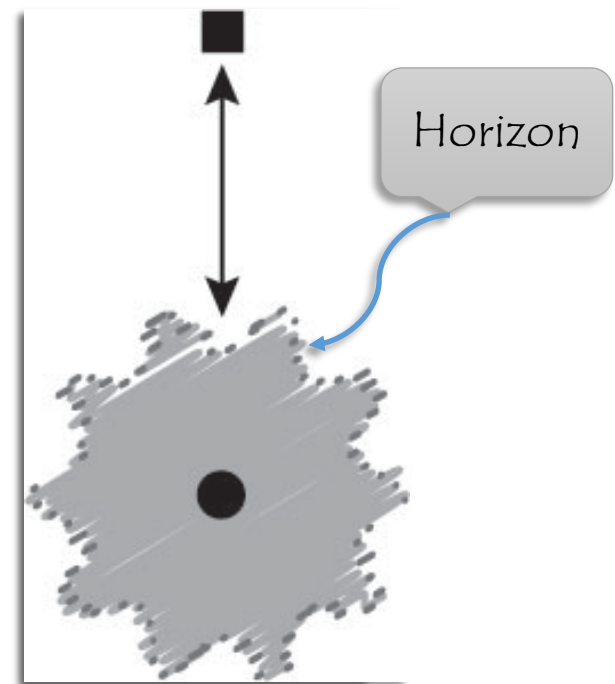
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Beyond Particles : String Theory

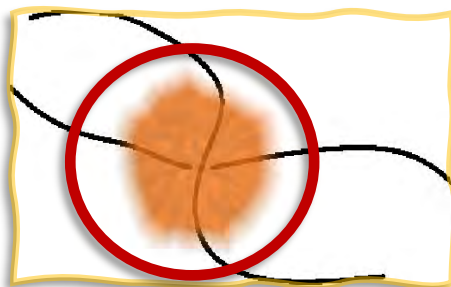
- **THUMB RULE:** a theory "WORKS" up to energies where $\alpha \approx 1 \rightarrow E \leq E_{Pl}$

- **GENERAL RELATIVITY:** $\alpha_G = \frac{G_N m^2}{\hbar c} \rightarrow \frac{G_N E^2}{\hbar c^5} \rightarrow E_{Pl} = \sqrt{\frac{\hbar c^5}{G_N}} \simeq 10^{19} \text{ GeV}$



- Energies **BILLIONS OF MILLIONS** of times **BEYOND** what we can probe ! **BUT TYPICAL** in our picture of the **EARLY UNIVERSE**

- **STRING THEORY: DILUTES** energy on (short) **STRINGS** (MODES \rightarrow PARTICLES; & $D=4 \rightarrow D=10!$)



$$\alpha_G = \frac{G_N m^2}{\hbar c} \sim \frac{G_N E^2}{\hbar c^5} \rightarrow \frac{G_N E^2}{\hbar c^5} \times \left(\frac{\Delta l}{l_s}\right)^2 = \frac{G_N}{\hbar c^5} \left(\frac{\hbar c}{l_s}\right)^2$$

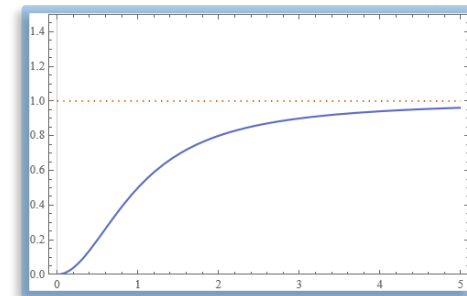
$$\text{Uncertainty Principle} \rightarrow \frac{\Delta l}{l_s} \sim \frac{\hbar}{p l_s} \sim \frac{\hbar c}{p c l_s} \simeq \frac{\hbar c}{E l_s}$$

- Interactions **SPREAD** within "clouds" determined by the **UNCERTAINTY PRINCIPLE**

- **GENERAL RELATIVITY (SUPERGRAVITY, actually) IS RECOVERED** at low energies!

BUT: the GROWTH of α_G STOPS at distances $O(l_s)$!

Most important FACT: GRAVITY is INEVITABLE in String Theory !



Vacuum Energy and Elementary Particles

Fermi

- BOSE (FERMI) OSCILLATOR:

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2 \longrightarrow \mathcal{E}_0 = (-1)^n \frac{\hbar \omega}{2}$$

$$\frac{\hbar \omega(k,m)}{2}$$

Ultraviolet CUT determined by E_{Pl} or ℓ_{Pl}

- FOR ALL PARTICLE SPECIES [UPPER END determined by E_{Pl}]

$$[V_{Pl} = \ell_{Pl}^3 = \left(\frac{c\hbar}{E_{Pl}}\right)^3]$$

$$\frac{E_0}{V} = \sum_i (-1)^{F_i} \int_{\frac{E_{Pl}}{c\hbar}} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\hbar c}{2} \sqrt{\mathbf{k}^2 + m_i^2}$$

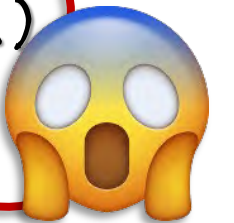
- COSMOLOGICAL CONSTANT ISSUE:

$$\frac{E_0}{V} \sim \frac{E_{Pl}}{V_{Pl}} \sum_i (-1)^{F_i} + \dots$$

- Exact SUPERSYMMETRY (BOSE \leftrightarrow FERMI) WOULD IMPLY $E_0 = 0$

BUT: Supersymmetry is NOT OBSERVED (up to what we can now explore ...)

- GENERICALLY, without SUPERSYMMETRY: E_0 / E_{macro} IS HUGE $\sim 10^{120}$



Vacuum Energy and String Theory

INTEGRAND \rightarrow CONSISTENT STRING SPECTRA

"CLOSED" STRINGS: "fat" loop diagram (LOOP \rightarrow TORUS: MODULAR INVARIANCE)

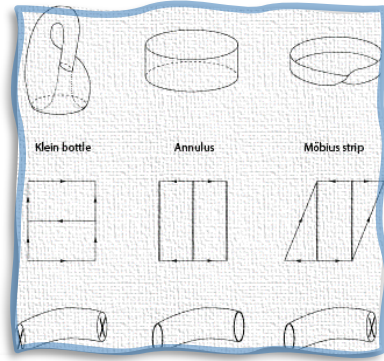
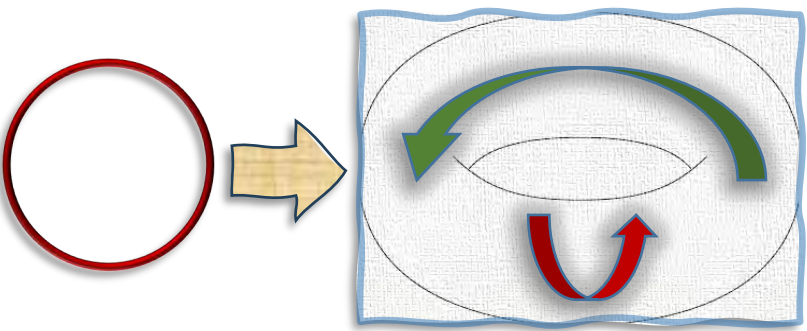
$$E_0 \sim \int_{\ell_{Pl}}^{\infty} d\tau f(\tau, \{M\})$$



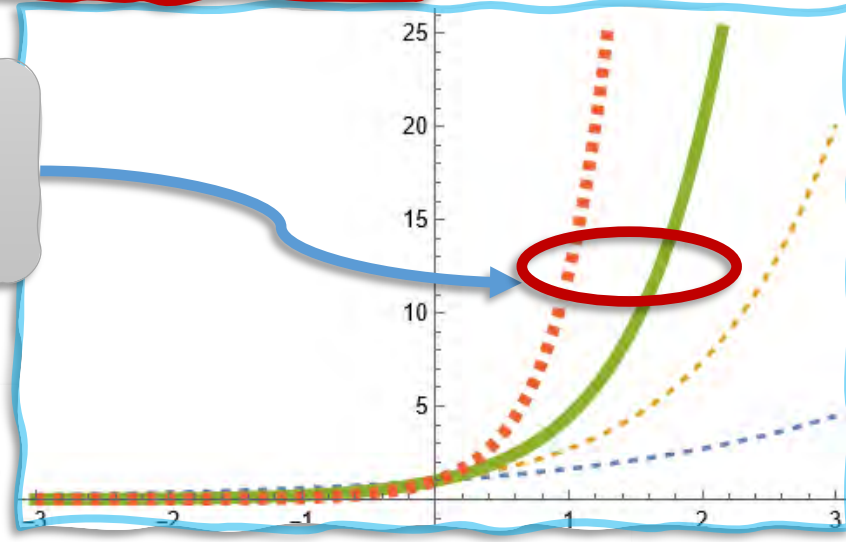
$$E_0 \sim \int_{\mathcal{F}} \frac{d^2\tau}{\text{Im}(\tau)^2} f(\tau, \bar{\tau}, \{M\})$$

Ultraviolet CUT determined by ℓ_s

- The extended nature of strings CUTS the ULTRAVIOLET: $\ell_{Pl} \rightarrow \ell_s$



An exponential Potential, with a "(super)critical" γ !

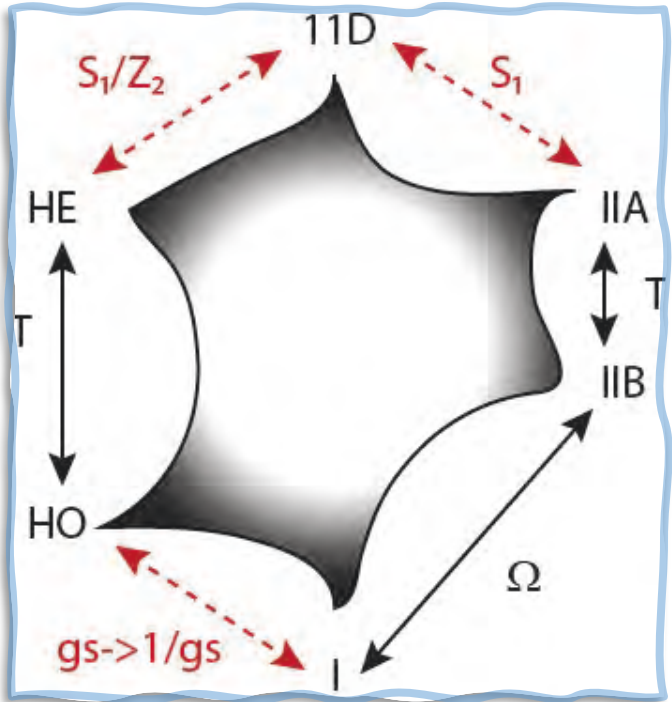


OPEN + CLOSED (ORIENTIFOLDS): more complicated BUT SIMILAR LESSONS

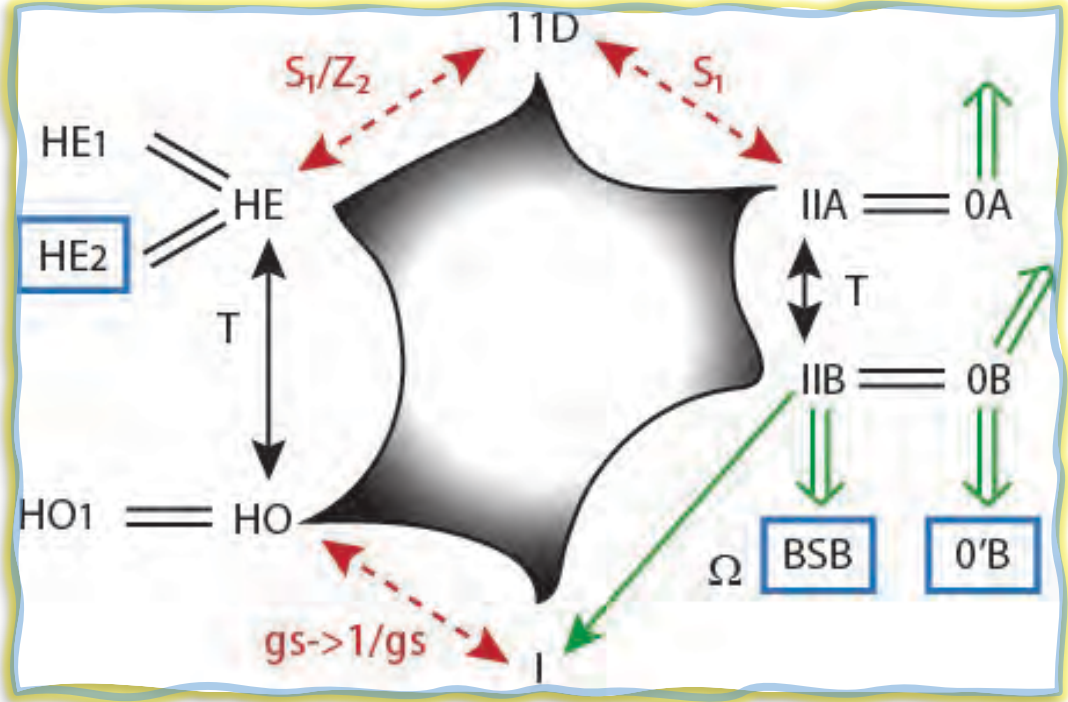
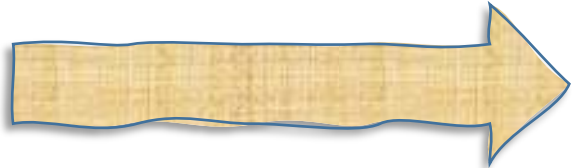
SUPERSYMMETRY: NO BACK-REACTION & $E_0=0$

- NO SUPERSYMMETRY $\rightarrow E_0$ is a "STEEP" SCALAR POTENTIAL $E_0 \sim e^{\gamma\phi} = (\alpha_s)^{\gamma/2}$

The 10D-11D String Zoo



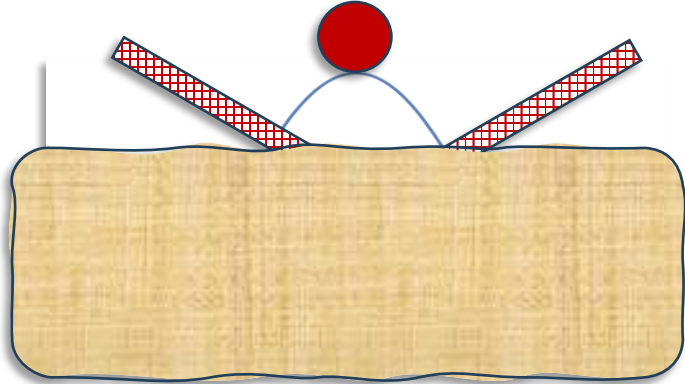
With SUPERSYMMETRY



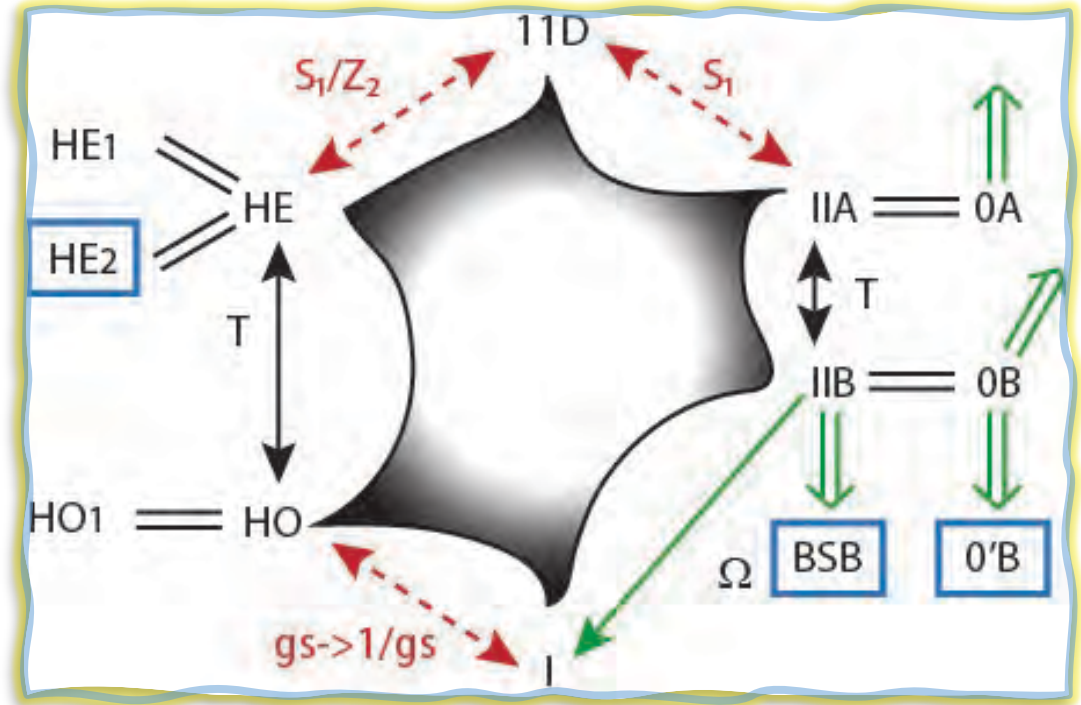
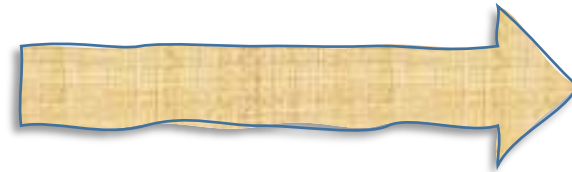
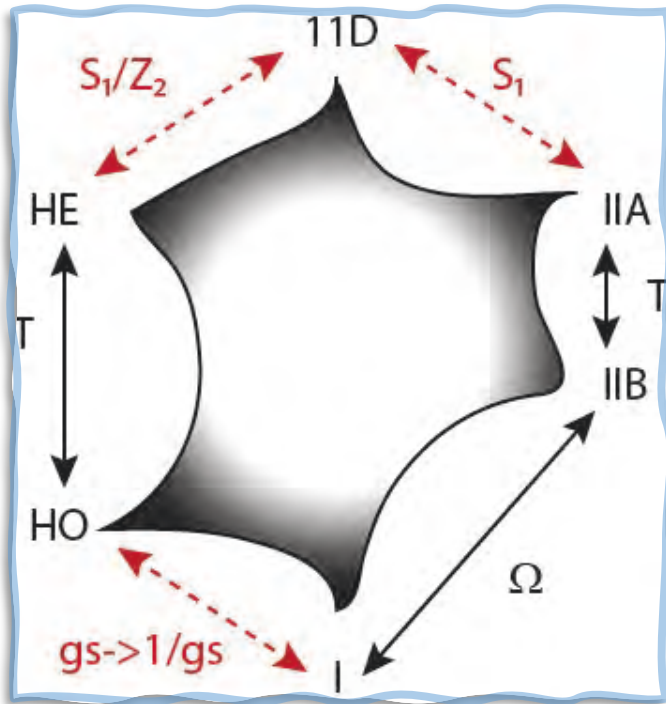
Also without SUPERSYMMETRY ($E_0 \neq 0$)

SUPERSTRING THEORIES "LIVE" IN 10 DIMENSIONS EQUIVALENT: BUT the underlying principle is ELUSIVE

- (Our) FOUR-DIMENSIONAL WORLD? "COMPACTIFICATION"



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With SUPERSYMMETRY

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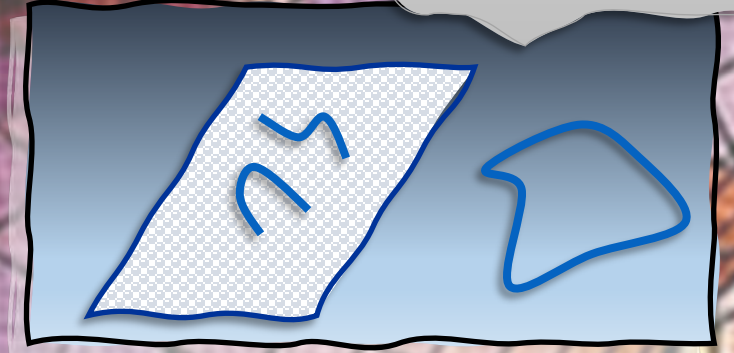
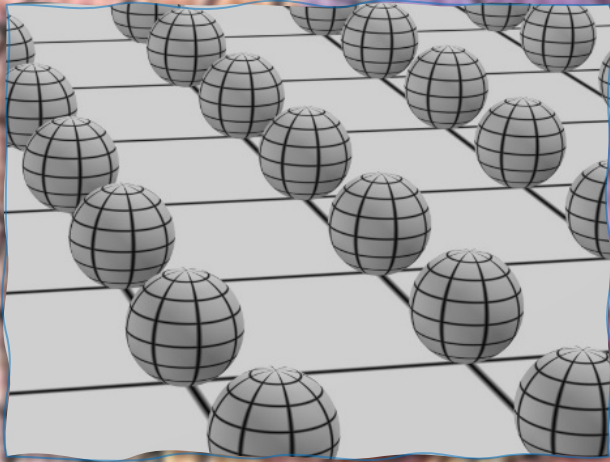
- (Our) FOUR-DIMENSIONAL WORLD? "COMPACTIFICATION"

Also w

One can at best try to find out **WHETHER** a compactification to a 4D spacetime **"LIKE OUR OWN"** is **POSSIBLE** and (locally) **STABLE** **POSTDICTION!** (& DIFFICULT)

10D → 4D : "Compactifications"

ALTERNATIVE :
Electromagnetism
"confined" to "branes"
& Gravity in "bulk"



SUPERSTRING THEORIES LIVE IN 10 DIMENSIONS:

- **FOUR-DIMENSIONAL WORLD** : recovered "effectively" **WITH SMALL** and **STABLE EXTRA DIMENSIONS**

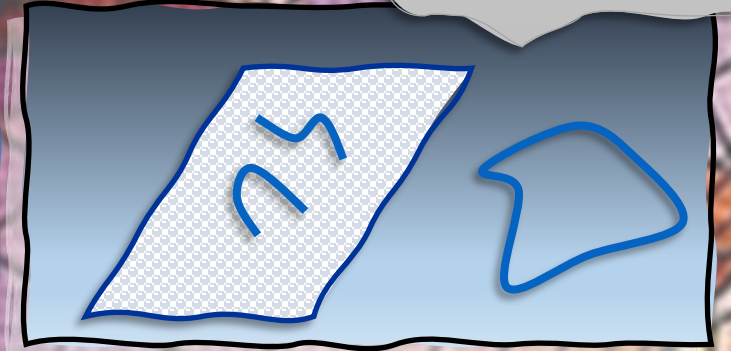
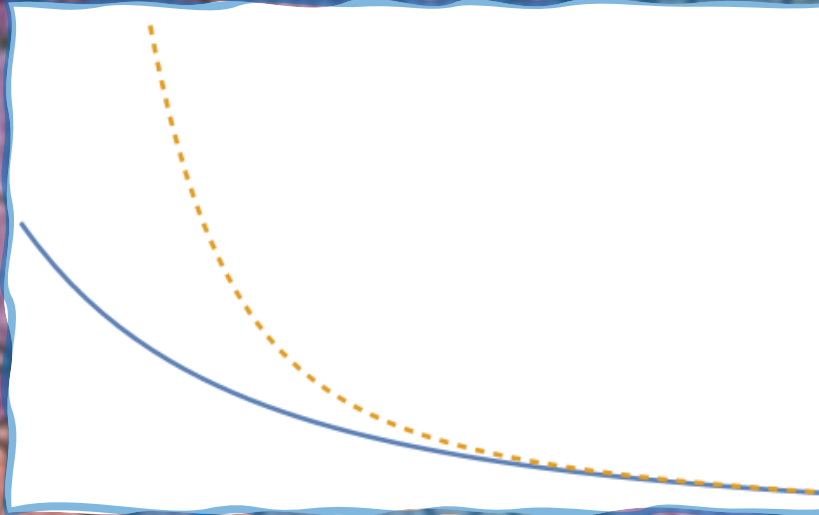
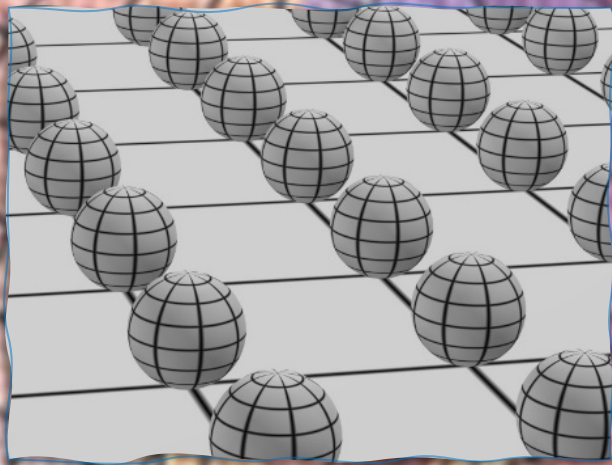
- **FORCES** are **MODIFIED** (at **SHORT DISTANCES**) :
$$\frac{G_{N,4} m^2}{r_{12}^2} \rightarrow \frac{G_{N,d} m^2}{r_{12}^{d-1}}$$

- **SUPERSYMMETRY → STABILITY** OTHERWISE: Only **ENCOURAGING EXAMPLES** (so far) ...

- **INTERNAL SPACE: INVISIBLE** BUT determines the **4D COUPLINGS** →
$$\alpha_4 = \frac{e^2}{\hbar c} = \frac{\alpha_{10}}{V_{int}}$$

10D → 4D : "Compactifications"

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Cosmology

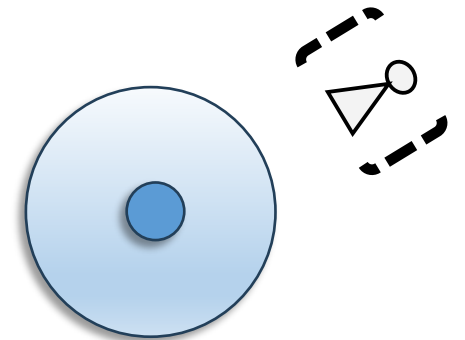
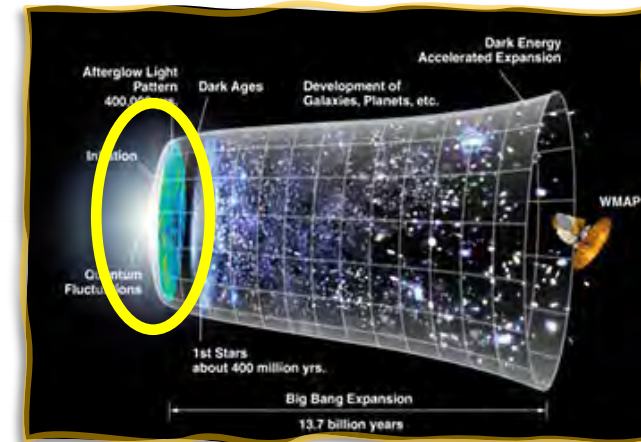
DIFFICULT TO COMBINE GRAVITY AND QUANTUM MECHANICS

- **COSMOLOGY:** (indirect) signals that it is POSSIBLE and NECESSARY
- The **UNIVERSE EXPANDS:** flat, homogeneous, isotropic (TOO MUCH SO!)
- ✓ **INFLATION:** EARLY UNIVERSE **BLOWN UP** by a "cosmological constant"!
- ✓ **CMB:** a "FOSSILE" of the remote past, a "FOSSILE" of **INFLATION**

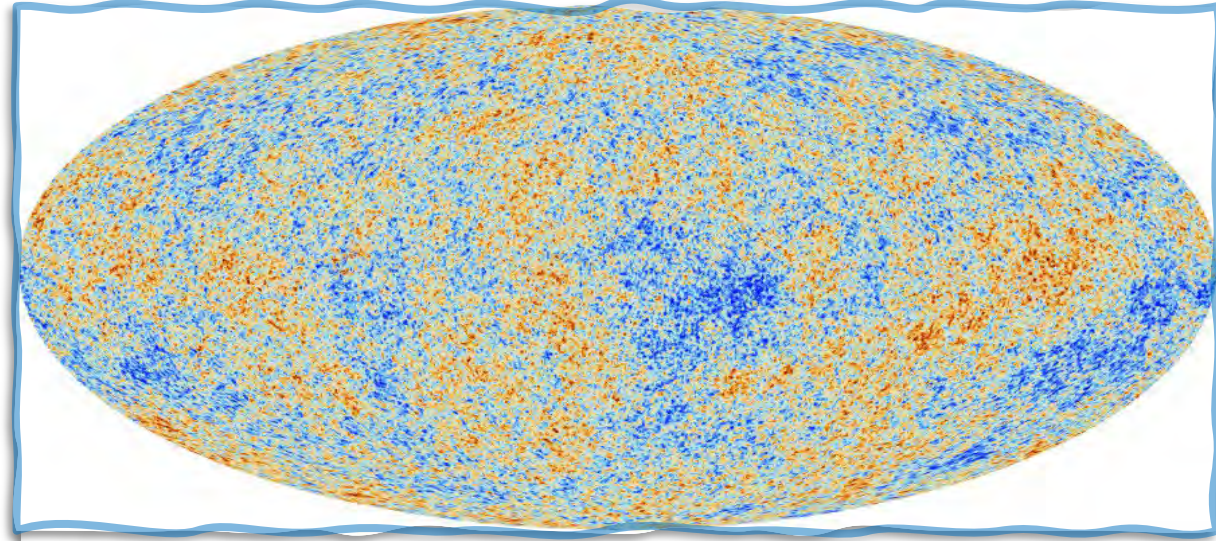
❖ PREDICTION:

The **UNCERTAINTY PRINCIPLE** induces **SMALL RIPPLES**
the **SEEDS** of

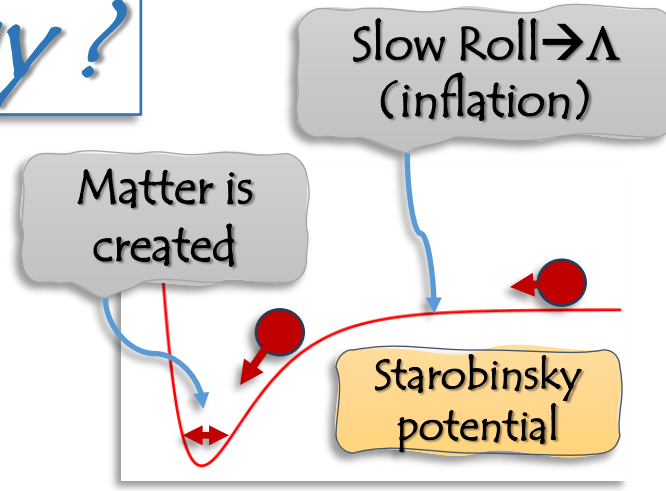
THE MARVELS IN THE SKY!



String Theory & Cosmology?



Why "Slow Roll" in an "exploding" Early Universe?



BROKEN SUPERSYMMETRY in STRING THEORY

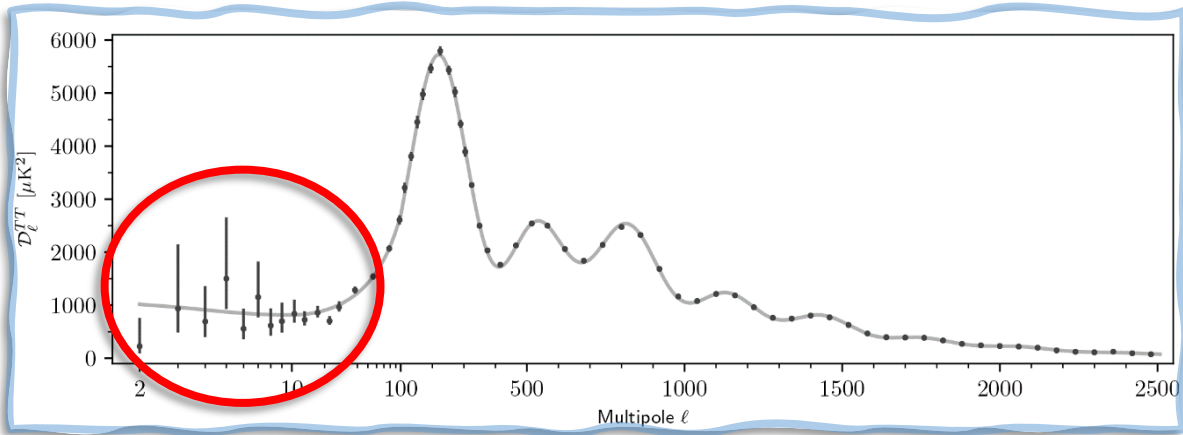
- Can add a "STEEP WALL" to the "STAROBINSKY POTENTIAL"
- Initial "FAST ROLL" → BOUNCE → "SLOW ROLL" & INFLATION
- POSTDICTION: low quadrupole (& some oscillations as above)
- [PREDICTIONS: enhanced "tensor-to-scalar ratio", ("low- ℓ non-gaussianity")]

• In Cosmology \exists Friction: the order parameter \bullet MOVES, losing ENERGY to expand the UNIVERSE

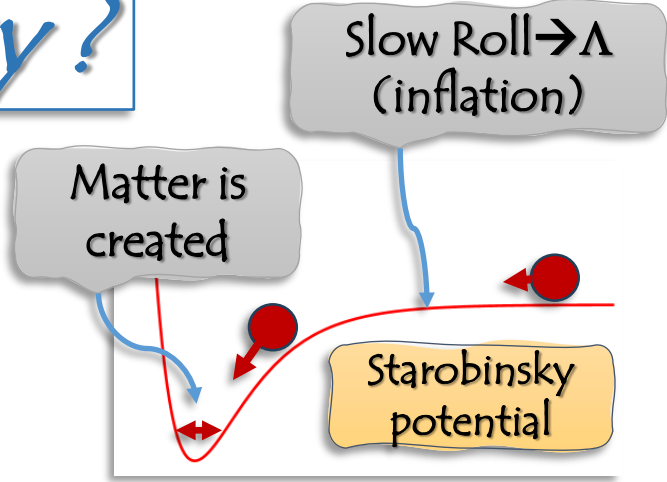
• (Super)critical exponentials ONLY allow "CLIMBING", as the UNIVERSE begins!

$\alpha_s \sim e^{2\langle\phi\rangle}$ BOUNDED & fast \rightarrow slow

String Theory & Cosmology?



Why "Slow Roll" in an "exploding" Early Universe?



BROKEN SUPERSYMMETRY in STRING THEORY

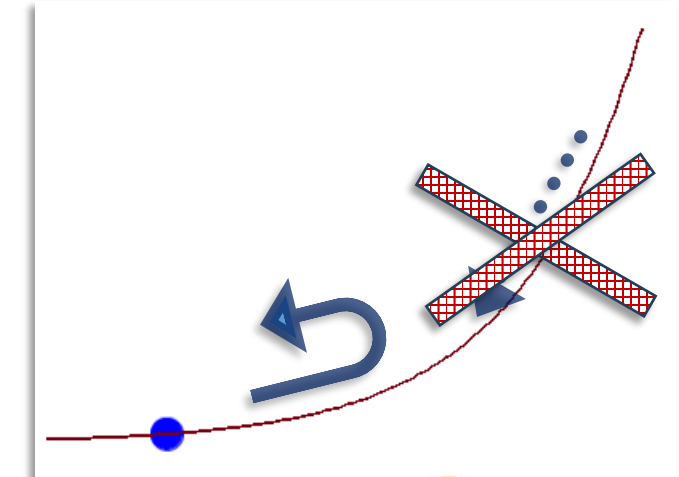
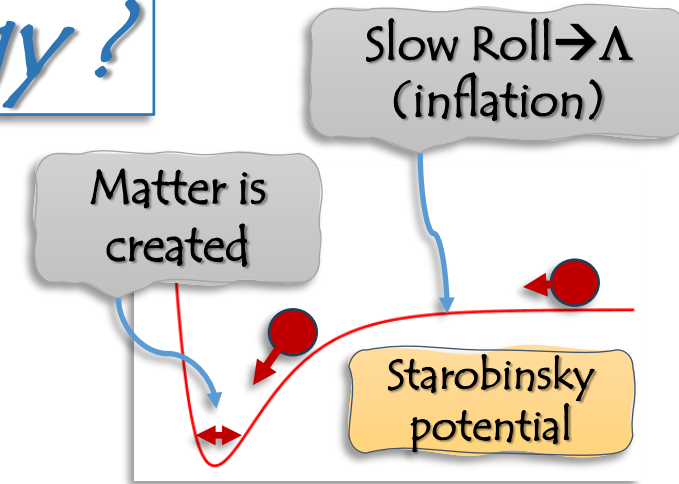
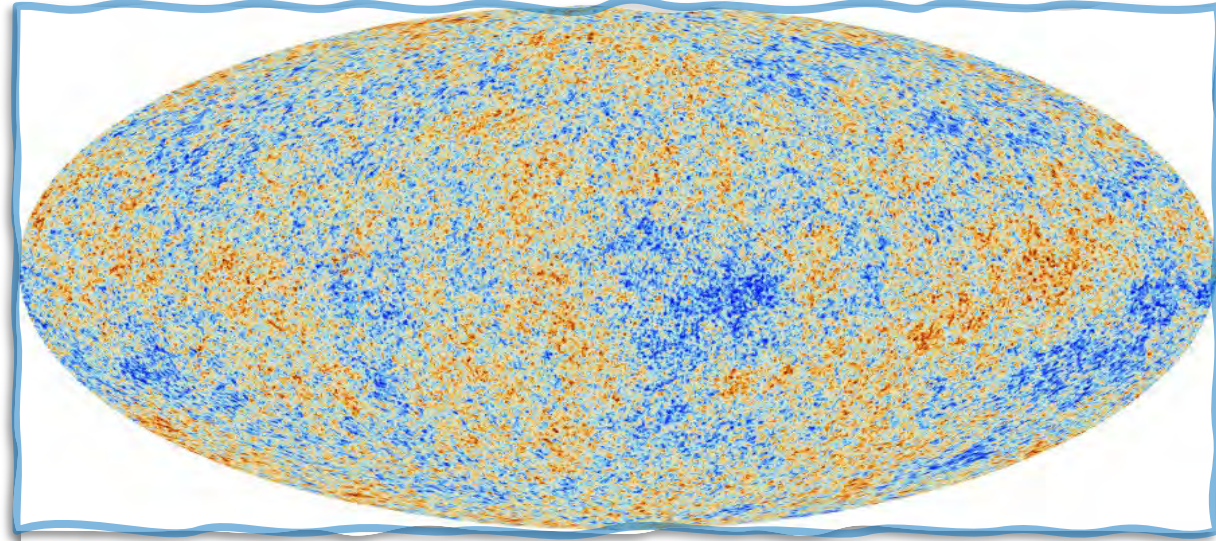
- Can add a "STEEP WALL" to the "STAROBINSKY POTENTIAL"
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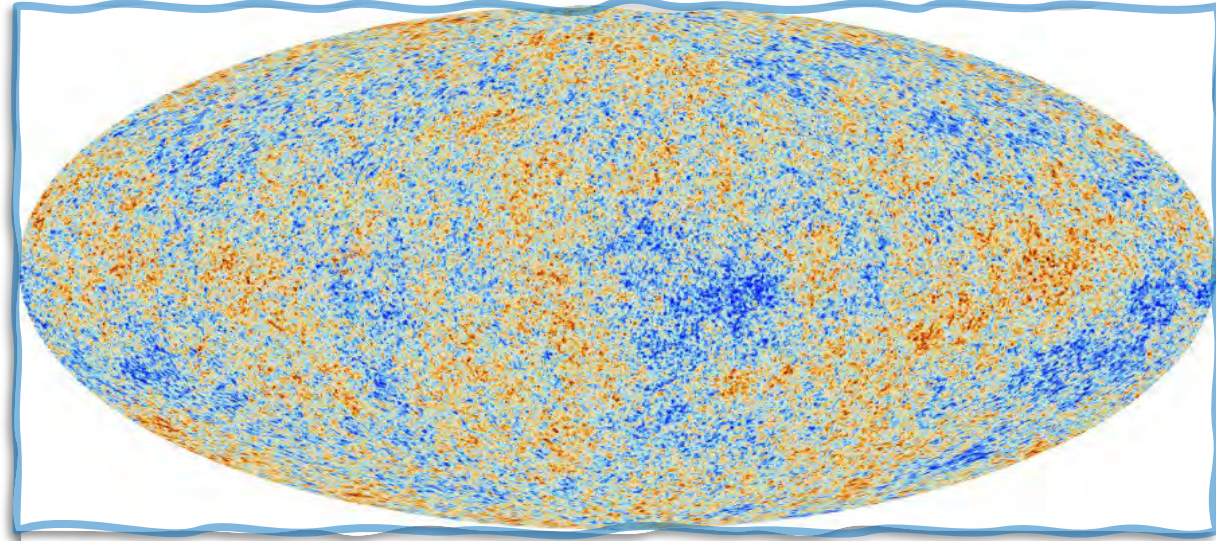


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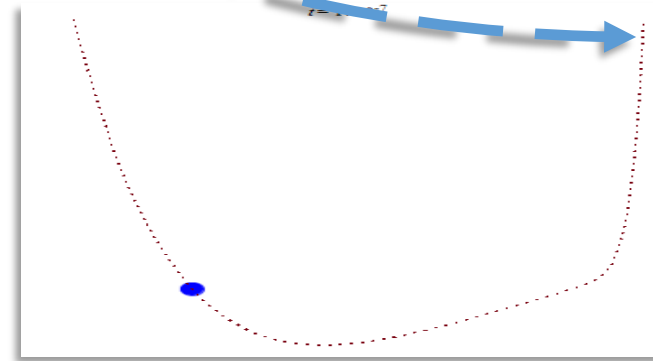
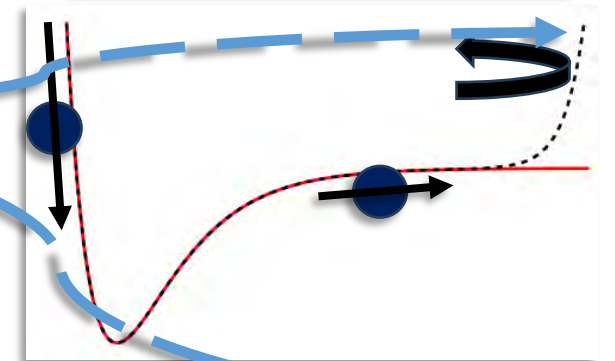
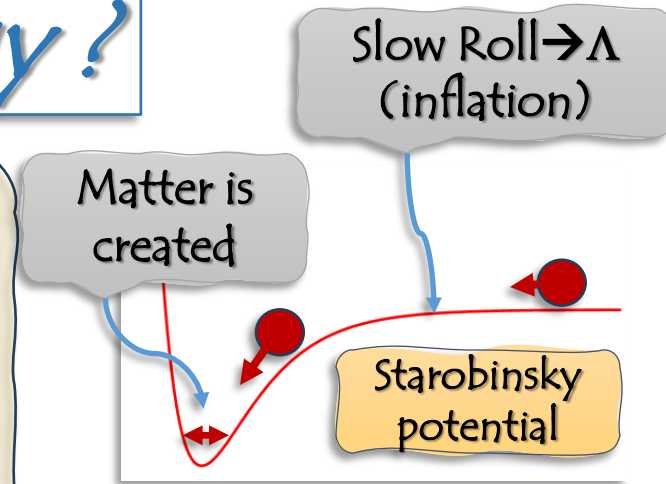
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Steep Potential
From
Broken
Supersymmetry



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Thank You

A. Sagnotti – 2024 Roberto Petronzio Lecture
U. Roma «Tor Vergata», April 5 2024