Topics in General Relativity

QFT and Gravity I - part II

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The aim of the course is to introduce the basic structures of the theory of General Relativity and to illustrate a few selected topics.

Number of hours: 20
Exam: problems and discussion.

References:

- Weinberg S, *Gravitation and Cosmology - principles and applications of the general theory of relativity* , (John Wiley & Sons, 1972).
- Hawking S W and Ellis G F R, *The Large Scale Structure of Space-Time* (Cambridge University Press, 1973).
- Wald R, *General Relativity* (The University of Chicago Press, 1984).
- Freedman D Z and Van Proyen A, *Supergravity* (Cambridge University Press, 2012).
- Dirac P A M, *Lectures on Quantum Mechanics* (Yeshiva U., New York, 1964).

Part I

□ Introduction

Forms of the Equivalence Principle (WEP, EEP, SEP). Motivations for general covariance. Space-Time in Galilean Mechanics and in Special Relativity. Lorentz transformations. Standard boost. Time dilations and Doppler effect. Vectors and tensors in SR.

□ Basics of GR

Curvilinear coordinates. Vectors and tensors under GCT. Parallel transport. Covariant derivatives. Relation between the metric and the affine connection. Geodesics and their Newtonian limit. Time-like, null and space-like curves. Riemann curvature tensor. Bianchi identities. Local inertial frames. Characterization of flat spaces. Geodesic deviation. Torsion. Weyl tensor. Conformal transformations of the metric tensor. Conformally flat spaces. Lagrangian for conformal gravity. Einstein-Hilbert action. Derivation of the equations of motion. Gauss-Bonnet term and Lovelock gravities. Minimal coupling to scalar fields and to Maxwell fields. Conformally coupled scalar fields [*problem*].

Part II

□ Fluctuations over Minkowski space-time: the non-geometrical perspective

Motivations. Weak fluctuations over flat space-time: linearisation of the Riemann tensor and its abelian gauge invariance. Equations of motion: de Donder gauge and gravitational waves. Particles and fields in SR. Irreps of the Poincaré group: Wigner's induced representation method. Massless particles: ISO(D-2) little group and gauge invariance [*problem*]. From relativistic massless spin-2 particles to full GR. Uniqueness of the quadratic Lagrangian. Noether method and non-linear completions. Belinfante's argument and symmetric energy-momentum tensor. Weinberg's Equivalence Principle from relativistic invariance of the S matrix. Spin and the sign of force [*problem*]. Massive gravity in the linear limit: Fierz-Pauli Lagrangian and vDVZ discontinuity.

□ Isometries and maximally symmetric spaces

Isometries: Invariance in form of the metric tensor. Killing equations. Maximal number of isometries. The S^2-sphere [*problem*]. The Lie derivative. Killing vectors and conservation laws. Conformal Killing vectors. Killing tensors.

Maximally symmetric spaces: Homogeneous and isotropic spaces. The example of Minkowski space. Characterisation of maximally symmetric spaces: curvature constant and signature. MSS as vacuum solutions to the EH equations with cosmological constant. Explicit construction from embedding in (D+1)-pseudo Lorentzian spaces: metric and Christoffel coefficients. FLRW metric.

□ The frame formulation: Fermionic couplings and Cartan-Weyl approach

Local inertial frames. The frame field and its relation to the metric field. Local Lorentz transformations. The spin connection. The vielbein postulate. Torsion constraint and second-order formulation. Coupling to Fermionic matter. Differential forms. Yang-Mills theory in terms of differential forms. Gravity as a Poincaré gauge theory: connection and curvature. First and second Cartan structure equations. Differences with YM. Cartan-Weyl first order Lagrangian.

□ Energy and Hamiltonian

Energy: Conserved quantities in gauge theories: the example of YM. Covariant conservation and ordinary conservation. Einstein-Hilbert equations for asymptotically flat metrics. Candidate for gravitational energy-momentum tensor. The superpotential. ADM energy. Generalization to arbitrary backgrounds. Ordinary conservation of densities for covariantly constant vectors.

Hamiltonian: Constrained Hamiltonian systems. Singular Lagrangians and primary constraints. Total Hamiltonian. Secondary constraints. First- and second-class constraints. Hamiltonian gauge invariance. Degrees of freedom count. Extended Hamiltonian. Reparametrization-invariant systems and Hamiltonian constraint. The relativistic point particle [*problem*]. Einstein-Hilbert Lagrangian in ADM variables.