A mountain pass theorem
(existence and bifurcation)

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Abstract. We present a new variational characterization of multiple critical points for even energy functionals corresponding to nonlinear Schrödinger equations of the following type:

\[
\begin{align*}
-\Delta u + V(x)u - q(x)|u|^{\sigma}u &= \lambda u, \quad (x \in \mathbb{R}^N) \\
u &\in H^1(\mathbb{R}^N) \setminus \{0\}.
\end{align*}
\]

We assume \( N \geq 3 \), \( q(x) \in L^\infty(\mathbb{R}^N) \), \( q(x) > 0 \) a.e. with \( \lim_{|x| \to \infty} q(x) = 0 \) and \( 0 < \sigma < \frac{4}{N-2} \). Our results cover the following 3 cases in a uniform way:

1. \( V(x) \equiv 0 \);
2. \( V(x) \) is a Coulomb potential and
3. \( V(x) \in L^\infty(\mathbb{R}^N) \) with \( V(x + k) \equiv V(x) \) for all \( k \in \mathbb{Z}^N \).

The eigenvalue \( \lambda \) thereby may or may not lie inside a spectral gap.

Our variational characterization is “simple” and well suited for discussing multiple bifurcation of solutions.

References


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