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Spatial coherence and index-profiling in optical fibres†

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Abstract. A simple relation is proved to exist between the coherence properties of the light emerging from the end cross-section of multi-mode optical fibres, excited by means of spatially incoherent sources, and their refractive index distributions. Experimental results showing the possibility of visualizing the index profile are reported.

1. Introduction

The coherence properties of the light propagating along multi-mode optical fibres have been the subject of some recent studies [1, 2]. In this paper we present the experimental results and a theoretical explanation of our recent investigations of some of the spatial coherence properties of light emerging from the end cross-section of multi-mode fibres illuminated by means of spatially incoherent sources.

We show that the linear dimensions of the coherence area are directly related to the local numerical aperture, and hence [3, 4] to the local refractive index. These results suggest a novel technique for measuring the index profile of optical fibres.

2. Relation between coherence and index profile

If the illumination of a multi-mode optical fibre is obtained by means of a spatially incoherent source, all the propagating modes are equally excited, and the light rays emerging from the point **r** of the fibre end cross-section uniformly fill a cone of vertex angle $A(\mathbf{r})$, i.e. a solid angle $\Omega(\mathbf{r})=\pi A^2(\mathbf{r})$, so that

$$B(\mathbf{r},\mathbf{s})=\begin{cases}\left(\dfrac{I(\mathbf{r})}{\pi A^2(\mathbf{r})}\right), & \mathbf{s}\subset\Omega(\mathbf{r})\\[2ex] 0, & \mathbf{s}\not\subset\Omega(\mathbf{r}),\end{cases}\qquad(1)$$

where $B(\mathbf{r},\mathbf{s})$ is the generalized radiance at the point **r** in the direction **s**, $I(\mathbf{r})$ being the field intensity at **r** [5] (see figure 1).

A simple relation has been proved to exist between the generalized radiance from a quasi-homogeneous planar source and the Fourier transform of the complex degree of spectral coherence $\tilde{\gamma}$ relative to two points of the same source [6, 7], namely

$$B(\mathbf{r},\mathbf{s})=k^2 I(\mathbf{r})\tilde{\gamma}(k\mathbf{s}_\perp)\cos\theta,\qquad(2)$$

where $\mathbf{s}_\perp=(s_x,s_y)$ is the two-dimensional vector obtained by projecting the unit vector $\mathbf{s}=(s_x,s_y,s_z)$ onto any plane perpendicular to the z axis, $\cos\theta=s_z=\sqrt{(1-s_\perp^2)}$, θ is the angle that the **s** vector makes with the positive z direction, $k=2\pi/\lambda$, and λ is the mean vacuum wavelength of the light propagating in the fibre.

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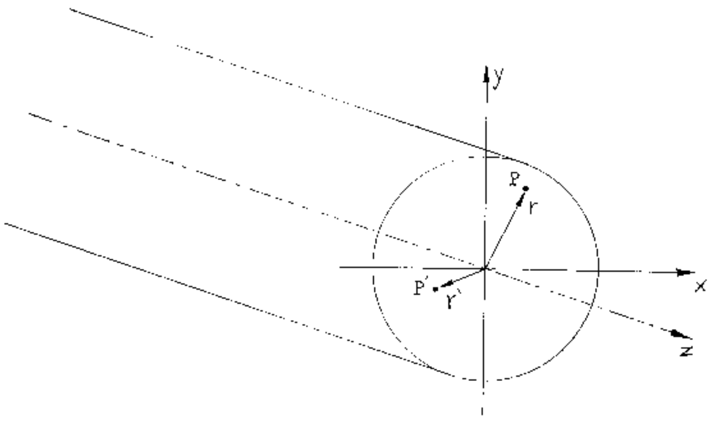


Figure 1. The geometry of the problem.

In practice, the source bandwidth is small enough for us to regard the light as monochromatic. This was the case in the experiment described here, in which an He–Ne laser was used.

Equating the expressions for $B(\mathbf{r}, \mathbf{s})$ given by equations (2) and (3) gives

$$\tilde{\gamma}(k\mathbf{s}_{\perp}) = \begin{cases} 1/(k^2 A^2(\mathbf{r}) \cos\theta), & \mathbf{s} \in \Omega(\mathbf{r}) \\ 0, & \mathbf{s} \notin \Omega(\mathbf{r}), \end{cases} \tag{3}$$

which is plausible if $A(\mathbf{r})$ is assumed to be a slowly varying function of $\mathbf{r}$ over the characteristic linear dimensions of the coherence area.

The complex degree of spectral coherence on the end cross-section of the fibre relative to two points symmetrically located with respect to a core diameter, written as $\gamma(x, y, -x, y)$ if (x, y) are rectangular coordinates in a transverse reference system centred on the fibre axis, can be simply derived from equation (4) by Fourier inversion.

Keeping in mind that, according to the derivation of (2) given in [6], r must be replaced in equation (3) by y, and in the limit of small values of $A(r)$, we can approximate $\cos\theta$ in equation (3) by unity, obtaining

$$\gamma(x, y, -x, y) = \frac{1}{\pi A^2(y)} \int_0^{\sin^{-1}(A(|y|))} d\theta \cos\theta \sin\theta \int_0^{2\pi} \exp(i2kx \sin\theta \cos\phi)\, d\phi \tag{4}$$

and thus

$$\gamma(x, y, -x, y) = 2\frac{J_1(2kxA(|y|))}{(2kxA(|y|))}. \tag{5}$$

Under different conditions, a similar result has been obtained by Pask and Snyder [2] for a step-index fibre.

We can immediately see that the points where $|\gamma| = \text{const.}$ obey the relation $2kxA(|y|) = \text{const.}$ This means that the locus of the points of constant visibility is made up by two families of curves whose relative distance is simply related to the local numerical aperture along the y diameter by the expression

$$d_c = 2x = d_{c0}\frac{A(0)}{A(|y|)}, \tag{6}$$

where d_{c0} is the distance between the points of intersections of the two curves with the x axis.

If we now consider a multi-mode optical fibre having a circularly symmetric index profile, the local numerical aperture is given [3] by

$$A(r) = [n^2(r) - n^2(a)]^{1/2}. \tag{7}$$

Inserting equation (7) in equation (6) gives

$$d_c = d_{c0}\frac{[n^2(0) - n^2(a)]^{1/2}}{[n^2(r) - n^2(a)]^{1/2}}. \tag{8}$$

The usual precautions for avoiding the presence of leaky modes should be taken if equation (8) is to be used to measure $n(r)$, or suitable corrections to equation (8) should be applied [8].

3. Experimental set-up and results

The experimental set-up is shown in figure 2. The light beam of an He–Ne laser is incident on a rotating ground-glass disc, which acts as a diffuser, and then falls on the input face of a short piece (4 m long) of optical fibre. By varying the size of the laser spot on the rotating disc, we can excite the fibre with light having an arbitrarily chosen degree of spatial coherence. In the case of completely incoherent excitation, the numerical aperture of the light emerging from the disc exceeds that of the fibre, so that all the modes are excited. The light emerging from the end cross-section of the fibre is collected by means of a microscope objective and sent on a reversing front interferometer [9], as shown in figure 2. This type of interferometer, as can be seen from the figure, superimposes two images of the exit face of the fibre, one of which is reversed around a line parallel to the prism corner. The two superimposed images give rise to a system of interference fringes whose visibility represents the absolute value of the complex degree of spectral coherence between points symmetrically located with respect to the inversion diameter.

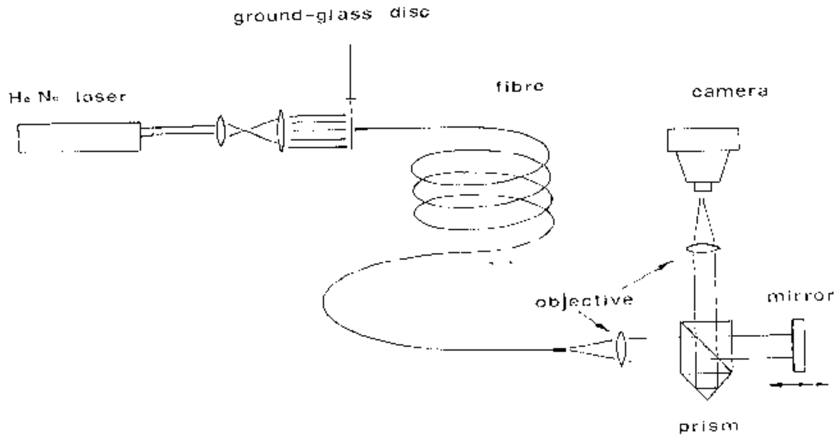
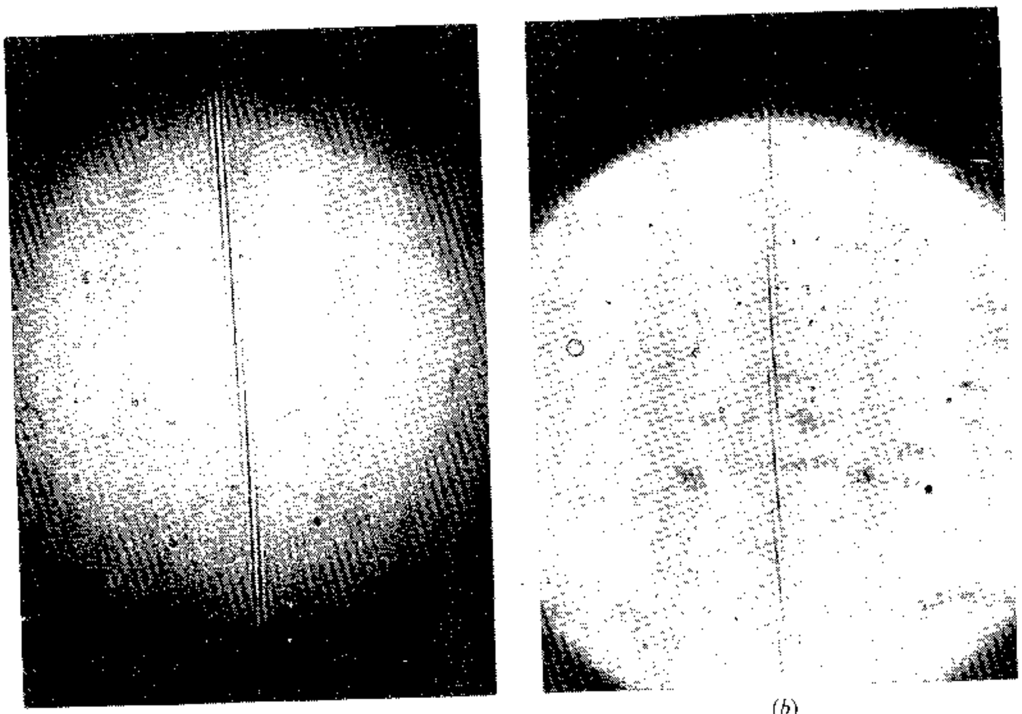


Figure 2. The experimental set-up.



(a) (b)

Figure 3. Superimposed images after the reversing front interferometer for (a) a graded-core fibre and (b) a step-index fibre.

Photographic records of the resulting image are given in figures 3 (a) and 3 (b) for two different types of fibre. The first picture was obtained by using a nearly parabolic, graded-core fibre with a core diameter of 50 μm, $n(0)=1{\cdot}425$ and $\Delta=(n^2(0)-n^2(a))/2n^2(0)=1{\cdot}3$ per cent, while the second was obtained by using a step-index fibre with a core diameter of 75 μm, $n_c=1{\cdot}50$ and $\Delta=1{\cdot}5$ per cent.

Both pictures show a clear, but still qualitative, agreement with the expected behaviour. In fact, as one can immediately see, d_c varies as a function of y for the graded-core fibre, while it is constant for the step-index one.

A quantitative analysis have been performed on the fibre used to obtain figure 3 (a), substituting a T.V. camera for the photographic one shown in figure 2.

By measuring the visibility of the fringes as a function of x for several values of y and plotting the locus of the points where $|\gamma|=0{\cdot}375$, one obtains the graph shown in figure 4, where the plotted points represent the experimental values and the continuous line is computed from equation (8), neglecting the presence of leaky modes and assuming that the refractive index varies as

$$n(r)=\begin{cases} n(0)\left(1-2\Delta\left(\dfrac{r}{a}\right)^{\alpha}\right)^{1/2}, & r<a \\ n(0)(1-2\Delta)^{1/2}, & r>a, \end{cases} \tag{8}$$

with $\alpha=2{\cdot}2$. Figure 5 shows a comparison between the measured values and the theoretical profile computed by means of equation (9).

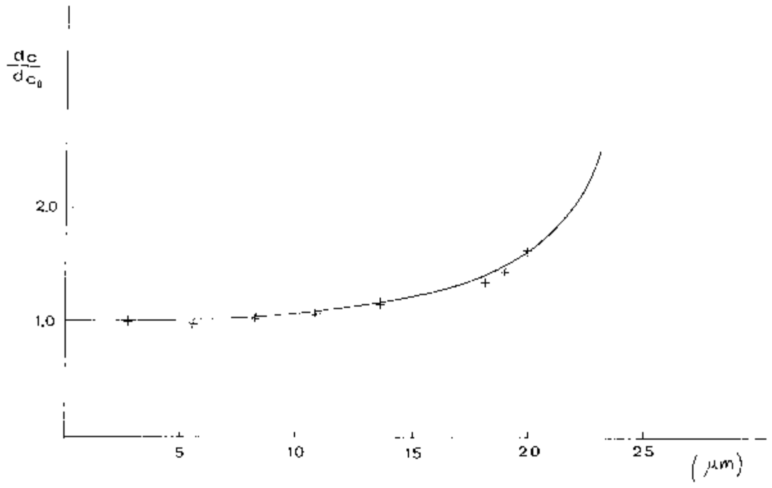


Figure 4. Plot of d_c/d_{c0} against y along a diameter for the graded-index fibre. The plotted points represent the measured values, while the continuous line is calculated from equation (8).

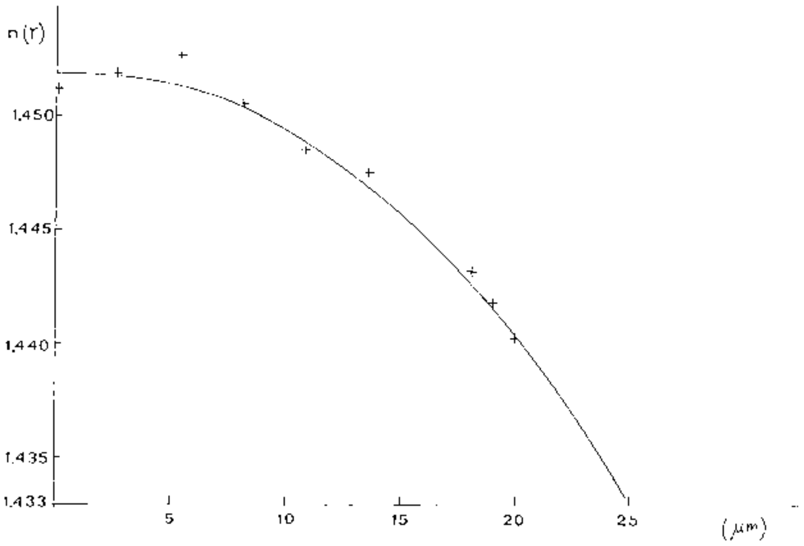


Figure 5. The calculated index profile of $n(r)$ against r; the plotted points represent experimental values.

4. Conclusions

By means of simple arguments we have shown that a direct relation exists between the complex degree of coherence of the light and the refractive index at every point of the end cross-section of a multi-mode optical fibre excited by a spatially incoherent source.

As a consequence, we have introduced an experimental technique which allows us to observe the behaviour of the refractive index in terms of fringe visibility on the exit face of the fibre.

Es wird gezeigt, daß zwischen den Kohärenzeigenschaften des Lichts, welches aus dem Ende einer mit räumlich inkohärenten Lichtquellen angeregten optischen Multimode-Faser austritt und der Brechungsindexverteilung in der Faser ein einfacher Zusammenhang existiert. Ferner werden experimentelle Ergebnisse mitgeteilt, welche die Möglichkeit der Sichtbarmachung der Brechungsindexprofile aufzeigen.

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