Constrained Hamiltonian Systems

Dario Francia

Scuola Normale Superiore - A.Y. 2013-2014
dario.francia@sns.it

The aim of the course is to provide the essential elements for the analysis of constrained Lagrangian and Hamiltonian systems with finite and infinite number of degrees of freedom, and to introduce the basic tools for the quantization of theories with constraints.

Number of hours: 25
Exam: problems and discussion.

References:

- Dirac P A M, *Lectures on Quantum Mechanics* (Yeshiva U., New York, 1964).
- Sudarshan E C G and Mukunda M, *Classical Dynamics: a Modern Perspective* (John Wiley and Sons, New York, 1974).
- Gitman D M and Tyutin I V, *Quantization of Fields with Constraints* (Springer Verlag, Berlin Heidelberg, 1990).
- Henneaux M and Teitelboim C, *Quantization of Gauge Systems* (Princeton U. Press, New Jersey, 1992).
- Gomis J, Paris J and Samuel S, *Antibracket, antifields and gauge theory quantization,* Phys. Rept. **259** (1995) [hep-th/9412228].
- Weinberg S, *The Quantum Theory of Fields*, voll. I-II (Cambridge U. Press, UK, 1995, 1996).
- Rothe H J and Rothe K D, *Classical and Quantum Dynamics of Constrained Hamiltonian Systems* (World Scientific, Singapore, 2010).

Covered topics:

□ Classical Lagrangian analysis

General introduction and motivations. Lagrangian formalism. Degenerate Lagrangians. Gauge symmetries and their meaning. The Sudarshan-Mukunda algorithm. Trivial gauge transformations. The gauge structure. Reducible gauge systems. Noether's theorem and local symmetries.

□ Classical Hamiltonian analysis

Legendre transform. Hamiltonian formalism. Primary and secondary constraints. Hamilton equations for constrained systems. Conditions on the constraint functions. Action principle. Hamiltonian action and Hamilton-Jacobi equations. Poisson brackets. Total Hamiltonian. First-class and second-class functions. First-class constraints and gauge invariance. Dirac's conjecture. Extended Hamiltonian.

Second-class constraints and Dirac brackets. Canonical gauges. Degrees of freedom: Hamiltonian and Lagrangian countings. Large gauge transformations.

□ **Fermions**

Why anticommutation rules? Grassmann algebra. Dynamical variables and superfunctions. Lagrangian and Hamiltonian formalism for non-degenerate Fermionic systems. Poisson brackets. Hamiltonian analysis for degenerate Fermionic systems.

□ **Quantization of constrained systems**

Dirac quantization: Prototype second-class and first-class systems. Reduced phase-space quantization. The Maskawa-Nakajima theorem. Operator quantization of systems with second-class constraints. Operator quantization of systems with first-class constraints. Non-degenerate Fermionic systems and negative-norm states.

Path-integral quantization: Yang-Mills theory. Faddeev-Popov procedure. BRST invariance. Observables and BRST cohomology. General procedure for BRST quantization. Batalin-Vilkovisky approach. Antifields and antibracket. Action in extended space and classical master equation. Proper solutions. Canonical and BRST transformations in extended space. The gauge-fixing Fermion. The quantum master equation.